Exam Notes:

- This is a closed books, closed notes exam. **No cheat sheets, please!**
- Show all work, clearly and in order. **Circle or otherwise indicate your final answers.**
- Make sure to **include units** in your answers, when numerical values are given.
- Always take a few moments to **double-check that your responses make sense.**
- Good luck!

Grade Table (for grader use only)

Part	Points	Score
A	16	12
B	14	14
C	12	12
D	14	13
E	14	13.5

Potentially useful equations and constants:

Maxwell's equations:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}\ ,\quad \oint \vec{B}\cdot d\vec{A} = 0\ ,\quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}\ ,\quad \oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

Energy in capacitor: $\frac{Q^2}{2C}$, Energy in inductor: $\frac{1}{2}LI^2$, *Inductance:* $L = \frac{\Phi_B}{i}$

L-R and *L-C* circuit in series: $\tau = \frac{L}{R}$, $\omega_0 = \sqrt{\frac{1}{LC}}$

L-R-C circuit in series: $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\tan\phi = \frac{X_L - X_C}{R}$

Speed of light in vacuum: $c = \frac{1}{\sqrt{\epsilon_0\mu_0}} = 3\times 10^8$ m/s

$$\epsilon_0 = 8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2}\ ,\quad \mu_0 = 4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}$$

Poynting vector and Intensity: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$, $I = S_{avg}$

Part A $\mathcal{E} - iR - L\frac{di}{dt} = 0$ $L\frac{di}{dt} = \mathcal{E} - iR$ $\frac{di}{\mathcal{E} - iR} = \frac{1}{L} dt$

1. (2 points) Starting from zero, an electric current is established in a circuit made of a battery of emf $\mathcal{E}$, a resistor of resistance R and an inductor of inductance L. The electric current eventually reaches its steady-state value. What would be the effect of using an inductor with a larger inductance in this circuit?

   A. The steady-state value of the current would be larger, but it would take the same amount of time to reach it.

   B. The steady-state value of the current would be the same, but it would take more time to reach it.

   C. The steady-state value of the current would be the same, but it would take the same amount of time to reach it.

   D. The steady-state value of the current would be larger, but it would take more time to reach it.

   E. The steady-state value of the current would be the same, but it would take less time to reach it.

2. (2 points) A capacitor, initially charged, and an inductor form an $L - C$ circuit. When the electric current in the circuit is equal to zero how much energy is stored on each device?

   A. Inductor: zero energy. Capacitor: zero energy

   B. Inductor: Half of the maximum energy. Capacitor: Half of the maximum energy

   C. Inductor: maximum energy. Capacitor: zero energy

   D. Inductor: zero energy. Capacitor: maximum energy

   E. Inductor: maximum energy. Capacitor: maximum energy

3. (2 points) The average energy dissipated in an inductor, when part of a series $R - L - C$ circuit, is

   A. zero.

   B. proportional to the inductance.

   C. proportional to both the inductance and frequency.

   D. proportional to the frequency.

   E. none of the above.

ac
$V = I\omega L \cos$
$i = I \sin$
$I^2 \omega L \cos \sin$

$\ln(\mathcal{E} - iR) = \frac{t}{L} + c$

$\mathcal{E} - iR = m e^{\frac{t}{L}}$

L bigger = t smaller so takes longer

4. (2 points) A 12V DC battery is connected to the *primary* coil of a *transformer*. If the *primary* coil consists of 5 turns, the voltage across the *secondary* coil with 10 turns is:

   A. greater than 12V
   B. equal to 12V
   C. less than 12V
   D. zero

   $\frac{12}{V} = \frac{5}{10}$ V = 24

5. (2 points) An electric field is created by a(n) ---------- , and an electromagnetic wave is generated by a(n) ---------- .

   ~~A. static charge / constant current~~
   ~~B. static charge / constant magnetic field~~
   ~~C. changing magnetic field / static charge~~
   D. static charge / accelerating charge
   E. constant magnet / accelerating charge

   because current needs to change to get B to change

6. (2 points) Which one of the following lists is a correct representation of electromagnetic waves from longer wavelength to shorter wavelength?

