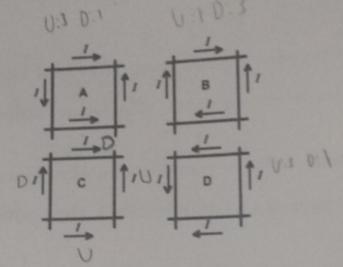
Part A

1. (2 points) The figure shows four different sets of insulated wires that cross each other at right angles without actually making electrical contact. The magnitude of the current is the same in all the wires, and the directions of current flow are as indicated. For which (if any) configuration will the magnetic field at the center of the square formed by the wires be equal to zero?

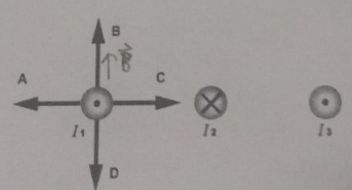


A. A
B. B
C. C
D. D
E. The field is not equal to zero in any of these cases.

2. (2 points) The figure shows three long, parallel current-carrying wires. The magnitudes of the currents are equal and their directions are as shown. Which of the arrows drawn near the wire carrying current 1 correctly indicates the direction of the magnetic force acting on that wire?

$F = I\,d\vec{l} \times \vec{B}$

$d\vec{l} \times \vec{B} = A$ if I_1 is positive

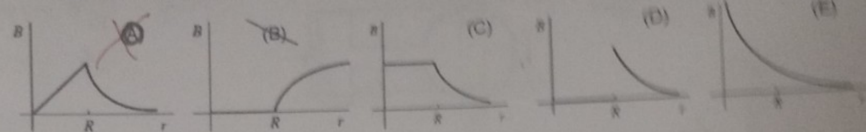


B_{total}

$I\,dl \times B =$

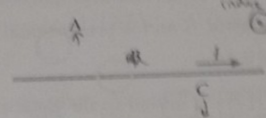
A. A
B. B
C. C
D. D
E. The field is not equal to zero in any of these cases.

3. (2 points) A very long, hollow, thin-walled conducting cylindrical shell (like a pipe) of radius R carries a current along its length uniformly distributed throughout the thin shell. Which one of the graphs shown in the figure most accurately describes the magnitude B of the magnetic field produced by this current as a function of the distance r from the central axis?



4. (2 points) The long straight wire in the figure carries a current I that is decreasing with time at a constant rate. The circular loops A, B, and C all lie in a plane containing the wire. The induced emf in each of the loops A, B, and C is such that

inducing $\vec{B}$ out of page



For B, flux through left = flux through right so no flux

A. no emf is induced in any of the loops.

B. a counterclockwise emf is induced in all the loops.

C. loop A has a clockwise emf, loop B has no induced emf, and loop C has a counterclockwise emf.

D. loop A has a counter-clockwise emf, loop B has no induced emf, and loop C has a clockwise emf.

E. loop A has a counter-clockwise emf, loops B and C have clockwise emfs.

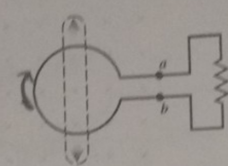
5. (2 points) A flexible loop of wire lies in a uniform magnetic field of magnitude B directed into the plane of the picture. The loop is pulled as shown, reducing its area. The induced current flows

$-\frac{d\Phi_B}{dt} = BA = \mathcal{E} = IR$

$I = \frac{A}{R} B$ proportional

$= kB$

increasing from B to oppose flux



A. downward through resistor R and is proportional to B.

B. upward through resistor R and is proportional to B.

C. downward through resistor R and is proportional to B^2.

D. upward through resistor R and is proportional to B^2.

E. None of the above is true.

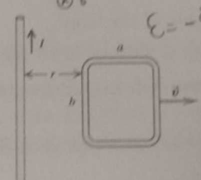
6. (2 points) The loop of wire is being moved to the right at constant velocity. A constant current I flows in the long, straight wire in the direction shown. The current induced in the loop is

$\mathcal{E} = vBL$ $BA = Bab$

$\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{dBA}{dt} = \frac{\mu_0 I \, d\ell \times \hat{r}}{4\pi r^2} A =$

