Exam Notes:

- This is a closed books, closed notes exam. **No cheat sheets, please!**
- Show all work, clearly and in order. **Circle or otherwise indicate your final answers.**
- Make sure to **include units** in your answers, when numerical values are given.
- Always take a few moments to **double-check that your responses make sense.**
- Good luck!

Grade Table (for grader use only)

Part	Points	Score
A	16	16
B	12	12
C	13	13
D	16	16
E	13	13

Potentially useful equations and constants:

Biot-Savart law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$, Lorentz force: $d\vec{F} = Id\vec{l}\times\vec{B}$ (or, $\vec{F} = q\vec{v}\times\vec{B}$)

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}$

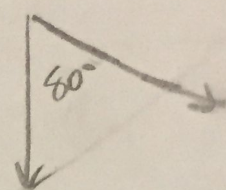
Maxwell's equations:

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$
$$\oint \vec{B}\cdot d\vec{A} = 0$$
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}$$

Motional *emf*: $d\mathcal{E} = \left(\vec{v}\times\vec{B}\right)\cdot d\vec{l}$

Part A

1. (2 points) A proton is moving at an angle of 80° to a uniform magnetic field. What is the relationship between the direction of the force on the proton and the direction of the magnetic field?



   ~~A. parallel to the magnetic field~~

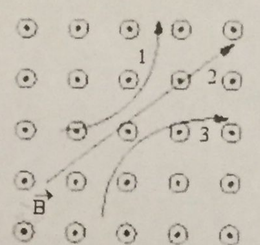
   B. at an angle of 10° to the magnetic field

   C. at an angle of 80° to the magnetic field

   D. at an angle of 180° to the magnetic field

   E. perpendicular to the magnetic field

2. (2 points) Three particles travel through a region of space where a uniform magnetic field is out of the page, as shown below. The electric charge of each of the three particles is, respectively,



   A. 1 is neutral, 2 is negative, and 3 is positive.

   B. 1 is neutral, 2 is positive, and 3 is negative.

   C. 1 is positive, 2 is neutral, and 3 is negative.

   D. 1 is positive, 2 is negative, and 3 is neutral.

   E. 1 is negative, 2 is neutral, and 3 is positive.

3. (2 points) Two long parallel wires placed side-by-side on a horizontal table carry identical size currents in opposite directions. The wire on your right carries current toward you, and the wire on your left carries current away from you. From your point of view, the magnetic field at the point exactly midway between the two wires

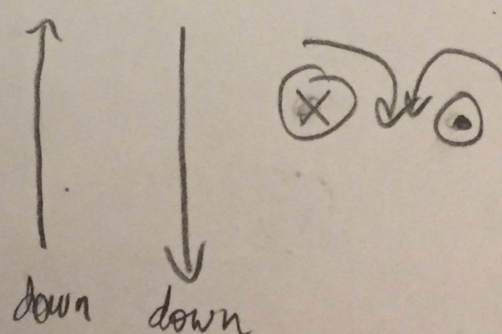
   A. points toward you.

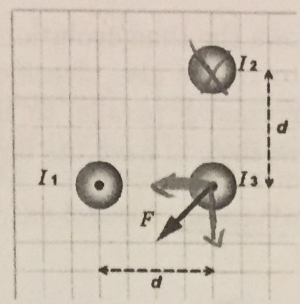
   B. points away from you.

   C. is zero.

   D. points up.

   E. points down.





4. (2 points) The figure shows three long, parallel, current-carrying wires. The current directions are indicated for currents I_1 and I_3. The arrow labeled F represents the net magnetic force acting on current I_3. The three currents have equal magnitudes. What is the direction of the current I_2?

   A. vertically upward

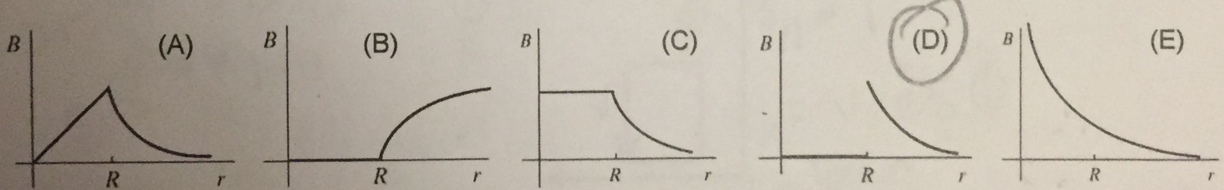
   B. vertically downward

   C. into the picture (in the direction opposite to that of I_1 and I_3)

   D. horizontal to the right

   E. out of the picture (in the same direction as I_1 and I_3)

5. (2 points) A very long, hollow, thin-walled conducting cylindrical shell (like a pipe) of radius R carries a current along its length uniformly distributed throughout the thin shell. Which one of the graphs shown in the figure most accurately describes the magnitude B of the magnetic field produced by this current as a function of the distance r from the central axis?



