Name: _____

Student ID: _____

Physics 1C - Winter 2022: Final Exam

March 18, 2022

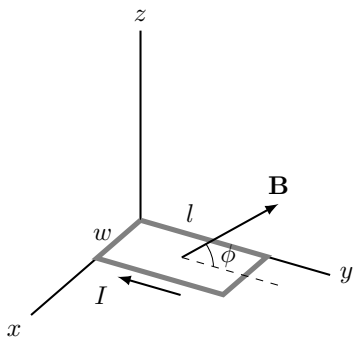
- Write your name and student ID at the top.
- Answer ALL 8 questions.
- Write your answers inside the borders on this handout. **Show all your work.** PLEASE write clearly so the graders can give you all the points you deserve.
- You are allowed to use the textbook and lecture notes, but you are not allowed to communicate with your classmates.
- You have 180 minutes. Upload your exam to Gradescope as soon as you are done. You will have 15 minutes after the end of the exam to upload your submission.

(extra space)

Problem 1

(28/200)

A rectangular loop of wire is $l = 0.500$ m long by $w = 0.300$ m wide and lies in the xy-plane, as shown in the figure below. A uniform magnetic field $\mathbf{B}$ with magnitude 1.50 T is directed into the loop at an angle of $\phi = 40.0°$ with respect to the plane of the loop, with the magnetic field lines parallel to the yz-plane. The loop carries a $I = 0.900$ A current in the direction shown.



(a) (8 points) What are the magnitudes of the individual forces on the four wire segments?

(b) (4 points) What is the magnitude and direction of magnetic moment $\boldsymbol{\mu}$ of the wire loop?

(c) (6 points) Find the magnitude of the torque τ on the wire loop due to the magnetic field.

(d) (4 points) About which axis will the wire loop rotate due to the torque?

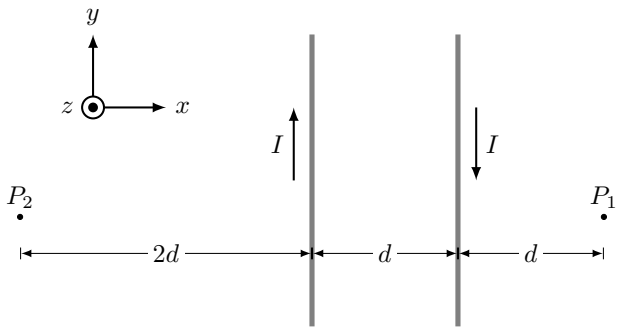
(e) (6 points) Now suppose the magnetic field begins increasing in strength at a rate of 0.5 T/s. Find the magnitude and direction (as viewed from above along the z-axis) of the emf $\mathcal{E}$ induced in the wire loop.

(problem 1 extra space)

Problem 2

(23/200)

Two wires are separated by a distance $d = 10.0$ cm, with each carrying a current of $I = 5.00$ A in opposite directions. (Note: $\mu_0 = 4\pi \times 10^{-7}$ T $\cdot$ m/A)



Find the magnitude and direction of the magnetic field at the following points:

(a) (6 points) At a point midway between the wires.

(b) (6 points) At point P_1 a distance 10.0 cm to the right of the right wire.

(c) (6 points) At point P_2 a distance 20.0 cm to the left of the left wire.

Now suppose a proton ($q = 1.6 \times 10^{-19}$ C) is moving upwards parallel to the wires and starts at point P_2 with velocity $\mathbf{v} = (5.0 \times 10^4 \text{ m/s})\hat{\mathbf{j}}$. At the same instant, a uniform electric field is activated that exactly cancels out the magnetic force on the proton due to the two wires.

(d) (5 points) Find the direction and magnitude of the electric field $\mathbf{E}$.

(problem 2 extra space)

Problem 3

(28/200)

A space probe is launched from Earth at a speed of $v = 0.700c$. The batteries that power the probe's data transmitter supply enough power to ensure that the probe continuously transmits data back to Earth. However, the batteries only have a lifetime of 15.0 years as measured in their rest frame. (Note: 1 ly = 9.46×10^{15} m, $c = 3.00 \times 10^8$ m/s)

(a) (6 points) How long do the batteries on the probe last as measured on Earth?

(b) (6 points) By the time the batteries fail, how far is the probe from Earth as measured by mission control?

(c) (6 points) How far is the probe from Earth as measured by its built-in odometer when its batteries fail?

(d) (10 points) What is the total time after launch for which data is received from the probe by mission control on Earth? (Hint: The signals sent by the probe travel at the speed of light and have to reach Earth from the probe's current position.)

