Name: [redacted]

Student ID: 005 513 759

Discussion section you attend ('X' one):

'X' here	Number	Time	TA
	3A	W3	Rathin Singha
	3B	W4	Rathin Singha
	3C	W5	Tushar Gopalka
	3D	F10	Rathin Singha
X	3E	F11	Rathin Singha

Problem	Points	Max
1		33
2		33
3		33
4		33
5		33
6		33
Free Point		2
Total		200

Physics 1C - Fall 2021: Final Exam

Thursday, December 9, 2021

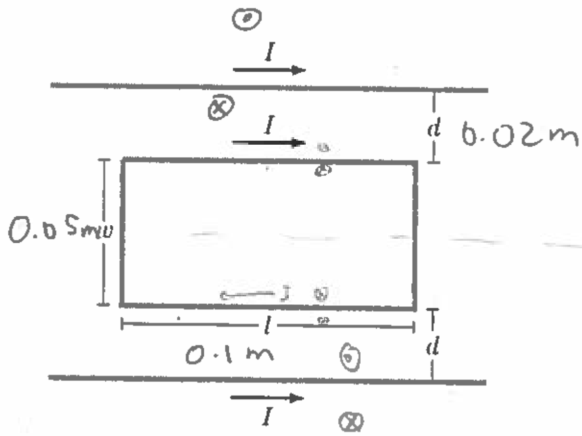
- Write your name and student ID at the top. Put an 'X' in the box to the left of your discussion section. This is worth two "free points."
- Answer ALL 6 questions.
- Write your answers inside the borders on this handout. **Show all your work.** PLEASE write clearly so the graders can give you all the points you deserve.
- You are allowed to use your cheat-sheet and a calculator, but no cell phones.
- You have 180 minutes.

(extra space)

Problem 1

(33/200)

A rectangular loop of wire is placed midway between two straight wires, as shown in the figure. The dimensions of the loop are $l = 10$ cm and $w = 5$ cm. The distance from either end of the loop to the nearest wire is $d = 2$ cm. There is a current of $I = 2.5$ A in both straight wires and in the loop. (Note: $\mu_0 = 4\pi \times 10^{-7}$ T·m/A)



(a) Find the magnitude and direction of the magnetic field at the position of the top section of the wire loop due to:

- (4 points) The top straight wire only.
- (4 points) The bottom straight wire only.
- (4 points) Both straight wires together.

$B = \frac{\mu_0 I}{2\pi r}$ (infinitely long straight wire)

$r_t = d \qquad r_b = w + d$

(b) Find the magnitude and direction of the force on the top section of the wire loop due to:

- (4 points) The top straight wire only.
- (4 points) The bottom straight wire only.
- (4 points) Both straight wires together.

(c) (9 points) Determine the magnitude and direction of the net force on the entire wire loop.

a) top straight wire:

$B_T = \frac{\mu_0 \cdot I}{2\pi \cdot 0.02m} = \frac{\mu_0 \cdot 2.5A}{2\pi \cdot 0.02m} = \frac{2\times10^{-7} \text{ T·m/A} \cdot 2.5A}{0.02m} = \boxed{2.5\times10^{-5}\,T, \text{ into page}}$

top wire

bottom straight wire:

$B_B = \frac{\mu_0 \cdot 2.5A}{2\pi \cdot (0.02+0.05)m} = \boxed{7.14\times10^{-6}\,T, \text{ out of page}}$

Both: $B = |B_T| - |B_B| = \boxed{1.79\times10^{-5}\,T, \text{ into page}}$

b) top only: $F_T = I\,\vec{l}\times\vec{B} = 2.5A \cdot 0.1m \cdot B_T = \boxed{6.25\times10^{-6}\,N, \text{ upwards}}$

Bottom only: $F_B = I\,\vec{l}\times\vec{B} = 2.5A \cdot 0.1m \cdot B_B = \boxed{1.79\times10^{-6}\,N, \text{ down}}$

Total: $F_{Tot} = |F_T| - |F_B| = \boxed{4.46\times10^{-6}\,N, \text{ upwards}}$

c) F_{bottom} is the same, but opposite (since this system is symmetric about the horizontal center line of the loop)

So $F_{bottom} = 4.46\times10^{-6}$ N, down and $F_{top} = 4.46\times10^{-6}$ N, upwards

Again due to the system's symmetry, the net force on the vertical ends of the loop due to the two straight wires is 0 for both.