6. (2 points) Magnetic flux depends upon

   A. the magnetic field.

   B. the orientation of the area with respect to the field.

   C. the area involved.

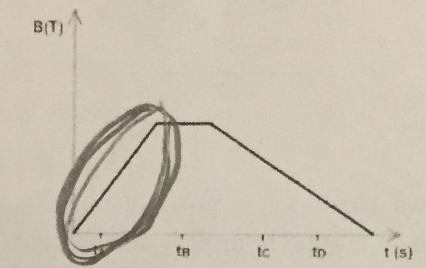
   D. none of the above

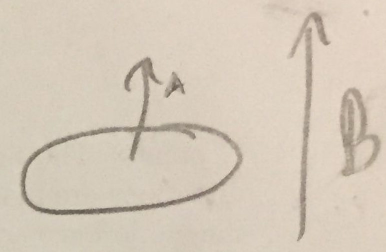
   E. all of the above

R

7. (2 points) The figure below shows the time evolution of a uniform magnetic field. Four particular instants labeled t_A to t_D are also identified on the graph. The field passes through a circular coil whose normal is parallel to the direction of the field. At what time does the current induced in the coil have the largest value?

$$\frac{d\Phi_B}{dt} = \frac{dB}{dt} A$$





A. The current is the same at all these times.

B. t_A

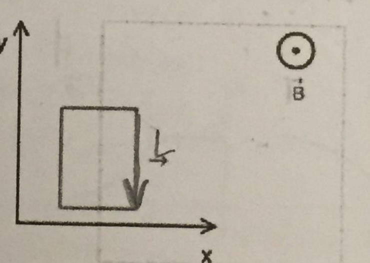
C. t_B

D. t_C

E. t_D

largest magnitude

8. (2 points) A metallic frame moving along the positive direction enters a region of space with a uniform magnetic field pointing in the positive z-direction as shown below. In what direction should a force be applied to the frame to keep it moving at a constant speed while it is entering the field?



enters so opposes

causes force to left, right to oppose

A. positive y

B. negative y

C. positive z

D. negative x

E. positive x

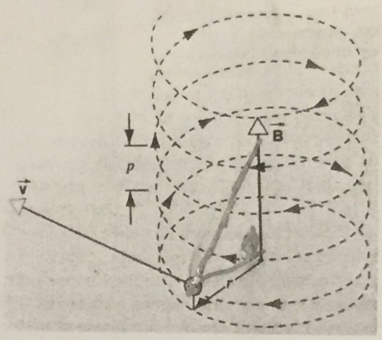
Part B

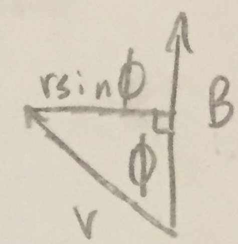
1. (12 points) A positron, i.e., a positively charged electron with charge e and mass m_e, is accelerated by an electric field acquiring final kinetic energy E. It is then projected into a uniform magnetic field $\vec{B}$ with its velocity vector making an angle ϕ with $\vec{B}$ (see figure below). Derive expressions for the following characteristics of the positron's helical path

   (a) (5 points) the period,

   (b) (4 points) the radius r, and

   (c) (3 points) the pitch p (=the "height" of one complete helix turn).