   ~~A. radio waves, infrared, microwaves, UV, visible, X-rays, gamma rays~~
   ~~B. radio waves, UV, X-rays, microwaves, infrared, visible, gamma rays~~
   ~~C. radio waves, microwaves, visible, X-rays, infrared, UV, gamma rays~~
   D. radio waves, microwaves, infrared, visible, UV, X-rays, gamma rays
   E. radio waves, infrared, X-rays, microwaves, UV, visible, gamma rays

7. (2 points) When light goes from one material into another material having a higher index of refraction,

   ~~A. its speed, wavelength, and frequency all decrease.~~
   B. its speed and wavelength decrease, but its frequency stays the same.
   C. its speed decreases but its wavelength and frequency both increase.
   D. its speed decreases but its frequency and wavelength stay the same.
   E. its speed increases, its wavelength decreases, and its frequency stays the same.

8. (2 points) A ray of light traveling in a material of index of refraction n_1 strikes the boundary with another transparent material of index of refraction n_2. If there is no transmitted ray and all of the light intensity is reflected, what can you conclude about the indices of refraction of these two materials?

   A. $n_1 \geq n_2$
   B. $n_1 > n_2$
   C. $n_1 = n_2$
   D. $n_1 \leq n_2$
   E. $n_1 < n_2$

   $n_1 \sin\theta = n_2 \sin 90$

   $\sin\theta = \frac{n_2}{n_1}$ $n_1 > n_2$

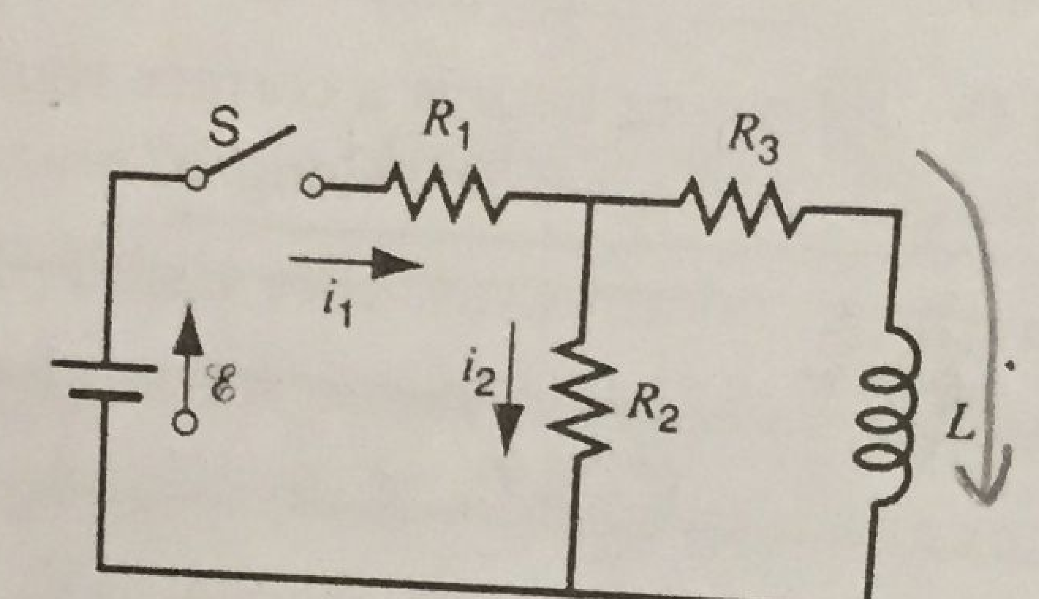
   if $n_1 = n_2$ then with act as if no barrier exists

Part B

1. (6 points) At $t = 0$, let $Q = Q_0$, and $I = 0$ in an LC circuit composed of only an inductor and a capacitor.
   - 3 (a) (3 points) At the first moment when the energy is shared equally by the inductor and the capacitor, what is the charge on the capacitor?
   - 3 (b) (3 points) How much time has elapsed in terms of the period T?

2. (8 points) In the figure below, $\mathcal{E} = 1$ V, $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$, and $L = 2$ H. Find the values of i_1, i_2
   - 2 (a) (2 points) immediately after switch S is closed, $i_1 = i_2 = \frac{\mathcal{E}}{R_1 + R_2} = \boxed{0.033\text{A}}$ ✓
   - 2 (b) (2 points) a long time later,
   - 2 (c) (2 points) immediately after switch S is opened again,
   - 2 (d) (2 points) a long time later.

inductor tries to keep current going



① ⓐ $q = Q_0 \cos\left(\sqrt{\frac{1}{LC}}\, t\right)$ $E = \frac{Q_0^2 \cos^2}{2C}$