So the net force $F_{loop} = |F_{bottom}| - |F_{top}| = \boxed{0\ N}$

(problem 1 extra space)

S'  0.8c in ~~aliens~~ frame S S'

S

S'S

inertial frames

Problem 2

S'S 0.6c relative to earth

(33/200)

An alien spaceship traveling at 0.600c towards the Earth launches a landing craft. The landing craft travels in the same direction with a speed of 0.800c relative to the mothership. As measured on the Earth, the spaceship is 0.200 ly from the Earth when the landing craft is launched. (Note: 1 ly = 9.46×10^{15} m, $c = 3.00 \times 10^{8}$ m/s)

$u = 0.6c$

(a) (8 points) What speed do the Earth-based observers measure for the approaching landing craft?

(b) (8 points) What is the distance to the Earth at the moment of the landing craft's launch as measured by the aliens?

(c) (10 points) What travel time is required for the landing craft to reach the Earth as measured by the aliens on the mothership?

(d) (7 points) If the landing craft has a mass of 4.00×10^{5} kg, what is its kinetic energy as measured in the Earth reference frame?

a) $$V_x = \frac{V_x' + u}{1 + uV_x'/c^2} = \frac{0.8c + 0.6c}{1 + 0.6c \cdot 0.8c/c^2} = \frac{1.4c}{1+0.6 \cdot 0.8} = \frac{1.4 \times 3 \cdot 10^8 \text{ m/s}}{1 + 0.6 \cdot 0.8} = \boxed{2.84 \times 10^8 \text{ m/s}}$$

b) $$x' = \gamma(x - ut) = 1.25\left(0.2\,\text{ly} \cdot \frac{9.46\times10^{15}\,\text{m}}{\text{ly}} - 0.6c \cdot 0\right) = \boxed{2.365 \times 10^{15}\,\text{m}}$$

$t=0$, when craft is launched

$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{1}{\sqrt{1-0.6^2 c^2/c^2}} = 1.25$$

c) travel time in frame S' : They measure $V'_x = 0.8c$, and $x' = 2.365 \times 10^{15}$ m

In their frame, it takes $$\Delta t' = \frac{x'}{V'_x} = \frac{2.365 \times 10^{15}\,\text{m}}{0.8 \times 3 \times 10^8\,\text{m/s}} = \boxed{9.85 \times 10^6\,\text{s}}$$

d) $$K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - V_x^2/c^2}} - 1\right) 4 \times 10^5\,\text{kg} \cdot c^2 = 2.083 \times 4 \times 10^5\,\text{kg} \cdot c^2 = \boxed{7.5 \times 10^{22}\,\text{J}}$$

speed of landing craft according to Earth

(problem 2 extra space)

Problem 3

(33/200)

The Very Large Array (VLA) is a set of 27 radio telescope dishes in Catron and Socorro counties, New Mexico. The antennas can be moved apart on railroad tracks, and their combined signals give the resolving power of a synthetic aperture 36.0 km in diameter. (Note: 1 ly = 9.46×10^{15} m) $1\,km = 10^3\,m$

(a) (9 points) If the detectors are tuned to a frequency of 1.40 GHz, what is the angular resolution of the VLA?

(b) (8 points) Clouds of interstellar hydrogen radiate at the frequency used in part (a). What must be the separation distance of two clouds at the center of the galaxy, 2.6×10^4 ly away, if they are to be resolved?

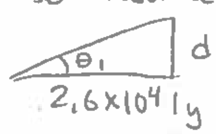
(c) (8 points) As the telescope looks up, a circling hawk looks down. Assume the hawk is most sensitive to green light having a wavelength of 500 nm and has a pupil diameter of 12.0 mm. Find the angular resolution of the Hawk's eye.