(problem 3 extra space)

Problem 4 (24/200)

Monochromatic light of wavelength $\lambda = 700$ nm is passed through two slits that are separated by a distance $d = 1.0$ cm. An interference pattern is formed on a screen that is 1.0 m away from the slits.

(a) (5 points) What is the distance from the $m = 2$ bright fringe to the center of the screen?

(b) (5 points) What is the phase angle ϕ for waves that fall on the screen at a distance of 10 cm away from the center of the screen?

(c) (6 points) In terms of the maximum intensity I_0 for the interference pattern, what is the intensity of the light a distance of 10 cm away from the center of the screen?

(d) (8 points) Suppose we now send monochromatic light of wavelength λ' through the slits such that the $m = 2$ dark fringe of the light with wavelength λ' coincides with the $m = 2$ bright fringe of the light with wavelength λ. What must the value of λ' be?

(problem 4 extra space)

Problem 5 (22/200)

An earth imaging satellite carries a camera with lenses 30 cm in diameter and with a focal length of 2.4 m.

(a) (6 points) According to Rayleigh's criterion, what is the angular resolution of the camera assuming that the wavelength of light passing through the aperture is 550 nm?

(b) (8 points) If the satellite looks down at Earth from a height of 650 km, what must be the minimum distance between two points on the ground that the camera on the satellite can barely resolve?

(c) (8 points) The lens in the camera projects images of the two points onto a light sensitive sensor at the focal plane of the lens. What is the distance between the two images projected onto the sensor?

(problem 5 extra space)

Problem 6

(29/200)

A linearly polarized microwave with wavelength $\lambda = 1.50$ cm propagates along the positive z axis. The electric field vector for the wave has a maximum value of $E_{\max} = 175$ V/m, and it points in the xy-plane. The magnetic field component of the wave can be written as

$$B = B_{\max} \sin(kz - \omega t).$$

(Note: $c = 3.00 \times 10^8$ m/s, $\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N $\cdot$ m^2, $\mu_0 = 4\pi \times 10^{-7}$ T $\cdot$ m/A)

(a) (4 points) What is the magnetic field amplitude $B_{\max}$?

(b) (5 points) What is the wavenumber k and angular frequency ω?

(c) (8 points) The electric field points in the direction $(\hat{\mathbf{i}} + \hat{\mathbf{j}})$. What is the corresponding direction for the magnetic field?

(d) (6 points) What is the direction and average value of the Poynting vector for this wave?

(e) (6 points) The wave is directed at normal incidence onto a perfectly reflecting sheet. What radiation pressure does it exert?

(problem 6 extra space)

Problem 7

(25/200)

A circuit contains an ac voltage source with a maximum voltage of $V_{\max} = 120$ V that operates at $f = 60.0$ Hz. The voltage source is connected in series to a resistor with $R = 60.0\ \Omega$ and a capacitor with $C = 30.0\ \mu$F.

(a) (4 points) What is the capacitive reactance of the circuit?

(b) (4 points) What is the impedance of the circuit?

(c) (5 points) Find the maximum current in the circuit.

(d) (4 points) Does the voltage of the circuit lag or lead the current?

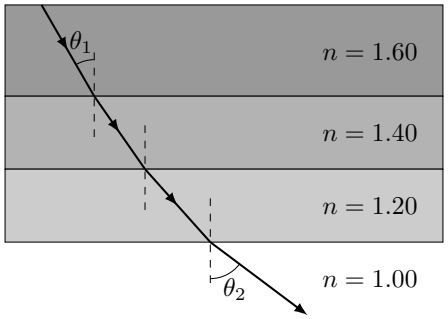
(e) (8 points) An inductor with inductance $L = 50$ mH is added to the circuit in series with the rest of the circuit elements. What effect does this have on the current?

(problem 7 extra space)

Problem 8

(21/200)

The figure below shows the path of a light beam as it passes through several slabs with different indexes of refraction.



(a) (8 points) If $\theta_1 = 30.0°$, what is the angle θ_2 of the beam after it has passed through all three layers?

(b) (6 points) For what incident angle θ_1 will there be total internal reflection of the light at the surface between the medium with $n = 1.20$ and $n = 1.00$?

The bottom layer with index of refraction $n = 1.20$ is removed so that only the top two layers remain. Visible light of wavelength $\lambda_0 = 550$ nm (in air) is incident normally from the top layer ($n = 1.60$) to the bottom layer ($n = 1.40$).

(c) (7 points) For what minimum value of t will the light reflected off of the bottom surface of the bottom layer constructively interfere with the light reflected off of the top surface of the bottom layer?

(problem 8 extra space)