a $\omega = 2\pi f$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$

$\frac{mv_\perp^2}{r} = qv_\perp B$

$\frac{mv_\perp}{r} = qB$

$v_\perp = \omega r$

$\frac{m\omega r}{r} = qB$

$\omega = \frac{qB}{m}$

$T = \frac{2\pi m}{qB}$

$\boxed{T = \frac{2\pi m}{eB}}$

+5

b $\frac{mv_\perp}{r} = qB$

$r = \frac{mv_\perp}{qB}$

$E = \frac{1}{2}mv^2$

$\sqrt{\frac{2E}{m}} = v$

$v_\perp = v\sin\phi$

$v_\perp = \sqrt{\frac{2E}{m}}\sin\phi$

$\boxed{r = \frac{m\sqrt{\frac{2E}{m}}\sin\phi}{eB}}$

+4

c how much vertical distance in one period so

$p = v_\parallel \cdot T$

$v_\parallel = v\cos\phi$

$T = \frac{2\pi m}{eB}$

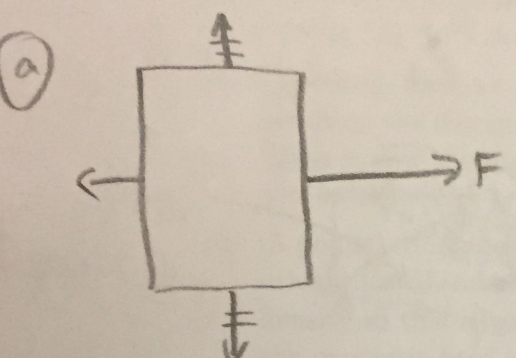
$\boxed{p = \frac{2\pi m}{eB}\sqrt{\frac{2E}{m}}\cos\phi}$

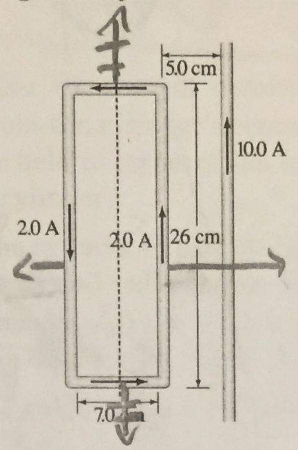
+3

Part C

1. (13 points) A rectangular loop of wire carries a 2-A current and lies in a plane which also contains a very long straight wire carrying a 10-A current as shown below. [*Hint*: The magnetic field strength away from a infinite straight conductor is $B = \frac{\mu_0 I}{2\pi r}$]

(a)





2 (a) (2 points) Draw on the figure the direction of the magnetic force on each side of the loop, if any.

5 (b) (5 points) What is the net force acted on the loop due to the straight wire (*magnitude* and *direction*)?

2 (c) (2 points) What is the loop's magnetic dipole moment (*magnitude* and *direction*)?

2 (d) (2 points) What is the net torque on the loop due to the straight wire?

2 (e) (2 points) By what angle about its long axis (dashed line in figure) would you rotate the loop to maximize the net torque?

(b) $F = I_1 L B$

$F = \frac{I_1 L \mu_0 I_2}{2\pi r}$

only the vertical wires matter so just subtract

$F = \frac{(2A)(10A)\mu_0(.26m)}{2\pi(.05m)} - \frac{(2A)(10A)\mu_0(.26m)}{2\pi(.12m)}$

$= \boxed{1.213 \times 10^{-5}\ N \text{ to the right}}$

toward the reader

(c) $\vec{\mu} = I\vec{A}$

$I = 2A$

$A = (.07m)(.26m) = 0.0182 m^2$

$\boxed{\mu = 0.0364\ Am^2}$ out of the page

(Extra Space)

(d) $\vec{\tau} = \vec{\mu} \times \vec{B}$

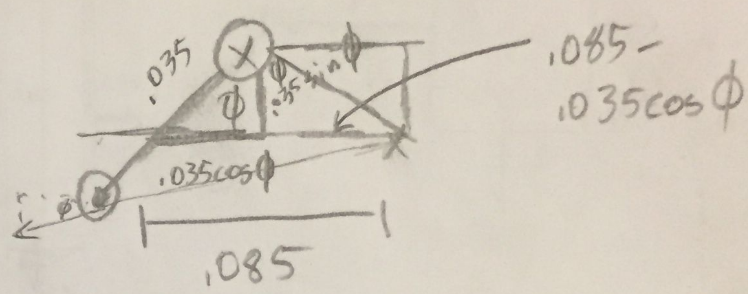
$\vec{\mu} \parallel \vec{B}$ so $\boxed{\vec{\tau} = \vec{0}}$ V

wrong (no points taken)