(d) (8 points) A mouse is on the ground 30.0 m below. By what distance must the mouse's whiskers be separated if the hawk can resolve them?

a) $\sin\theta_1 = 1.22\,\lambda/D$ $\quad c = \lambda \cdot f \Rightarrow \lambda = 3\times10^8\,m/s \cdot \frac{1}{1.40\times10^9\,Hz} = 0.214\,m$

$\theta_1 = \sin^{-1}(1.22 \cdot 0.214\,m / 36 \cdot 10^3\,m) = \boxed{4.16\times10^{-4\,\circ}}$

b)  to be resolved, need angular res of $4.16\times10^{-4\,\circ}$

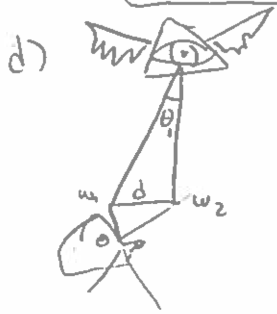


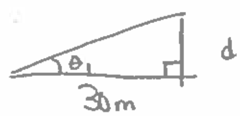
$\tan\theta_1 = \frac{d}{2.6\times10^4\,ly}$

$d = 2.6\times10^4\,ly \cdot 9.46\times10^{15}\frac{m}{ly} \cdot \tan\theta_1 = \boxed{1.79\times10^{15}\,m}$

c) $\sin\theta_1 = 1.22 \cdot 500\times10^{-9}\,m / 12\times10^{-3}\,m$

$\theta_1 = \boxed{2.91\times10^{-3\,\circ}}$

d) 



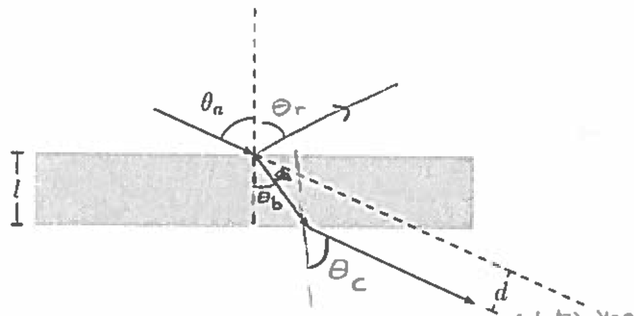
$d = 30m \cdot \tan\theta_1 = \boxed{1.53\times10^{-3}\,m}$

(problem 3 extra space)

Problem 4

(33/200)

Light is incident in air ($n_{air} = 1$) at an angle θ_a on the upper surface of a transparent plate, with the surfaces of the plate being plane and parallel to each other. Assume an incident angle of 66° and a slab of glass ($n_{glass} = 1.52$) with thickness $l = 2.40$ cm. $= 0.024\,m$



w/ respect to the (normal)

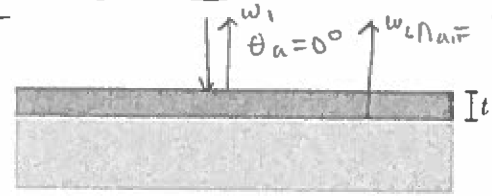
(a) (6 points) What is the angle of the light inside the plate?

(b) (5 points) What is the angle of the outgoing light with respect to the normal?

(c) (12 points) Calculate the lateral displacement d of the emergent beam.

(d) (4 points) For which incident angle will the reflected light be completely polarized? light reflected off surface of glass

Now suppose we place a thin film of MgF_2 ($n_{MgF_2} = 1.38$) of thickness t on top of the glass. Visible light is incident normally on the thin film.