$i = -Q_0 \sqrt{\frac{1}{LC}} \sin\left(\sqrt{\frac{1}{LC}}\, t\right)$ $E = \frac{1}{2} Q_0^2 \sin^2 L \cdot \frac{1}{LC}$

$\frac{Q_0^2 \cos^2}{2C} = \frac{1}{2} \sin^2 \cdot \frac{1}{C} Q_0^2$

ⓑ $\cos^2\left(\sqrt{\frac{1}{LC}}\, t\right) = \sin^2\left(\sqrt{\frac{1}{LC}}\, t\right)$ $q = Q_0 \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{Q_0\sqrt{2}}{2}}$ ✓

when $\frac{\pi}{4} = \frac{1}{\sqrt{LC}}\, t$

$\frac{\pi}{4\omega} = t$ $\omega = \frac{2\pi}{T}$

$\frac{\pi}{\frac{8\pi}{T}} = t$ $\boxed{t = \frac{T}{8}}$ ✓

② on next page

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(2) (b) $R_{eff} = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = 22\Omega$ $i_1 = \frac{\varepsilon}{22\Omega} = \boxed{0.045A}$ ✓

After R_1, $V_{left} = 1 - 0.045(10) = .55V$

$i_2 = \frac{.55V}{20\Omega} = \boxed{0.0272A}$ ✓

(c) $\boxed{i_1 = 0A}$ ✓ $i_2 = \frac{.55V}{30\Omega} = \boxed{-0.0182A}$ ✓ negative bc going against the arrow