$\mathcal{E} = -\frac{d\Phi_B}{dt}$

flux down increases to oppose decreasing



$B = \frac{\mu_0 I}{2\pi r}$

$dB = \frac{\mu_0}{2} \frac{I \, d\ell \times \hat{r}}{4\pi r^2}$

$\underline{I \, d\ell \times \hat{r}}$

A. clockwise and proportional to I.

B. counterclockwise and proportional to I.

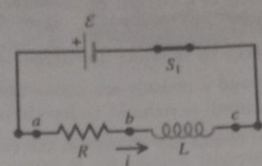
C. clockwise and proportional to I^2.

D. counterclockwise and proportional to I^2.

E. zero.

7. (2 points) An inductor (inductance L)and a resistor (resistance R) are connected to a source of emf as shown. When switch S_1 is closed, a current begins to flow. The final value of the current is

$\frac{\mathcal{E}}{R} = I$



A. directly proportional to RL.

B. directly proportional to R/L.

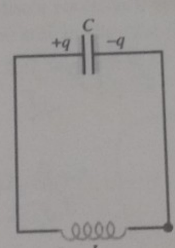
C. directly proportional to L/R.

D. directly proportional to $1/(RL)$.

E. independent of L.

8. (2 points) An inductor (inductance L) and a capacitor (capacitance C) are connected as shown. If the values of both L and C are doubled, what happens to the time required for the capacitor charge to oscillate through a complete cycle?

$\omega \sqrt{\frac{1}{LC}}$ $\sqrt{\frac{1}{4LC}}$ $\frac{1}{2}\sqrt{\frac{1}{LC}}$



$\omega = \frac{2\pi}{f}$

$f = \frac{2\pi}{\frac{\omega}{2}}$ $f = \frac{4\pi}{\omega}$

$\omega' = \frac{1}{2}\omega$

lower freq.
longer time

$T = \frac{\omega}{2\pi} = \frac{1}{2}\frac{\omega}{2\pi}$ = less period

$\omega = \frac{2\pi}{f} = 2\pi T$ $\omega = 2\pi T$

$\frac{1}{2} \times T$ = smaller pe.

~~A. It becomes four times longer.~~

B. It becomes twice as long.

~~C. It is unchanged.~~

D. It becomes half as long.

~~E. It becomes one-fourth as long.~~

9. (2 points) Which of the following statements about inductors are correct? There may be more than one correct choice.

   A. When it is connected in a circuit, an inductor always resists having current flow through it. $t \to \infty$ $I = I_f = \frac{\mathcal{E}}{R}$, no more resistance

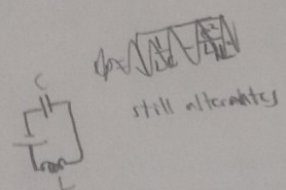
   B. ~~Inductors~~ Capacitors store energy by building up charge. mag. field

   C. When an inductor and a resistor are connected in series with a DC battery, the current in the circuit is reduced to zero in one time constant.

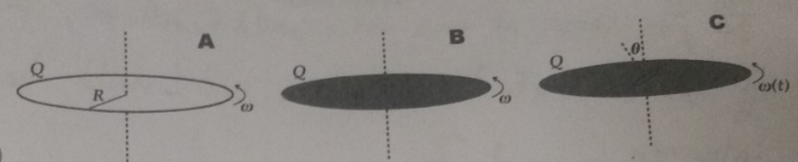
   DC means one way would alternate

   D. An inductor always resists any change in the current through it.

   E. When an inductor and a resistor are connected in series with a DC battery, the current in the circuit is zero after a very long time. $\lambda = I_0$



Part B



1. (15 points)

   (a) (4 points) A ring of radius R has charge Q uniformly distributed along its circumference. If the ring rotates at an angular frequency ω about its axis (see Fig.A), what is the magnetic field (*magnitude* and *direction*) at the center of the rotating ring?

   (b) (5 points) Now, the same charge Q is distributed uniformly over the surface of a thin plastic disc of the same radius R (see Fig.B). Again, the disc rotates at an angular frequency ω about its axis. What is the magnetic field at the center of the rotating disc?