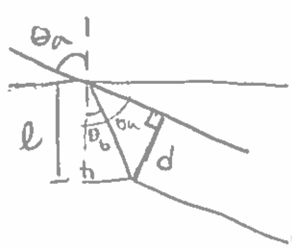


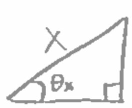
(e) (6 points) For what minimum value of t will the reflected light of wavelength $\lambda = 540$ nm (in air) be destructively reflected?

a) $1 \cdot \sin 66° = 1.52 \cdot \sin\theta_b \Rightarrow \theta_b = \sin^{-1}\left(\frac{\sin 66°}{1.52}\right) = \boxed{36.9°}$ w/ respect to the normal

b) 

$1.52 \cdot \sin\theta_b = 1 \cdot \sin\theta_c = \boxed{66°}$

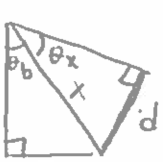
c) 



d: $\theta_x = \theta_a - \theta_b$

$\frac{d}{x} = \sin\theta_x$

$d = x\sin\theta_x = \boxed{1.46 \times 10^{-2}\,m}$



$\frac{l}{x} = \cos\theta_b$

$x = \frac{l}{\cos\theta_b}$

$= 3 \times 10^{-2}\,m$

$\lambda = \frac{\lambda_0}{n}$

d) $\tan\theta_p = \frac{n_b}{n_a} = \frac{n_{glass}}{n_{air}}$

$\theta_p = \tan^{-1}\left(\frac{1.52}{1}\right) = \boxed{56.7°}$

e) $n_{air} < n_{MgF_2}$ = wave reflected at first interface has half-cycle shift

$n_{MgF_2} < n_{glass}$, wave off 2nd interface has half-cycle phase shift

No relative phase shift

$2t = (m + \frac{1}{2})\lambda$

Note: λ here is λ in MgF_2, not air.

$\frac{1}{2} \cdot 2t = (m + \frac{1}{2}) \cdot \frac{\lambda_0}{n} \cdot \frac{1}{2}$

$t = \frac{1}{2} \cdot (0 + \frac{1}{2}) \cdot \frac{540 \times 10^{-9}\,m}{1.38}$

$= \boxed{9.78 \times 10^{-8}\,m}$

(problem 4 extra space)

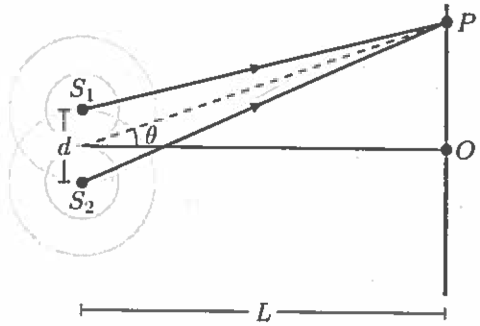
Problem 5

(33/200)

two-source $=100\times10^6$ Hz

Suppose we have two antennas broadcasting a signal at $f = 100.0$ MHz, separated by a distance $d = 120$ m. A highway parallel to the antennas is a perpendicular distance $L = 1.2$ km away. $= 1.2\times10^3$ m

$$\lambda = \frac{c}{f} = \frac{3\times10^8 \text{ m/s}}{100\times10^6 \text{ Hz}} = 3\text{m}$$



(a) (4 points) Calculate the phase difference between the waves broadcasted by the antennas arriving at a point P on the highway when $\theta = 0.500°$.

(b) (5 points) Calculate the phase difference between the waves broadcasted by the antennas arriving at a point P on the highway when $y = 5$ m.

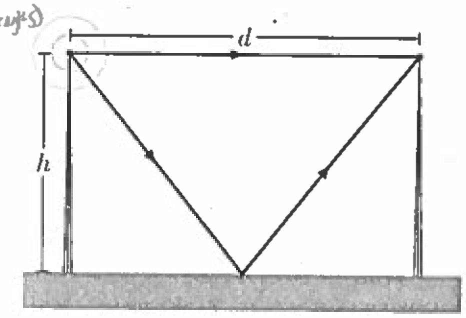
(c) (5 points) What is the value of θ for which the phase difference is 0.333 rad?

(d) (5 points) What is the value of θ for which the path difference is $\lambda/4$?

One of the antennas stops broadcasting its signal and instead is set to receive signals. The other antenna begins transmitting signals of various wavelengths. The receiver antenna can receive signals both directly from the transmitter and indirectly from signals that reflect from the ground. Assume the ground is level between the transmitter and the receiver, the antennas both have height $h = 70.0$ m, and a 180° phase shift occurs upon reflection.