(e) rotate by 90° because max when $\vec{\mu} \perp \vec{B}$ confirms

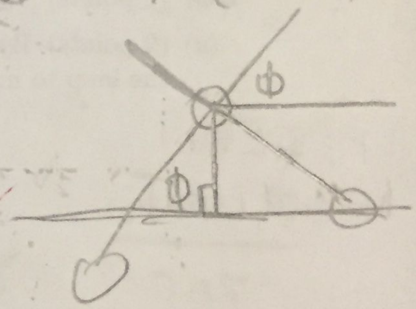
→

↓



$$r_1 = \sqrt{(.085 - .035\cos\phi)^2 + (.035\sin\phi)^2}$$

$$r_2 = \sqrt{(.085 + .035\cos\phi)^2 + (.035\sin\phi)^2}$$



$\tau = F\sin\phi$

$$\tau = \frac{2 \cdot 10 \cdot \mu(.26)\sin\phi}{2\pi r_1} + \frac{2 \cdot 10\mu \cdot .26\sin\phi}{2\pi r_2}$$

when $\frac{\sin\phi}{r_1} + \frac{\sin\phi}{r_2}$ is maximal

$\frac{d}{d\theta} = 0$

Part D

$\hat{\phi} = \odot$

1. (16 points) A long, straight, solid cylindrical conductor of radius R_1 is oriented with its axis in the z-direction, and it carries a uniformly distributed total current I_0.

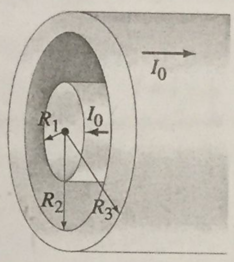
   (a) (4 points) Determine the magnetic field in terms of I_0 inside $(0 \le r \le R_1)$ and outside $(R_1 \le r)$ the cylinder.

   (b) (4 points) Assume now that the current density is not uniform anymore but depends on the distance r from the cylinder's center as $\vec{J}_1(r) = C_1 r\hat{k}$, for $0 \le r \le R_1$. What is now the magnetic field in terms of the total current I_0 inside $(0 \le r \le R_1)$ and outside $(R_1 \le r)$ the cylinder?

   (c) (5 points) Imagine that the cylinder of part (b) is surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3 (see figure below). If the current density in this tube is given by $\vec{J}_2(r) = -C_2 r\hat{k}$ with a total current I_0, determine the magnetic field in terms of I_0 for $R_2 \le r \le R_3$

   (d) (3 points) For the coaxial cable of part (c) (see figure), determine the magnetic field in terms of I_0 for $R_3 \le r$.

Note that the constants C_1 and C_2 are not known.



(a) $\oint \vec{B} \cdot d\vec{l} = I_0 \mu_0$

$R_1 < r$: $2\pi r B = I_0 \mu$

$$\boxed{\vec{B} = \frac{I_0 \mu_0}{2\pi r}\hat{\phi}}$$

B goes in

$0 < r < R_1$: $2\pi r B = \frac{I_0}{\pi R_1^2} \cdot \pi r^2 \mu_0$

$$\boxed{\vec{B} = \frac{I_0 r \mu_0}{R_1^2 2\pi}\hat{\phi}}$$

same direction

(b) $R_1 < r$:

$$\boxed{\vec{B} = \frac{I_0 \mu_0}{2\pi r}\hat{\phi}} \leftarrow \text{same}$$

$0 < r < R_1$:

each ring contains current $= 2\pi r dr J$

$$\Sigma I = \int_0^r 2\pi r C_1 r dr$$

$$= \frac{2}{3} r^3 \pi C_1 = I$$

$$2\pi r B = \frac{2}{3} r^3 \pi C_1 \mu_0$$

$$B = \frac{r^2 C_1 \mu_0}{3}$$

same direction

cont. on back →

(Extra Space)

$$I_0 = \int_0^{R_1} 2\pi r^2 C_1 \, dr$$

$$= \frac{2}{3}\pi R_1^3 C_1 = I_0$$

$$C_1 = \frac{3I_0}{2\pi R_1^3}$$

$0 < r < R_1$:

from last page

$$B = \frac{r^2 \mu_0}{3} \cdot \frac{3I_0}{2\pi R^3}$$

$$\boxed{\vec{B} = \frac{r^2 \mu_0 I_0}{2\pi R_1^3} \hat{\phi}}$$ same direction

(d) $\oint \vec{B} \cdot d\vec{l} = I_{enc} \mu_0$

$I_{enc} = I_0 - I_0 = 0$

$$\boxed{\vec{B} = \vec{0}}$$

(c) $$I_0 = \int_{R_2}^{R_3} 2\pi r^2 C_2 \, dr$$

$$I_0 = \frac{2}{3}\pi C_2 (R_3^3 - R_2^3)$$

$$C_2 = \frac{3I_0}{2\pi (R_3^3 - R_2^3)}$$

$$I_{enc.} = \int_{R_2}^{r} = \frac{2\pi}{3} C_2 (r^3 - R_2^3)$$

total $I_{enc.}$: $I_0 - \frac{2\pi}{3} C_2 (r^3 - R_2^3)$

$$2\pi r B = I_0 - \frac{\cancel{2\pi}}{\cancel{3}} (r^3 - R_2^3) \cdot \frac{\cancel{3} I_0}{\cancel{2\pi}(R_3^3 - R_2^3)}$$

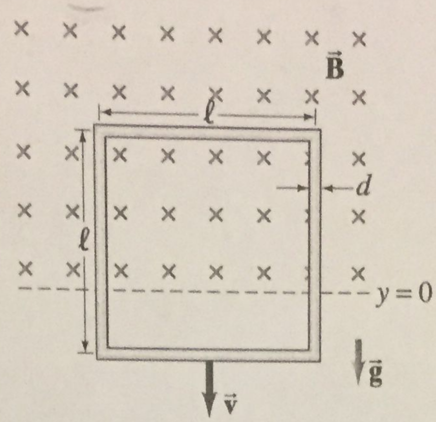
$$2\pi r B = I_0 - \frac{I_0 (r^3 - R_2^3)}{(R_3^3 - R_2^3)}$$

$$\boxed{\vec{B} = \frac{I_0 \left(1 - \frac{(r^3 - R_2^3)}{(R_3^3 - R_2^3)}\right)}{2\pi r} \hat{\phi}}$$

in same direction as before

Very nice!

Part E



1. (13 points) In a certain region of space near Earth's surface, a uniform horizontal magnetic field of magnitude B exists above a level defined to be $y = 0$. Below $y = 0$, the field abruptly becomes zero (see Figure). A vertical square wire loop has resistance R, uniformly distributed mass m, diameter d, and side length ℓ. It is initially at rest with its lower horizontal side above $y = 0$ and is then allowed to fall under gravity, with its plane perpendicular to the direction of the magnetic field.

   (a) (2 points) Find the magnitude of the *emf* induced in the loop, before its lower horizontal side reaches $y = 0$.

   (b) (2 points) Repeating the same experiment but this time releasing the loop from rest with its lower horizontal side at $y = 0$, in what direction does current flow, if any? Briefly explain.

   (c) (6 points) While the loop is still partially immersed in the magnetic field (as it falls into the zero-field region), determine the magnetic "drag" force that acts on it at the moment when its speed is v.

   (d) (3 points) If you know that the loop reaches a terminal velocity v_T before its upper horizontal side exits the field, find an expression for v_T.

(a) $\mathcal{E} = -\frac{d\Phi_B}{dt} = 0$ $\boxed{\mathcal{E} = 0}$ no change in flux bc no angle change, motion, field change

(b) Due to Lenz law! the current will flow in the clockwise direction.
This is because there is now less field passing thru the rect. into the page. The current tries to compensate by pushing $\vec{B}$ into the page so the flux is constant. It creates inward flux by circulating current clockwise.

(Extra Space)

(c) $\frac{d\Phi_B}{dt} = B\underbrace{vL}_{\frac{dA}{dt}} = \mathcal{E}$

$\frac{\mathcal{E}}{R} = \frac{BvL}{R} = I \quad V = IR$

$F = ILB$

$F = \frac{BvLLB}{R}$

$\boxed{\vec{F} = \frac{vB^2\ell^2}{R}\hat{j}}$ ✓

points up

(d) $\sum F = mg - \frac{vB^2\ell^2}{R}$

V_t when $a, F = 0$

$0 = mg - \frac{vB^2\ell^2}{R}$

$\frac{vB^2\ell^2}{R} = mg$

$\boxed{V_t = \frac{mgR}{B^2\ell^2}}$ ✓