   (c) (2 points) What would have to change in your calculation in part (b) if the charge Q was distributed on the disc surface according to a surface charge density $\sigma(r)$ instead, where r is the distance from the disc's center? Briefly explain.

   (d) (4 points) Assume that in the scenario of part (b), a small conducting ring of area S is placed near the center of the disc with its axis tilted from the disc axis by an angle θ, as shown. If the disc's rotation starts speeding up at some $t = 0$ so that $\omega(t) = \omega_0 + kt^2$, what is the emf in the ring (ignore the ring's self-inductance)?

$\oint B \cdot dl = \mu_0 I_{encl}$

$\frac{dq}{dt} = q \frac{d\theta}{dt} \frac{Q}{2\pi r} = \frac{Q}{2\pi r}\omega$

$B 2\pi r = \mu_0 \frac{dq}{dt}$

$B = \mu_0 \frac{Q\omega}{4\pi^2 r^2} = \frac{\mu_0 Q \omega}{4\pi^2 r^2}$

$\boxed{B = \frac{\mu_0 Q \omega}{4\pi^2 r^2}}$

$B \circ \pi r^2 = \mu_0 I_{encl}$

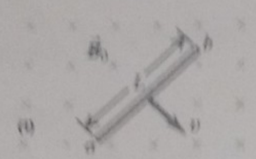
$= \mu_0 \frac{dQ}{dt}$

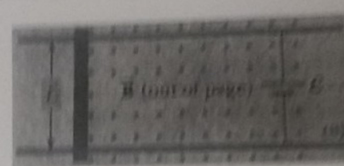
$\frac{dQ}{dt} = \frac{Q}{\pi r^2}\frac{d\theta}{dt} = \omega \frac{Q}{\pi r^2}$

$\boxed{B = \frac{\mu_0 Q \omega}{\pi^2 r^4}}$

Part C

1. (16 points) A conducting rod of length L, mass m, and resistance R can move freely in a region of space where there is a magnetic field $\vec{B}_0$ (Fig.(i)).





(a) (3 points) If the rod moves with constant velocity $\vec{v}$ as shown in fig.(i), what's the induced emf between the ends of the rod, if any?

(b) (4 points) Assume, now, that the rod in (a) pivots about one of its ends instead with constant angular speed ω, and the plane of rotation remains perpendicular to $\vec{B}_0$. What's the induced emf between the ends of the rod, if any?

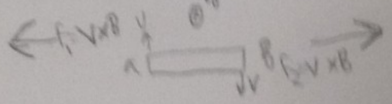
(c) (6 points) The same rod is then placed in a different magnetic field $\vec{B}$ and on parallel frictionless rails, as shown in Fig.(ii). A battery maintains a constant $\mathcal{E}$ between the rails. Assuming that the bar starts from rest, derive an expression for the rod's speed as a function of time.

(d) (3 points) Plot a *speed* vs. *time* diagram, assuming that the battery is $\mathcal{E} = 12$V, $B = 2$T, $L = 0.5$m, $m = 0.5$kg, and $R = 3\Omega$.

a) $\mathcal{E} = vBL = \boxed{vB_0L}$ ✓+3

b) 0 b/c force due to flux is opposite on each side of the rod.

$F_1 = F_2$ so no force on electrons



c) $\mathcal{E}_0 = vBL$ +1 $\qquad v = \frac{\mathcal{E}}{BL}$ $\qquad \mathcal{E} = -\frac{d\Phi_B}{dt}$

~~$\mathcal{E} = \mathcal{E}_{induced} - \mathcal{E}_{total}$~~

$v = \frac{\mathcal{E}}{BL}$

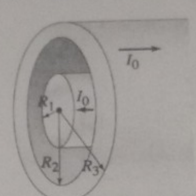
$V(t) = \frac{\mathcal{E}}{R} e^{-BLt}$ $\qquad V_0 = 0 \qquad \frac{dv}{dt} = -vBL$

$\int dv = \int BL\,dt$

10

Part D

1. (15 points) A coaxial cable consists of a solid inner conductor of radius R_1, surrounded by a concentric cylindrical tube of inner radius R_2 and outer radius R_3 (see below). The conductors carry equal and opposite total current I_0 with current densities $J_1(r) = C_1 r$ and $J_2(r) = C_2 r$ for the inner and outer conductors, respectively, where r is the distance from the center of the cable. The constants C_1 and C_2 are not known.