(d) $\boxed{i_1 = i_2 = 0A}$ ✓

ELI ICE

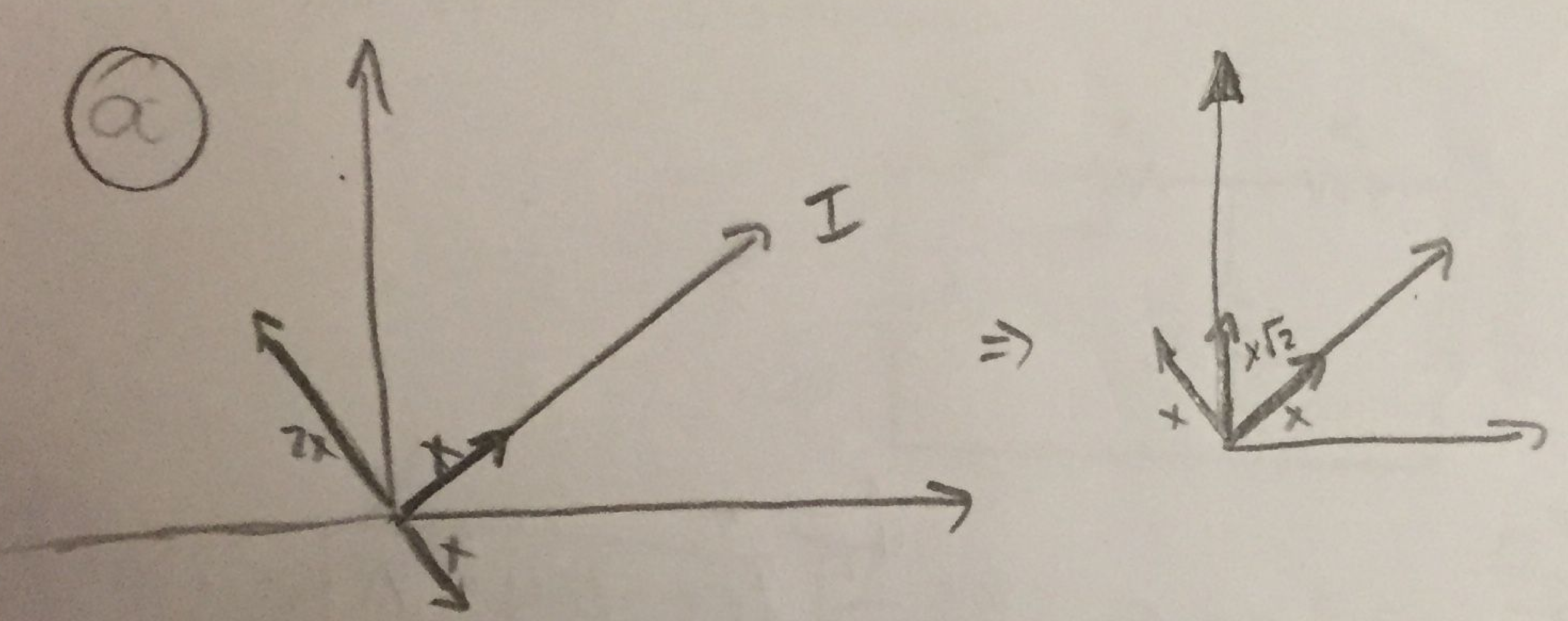
Part C

1. (12 points) In a certain *RLC* circuit, operating at 60 Hz, the maximum voltage across the inductor is twice the maximum voltage across the resistor, while the maximum voltage across the capacitor is the same as the maximum voltage across the resistor.
   (a) (4 points) What is the phase relationship between the voltage across the *RLC* circuit and the current running through it?
   (b) (4 points) If the AC source is actually a US wall power plug ($V_{rms} = 120$ V), what should be the circuit's resistance in order to obtain a maximum current of 300 mA?

2. (4 points) In a certain *RLC* circuit, when the AC voltage source has a particular angular frequency ω, the peak voltage across the inductor is 6 times greater that the peak voltage across the capacitor. Determine ω in terms of the resonant frequency ω_0 of this circuit.

(1)

(a)



off by 45° with voltage leading

+4

(b) $V_{rms} = 120$

$V\frac{\sqrt{2}}{2} = V_{rms}$

$V = 169.7V$

$\frac{V}{I} = Z$

R & X_{cL} are equal because voltages are the same

so

$\frac{V}{I} = \sqrt{2R^2}$

+4

R = 400 Ω

(2) $V_L = i\omega L$ $V_L = 6V_c$

$V_c = \frac{i}{\omega C}$

$i\omega L = \frac{6i}{\omega C}$

$\omega L = \frac{6}{\omega C}$ +4

$\omega^2 = \frac{6}{LC}$

$\omega = \sqrt{\frac{6}{LC}} = \sqrt{6}\,\omega_0$

Part D

1. (14 points) He-Ne lasers are very commonly used in physics demonstrations. They produce light of wavelength ~630 nm with typical electric field amplitude of ~1 kV/m.
   (a) (2 points) What is the frequency of this laser beam?
   (b) (3 points) If $E = E_{max}\sin(kx + \omega t)$ and $\vec{E} \parallel \hat{y}$, what are the values of k and ω?
   (c) (4 points) Write the vector equation for $\vec{B}(\vec{r}, t)$, the magnetic field of the laser beam.
   (d) (2 points) Does the laser beam transport any energy? If yes, in what direction?
   (e) (3 points) What is the intensity of this laser beam?

(a) $c = f\lambda$ $f = \frac{c}{\lambda} = 4.76 \times 10^{14}$ Hz +2

(b) $k = \frac{2\pi}{\lambda} = 9973310\ m^{-1}$ $\omega = 2\pi f = 2.99 \times 10^{15}$ Hz +3

(c) $E = cB$ $B = \frac{E}{c} = 3.33 \times 10^{-6}$ +3

$\vec{B}(\vec{r}, t) = \hat{z}(3.33 \times 10^{-6}\ T)\sin(kx - \omega t)$

y, z aren't inputs in this eq. so left in terms of x

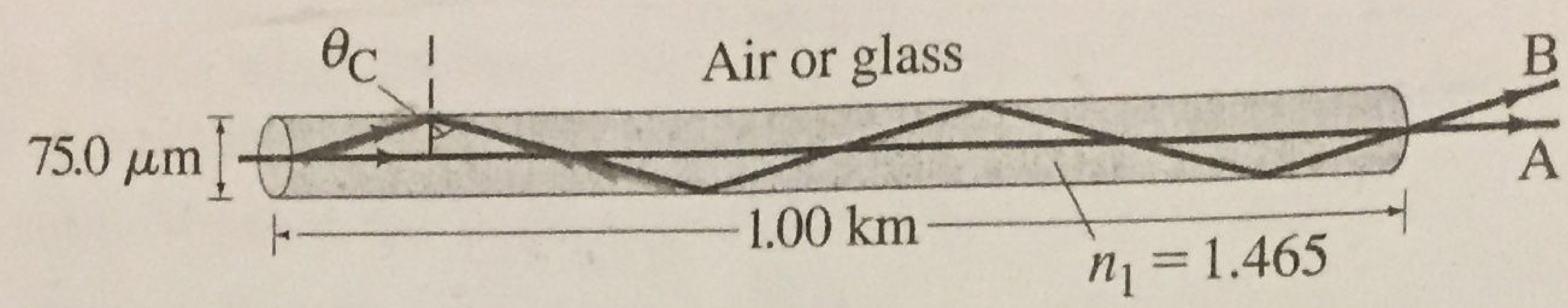
(d) Yes, it transports energy along the x-axis, in direction of wave +2