$\frac{d\sin\theta}{\lambda} = m$ | m is # of cycles in path difference (full wavelengths)

$m \text{ cycles} \cdot \frac{2\pi \text{ rad}}{1 \text{ cycle}} = 2\pi m$ rad



Transmitter Receiver

(e) (7 points) Determine the longest wavelengths that interfere constructively.

(f) (7 points) Determine the longest wavelengths that interfere destructively.

a) phase-difference: $\frac{d\sin\theta}{\lambda} = \frac{\text{path-length difference}}{\lambda} = \frac{120\text{m}\cdot\sin 0.5°}{3\text{m}} = 0.35$ cycle

$0.35 \text{ cycle} \cdot \frac{2\pi \text{ rad}}{1 \text{ cycle}} = \boxed{2.20 \text{ rad}}$

b) $y = L\tan\theta \rightarrow \theta = \tan^{-1}\frac{y}{L} = 0.24°$

$\frac{d\sin\theta}{\lambda} = 40\cdot\sin 0.24° = 0.167$ cycle $\cdot$

$0.167 \text{ cycle} \cdot \frac{2\pi \text{ rad}}{1 \text{ cycle}} = \boxed{1.05 \text{ rad}}$

c) $\frac{d\sin\theta}{\lambda} \cdot \frac{2\pi \text{ radians}}{\text{cycle}} = 0.333$ rad

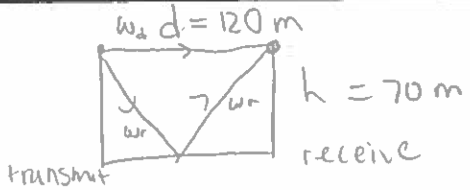
$\sin\theta = \frac{0.333}{2\pi}\cdot\frac{\lambda}{d} = \frac{0.333}{2\pi}\cdot\frac{3\text{m}}{120\text{m}} = 1.32\times10^{-3}$

$\theta = \sin^{-1}(1.32\times10^{-3}) = \boxed{7.59\times10^{-2}\,°}$

d) path-difference $= d\sin\theta = m\lambda = \frac{\lambda}{4}$

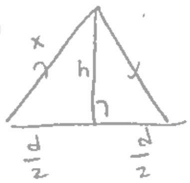
$\sin\theta = \frac{\lambda}{4}\cdot\frac{1}{d} = \frac{3\text{m}}{4}\cdot\frac{1}{120\text{m}}$

$\boxed{\theta = 3.58\times10^{-1}\,°}$



(problem 5 extra space)

e)



$x = \sqrt{h^2 + \frac{1}{4}d^2} = \sqrt{70^2 + (120/2)^2} = 92.2 \text{ m}$

$2x = 184.4 \text{ m}$

w_r has 180° phase-shift, so to be constructive, it must be

$$2x_r - d_a = \left(m + \tfrac{1}{2}\right)\lambda$$

, longest wavelength is when m is minimal (0)

$184.4\text{m} - 120\text{m} = \frac{1}{2} \cdot \lambda$

$\lambda = 2 \cdot (184.4 - 120)\text{ m} = \boxed{129 \text{ m}}$

f) For destructive, we need $2x - d = m\lambda$, which will maximize λ at the minimal $m = 1$ (m=0 is an invalid equation)

$\lambda = (184.4 - 120)\text{ m} = \boxed{64.4 \text{ m}}$

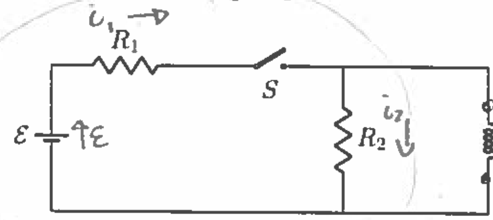
Problem 6

(33/200)

At $t = 0$, the open switch in the figure is thrown closed. Let the current through the inductor be called i and choose it to be downward through the inductor in the figure. Identify i_1 as the current to the right through R_1 and i_2 as the current downward through R_2.