(a) (5 points) Determine the magnetic field in terms of I_0 for $0 \le r \le 2R_3$.

(b) (4 points) Consider the case where the coaxial cable is build with two concentric thin cylindrical shells instead, with radii R_1 and R_2 respectively. If they carry uniformly distributed equal and opposite current I_0, sketch a plot of magnetic field vs. distance from the center r, for $0 \le r \le 2R_2$.

(c) (6 points) For a length ℓ of the coaxial cable in part (b), what is the total energy stored in its magnetic field? (*Hint*: The cable has some self-inductance!)

$J = \frac{I}{A}$ $I = J$

$I_{encl} = I_0 - I_0 = 0$ $R_3 \le r \le 2R_3$

$I = C_1 \pi r^3$

$\oint B \cdot dl = \mu_0 I_{encl}$

$\oint B \cdot dl = 0$ $B = 0$

$0 \le r \le r_1$

$\oint B \cdot dl = \mu_0 I_{encl}$ $I_{encl} = C_1 \pi r^3$

r_2

$\oint B \cdot dl = \mu_0 I_{encl} =$ $I_{encl} = I_0$

$B \cdot 2\pi r = \mu_0 C_1 \pi r^3$

$B = \frac{\mu_0 C_1 \pi r^3}{2\pi r} = \frac{\mu_0 C_1 r^2}{2}$

$B \cdot 2\pi r = \mu_0 I_0$

$B = \frac{\mu_0 I_0}{2\pi r}$

$r_2 \le r \le r_3$ $I = JA = C_2 r \pi r^2$

$\oint B \cdot dl = \mu_0 I_{encl}$ $I_{encl} = I_0 - C_2 \pi r^3$

$\frac{\mu_0 C_1 r^2}{2}$ 0.5 $0 \le r \le R_1$

$B \cdot 2\pi r = \frac{\mu_0 (I_0 - C_2 \pi r^3)}{2\pi r}$

$\frac{\mu_0 I_0}{2\pi r}$ ✓ +1 $R_1 \le r \le R_2$

$B = \frac{\mu_0 I_0}{2\pi r} - \frac{\mu_0 C_2 r^2}{2}$

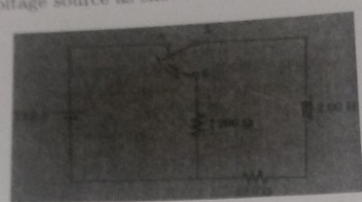
$\frac{\mu_0 I_0}{2\pi r} - \frac{\mu_0 C_2 r^2}{2}$ $R_2 \le r \le R_3$

$2R_3$

Part E

1. (8 points) One application of an $R-L$ circuit is the generation of time-varying high voltage from a low voltage source as shown below.

$= I_0($

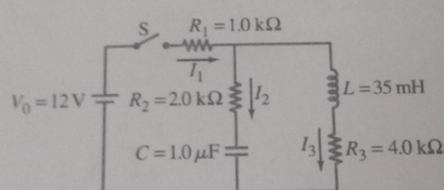


$\mathcal{E} = -d\phi$

   (a) (4 points) After the switch has been in position a for a long time, it is thrown quickly to b. Compute the initial voltage across each resistor and across the inductor.

   (b) (4 points) How much time elapses before the voltage across the inductor drops to 12 V?

2. (8 points) For the $R-L-C$ circuit shown below, at time $t=0$, the switched is closed.



   (a) (5 points) After a sufficiently long time, steady currents flow through the resistors. Determine these three currents.

   (b) (3 points) If the switch is opened again, after the situation described in (a) has been reached, do you expect the charge on the capacitor to oscillate with time? Briefly explain.