(e) $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{\hat{x}(0.0033(kV \cdot T)\sin^2(kx - \omega t))}{\mu_0}$

intensity, $I = S_{average} = \frac{(0.0033)}{2\mu_0} = 1313\ W/m^2$ +3

for $\sin(kx - \omega t)$, k, ω values listed in (b)

Part E



1. (14 points) Two light rays A and B of identical lasers travel down a cylindrical optical fiber of diameter $d = 75\mu$m, length $\ell = 1$km, and index of refraction $n_1 = 1.465$. Ray A travels a straight path down the fiber's axis, whereas ray B propagates down the fiber by repeated reflections at the critical angle each time it impinges on the fiber's boundary.

   (a) (2 points) What is the speed of light in the optical fiber?

   (b) (2 points) If the light has a wavelength of 440 nm in the optical fiber, what is the actual wavelength of our laser (measured in air)?

   (c) (6 points) Determine the extra time it takes for ray B to travel down the entire fiber in comparison with ray A, when the fiber is surrounded by air. $n=1$

   (d) (2 points) If, instead, the optical fiber is surrounded by a cylindrical glass "cladding" ($n_2 = 1.460$), how do you expect the time calculated in (c) to change? Explain.

   (e) (2 points) Would you expect your answer in (c) to be different if a green laser was used instead of a red one? Explain.

(a) $c = \lambda f$ $\quad \lambda = \frac{\lambda_0}{n}$

$v = \lambda f$

$c = \lambda_0 f$

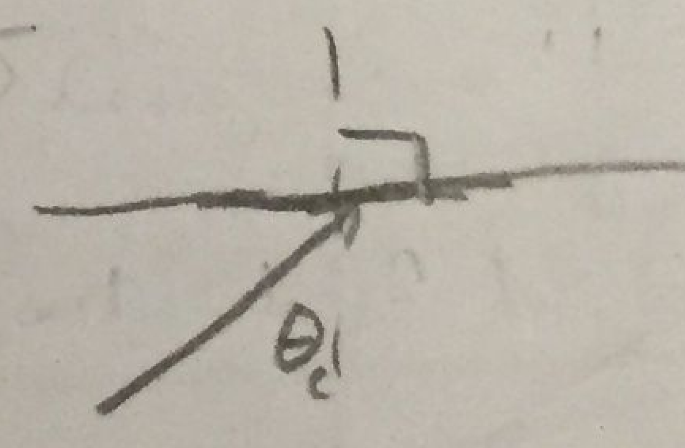
$v = \frac{\lambda_0}{n} f$

$v = \frac{c}{n} = $ **204778157 m/s** ✓

(b) $\lambda = \frac{\lambda_0}{n}$

$\lambda n = \lambda_0 = $ **6.446×10^{-7} m** ✓

(c) ray A $t_1 = \frac{\ell}{v} = 4.88 \times 10^{-6}$ s ✓



$1.465 \sin\theta_c = 1$

$\theta_c = 43.05°$ ✓

How many hypotenoses? g

$\tan(43.05°) = \frac{a}{\frac{1}{2}(75\mu m)}$ $\quad a = 3.50 \times 10^{-5}$ m

$\frac{\ell}{a} = g = 28546514$

hypotenose = h

$\cos\theta_c = \frac{\frac{1}{2}(75\mu m)}{h}$

distance = hg $\quad h = 5.1317 \times 10^{-5}$ m

$t_2 = \frac{hg}{v} = 7.15 \times 10^{-6}$

next page →

(Extra Space)

$\Delta t = t_2 - t_1 = \boxed{2.73 \times 10^{-6}\ s}$

+5.5

(d) -if n_2 gets higher then the critical angle will increase (Snell's law: $n_1 \sin\theta_c = n_2$)
⇒ if θ_c increases then the reflection path models a ~~straight line~~ ~~more~~ which means t_2 is close to t_1, so this time difference will decrease

(e) -1 different, n will change because based on wavelength

- this means the speeds of the waves will change, and so will the critical angles
- if the critical angle changes, the distance B travels will change. This means the ratio of distances will change (along with the velocity)
- because the angle is based on sine/tangent/cosine which are not linear but a change in v will be linear, the time difference will change

Yes