$i_2 = i_1 - i$

$\mathcal{E}_L = -L\frac{di}{dt}$



$V = IR$

(a) (5 points) Use Kirchoff's junction rule to find a relation among the three currents.

(b) (5 points) Use Kirchoff's loop rule around the left loop to find another relationship.

(c) (5 points) Use Kirchoff's loop rule around the outer loop to find a third relationship.

(d) (12 points) Eliminate i_1 and i_2 among the three equations to find a differential equation involving only the current i.

$i_2 = i_1 - i$

(e) (6 points) Using the previous result, what is the current $i(t)$ in the inductor as a function of time? You should get a differential equation for i in part (d) that is similar (but not exactly equal) to the following:

$i_1 - i - i_2 = 0$

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \quad \rightarrow \quad i(t) = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L})$$

a) $\boxed{i_1 = i + i_2}$ b) $\boxed{\mathcal{E} - i_1R_1 - i_2R_2 = 0}$ c) $\boxed{\mathcal{E} - i_1R_1 - L\frac{di}{dt} = 0}$

d) $\begin{cases} \mathcal{E} - i_1R_1 - i_2R_2 = 0 \\ \mathcal{E} - i_1R_1 - L\frac{di}{dt} = 0 \end{cases}$ $i_1 = i + i_2$

$-L\frac{di}{dt} + i_2R_2 = 0$

$-L\frac{di}{dt} + i_1R_2 - iR_2 = 0$

$-L\frac{di}{dt} + \frac{\mathcal{E} + iR_2}{R_1 + R_2}R_2 - iR_2 = 0$

$\mathcal{E} - i_1R_1 - (i_1 - i)R_2 = 0$

$\mathcal{E} - i_1(R_1 + R_2) + iR_2 = 0$

$i_1 = \frac{\mathcal{E} + iR_2}{R_1 + R_2}$

$-iR_2 + L\frac{di}{dt} = -i_1R_2$

$\mathcal{E} - i_1R_1 - (i_1 - i)R_2 = \mathcal{E} - i_1(R_1 + R_2) + iR_2 = 0$

$iR_2 = \mathcal{E} - i_1(R_1 + R_2)$

$i_1 = \frac{\mathcal{E} + iR_2}{R_1 + R_2}$

$-L\frac{di}{dt} + (i_1 - i)R_2 = 0$

$i_1R_2 - iR_2 - L\frac{di}{dt} = 0$

$$\boxed{\frac{\mathcal{E} + iR_2}{R_1 + R_2} \cdot R_2 - iR_2 - L\frac{di}{dt} = 0}$$

Check d: $\begin{cases} \mathcal{E} - i_1 R_1 - i_2 R_2 = 0 \\ \mathcal{E} - i_1 R_1 \; L\dfrac{di}{dt} = 0 \end{cases}$

$-L\frac{di}{dt} + i_2 R_2 = 0 \rightarrow -L\frac{di}{dt} + i_1 R_2 - i R_2 = 0$

$\mathcal{E} - i_1 R_1 - (i_1 - i) R_2 = \mathcal{E} - i_1 (R_1 + R_2) + i R_2 = 0$

$i_1 = \dfrac{\mathcal{E} + i R_2}{R_1 + R_2}$ $\qquad i_2 R_2 = (i_1 - i) R_2$

(problem 6 extra space)

e) $$\frac{\mathcal{E} + i R_2}{R_1 + R_2} R_2 - i R_2 - L\frac{di}{dt} = 0$$

Note: $i(t) = \dfrac{\mathcal{E}}{R}(1 - e^{-Rt/L})$

for

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

Our $R' = R_2$, $\mathcal{E}' = \dfrac{\mathcal{E} + i R_2}{R_1 + R_2} R_2$ $\quad i' = i$

$$i'(t) = \frac{\mathcal{E}'}{R'}(1 - e^{-R't/L})$$

$$= \boxed{\frac{\mathcal{E} + i R_2}{R_1 + R_2}(1 - e^{-R_2 t/L})}$$