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## PHYSICS IC: Electrodynamics, Optics, and Special Relativity

## Fall 2018, Lecture Series 4

## Midterm 2—Thursday 15th November

Version:  $\Delta$ 

Time allowed: 50 minutes

Answer all questions.

Calculators are permitted in this exam

The numbers in the margin indicate the weight that the examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.



1. The figure below shows four different DC circuits (on the left) and five different plots of current versus time (on the right). Draw a line from each circuit to the plot that shows the corresponding current as a function of time, after the switch at  $S$  has been closed. [The capacitor begins fully charged in each case. There will be one plot left over).



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2. If magnetic monopoles were ever discovered experimentally, Maxwell's Equations would need to be modified. Write down the new form that the equations would take in this case, briefly explaining any new terms you add. [You do not need to include the correct coefficients; just the qualitative form of the

 $\oint \vec{E} \cdot d\vec{A} = \frac{9}{6}$  $(MEI)$  $\oint \vec{B} \cdot d\vec{A} = c$  penci where est c is just a coefficient and each the gener is the quantity of Not dipoles in the enclosed area, with North dipoles being + and South dipoles being -.

 $[7]$ 

 $\oint \vec{E} \cdot d\vec{l} = -\frac{dF_B}{dt}$ 

 $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\vec{E}}{dt}$ 

3.

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(a) You take a 12:30am flight to Africa, the moonlit wings reflecting the stars. Estimate the radiative force on the wings of the plane due to the light from the moon, assuming the moon is full and is directly overhead, and that the totally reflecting wings have an area of  $500 \text{ m}^2$  perpendicular to the rays of light from the moon. The full moon emits radiation with a total power of  $1.6 \times 10^{15}$  W from its illuminated side, and the average distance between the Earth and the Moon is  $3.8 \times 10^8$  m. [Ignore effects due to the atmosphere].











 $P = \frac{F}{A}$ 

$$
= \frac{2L}{4\pi r^{2}C} \cdot A
$$
  
= 
$$
\frac{2(1.6\times10^{15})}{4\pi (3.8\times10^{8})^{2}(3\times10^{8})} \cdot (500) N
$$

 $[7]$ 

(b) The next day, you observe Mount Kilimanjaro rising above the Serengeti, at a horizontal distance of  $W = 10$  km away. However, a dense layer of air with refractive index  $n = 1.02$  and depth  $d = 3$  km is distorting the view, making the mountain appear taller than it really is (sec the path of a ray in the figure below). You observe its height to be  $h' = 6130$  m. What is its true height h? [Assume that you are viewing the mountain from ground level. You may continue on the next page.] [10]

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$$
\frac{d}{dx} = \frac{h'}{w} \times \frac{d}{h} = \frac{4443}{h} \times \frac{d}{w} = \frac{4443}{h} \times \frac
$$

 $\overline{5}$ 

$$
h=3000)^{2} = 8404082.336
$$
  
\n $h=3000 = 2898.9795$   
\n $h = 5898.98$  m  
\n $35.5$   
\n5900 m

 $\overline{6}$ 

(c) On your final day in Africa, it starts to rain heavily and you observe a double rainbow. The figures below show a ray of white light from the sun striking a spherical raindrop during the formation of a primary rainbow (top) and secondary rainbow (bottom). Draw arrows to show the path of the ray through the droplet in each case. Add rays corresponding to red and violet light leaving the droplet in each figure, indicating their order.



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4. The figure below shows an AC circuit with a resistor (with resistance  $R$ ) and inductor (with inductance L) connected to an AC source in parallel. The voltage from the source is  $v_s(t)$  and the current from the source is  $i_s(t)$ . The voltage dropped across the resistor is  $v_R(t)$ , the voltage dropped across the inductor is  $v_L(t)$ , while the currents through each component are  $i_R(t)$  and  $i_L(t)$ , as shown in the figure.



(a) Use Kirchhoff's circuit law for currents to obtain an expression relating the current from the source  $i_s(t)$  to currents through the resistor and the inductor. Use Kirchhoff's circuit law for voltages to obtain expressions relating the voltage from the source  $v_s(t)$  to voltages across the resistor and across the inductor.

(Hint: Most of the AC circuits we considered in class were series circuits, whereas this is a parallel circuit. For this reason, the expressions you obtain may look different from what you are used to.]

$$
i_5(+) = i_7(+) + i_1(+) =
$$

Vologerule:

$$
V_{s}(t) - V_{R}(t) = 0 \Rightarrow V_{s}(t) = V_{R}(t)
$$
  
\n $V_{s}(t) - V_{L}(t) = 0 \Rightarrow V_{s}(t) = V_{L}(t)$   
\n $-V_{R}(t) + V_{L}(t) = 0$ 

 $[5]$ 

(b) Assume that the voltage from the source takes the sinusoidal form

$$
v_s(t) = V_0 \cos(\omega t).
$$

Given this input voltage, obtain expressions for the current through the resistor  $i_R(t)$  and the current through the inductor  $i_I(t)$ .

Hence, by drawing a phasor diagram for currents and using your answer to part (a) (or otherwise), obtain an expression for  $I_s$ , the amplitude of the current from the source in terms of  $V_0$ , R, L and  $\omega$ .

What is the impedance of the circuit  $Z$ ? Does the voltage from the source lead or lag the current from the source? [10]

 $V_{L} = \frac{1}{\mu} \omega L \cos(\omega t + 10^{\circ})$ <br>  $V_{R} = \frac{1}{\mu} R \cos(\omega t)$  $i(1)$  $cos(\omega t) = \sqrt{L}f cos(\omega t)$  $J_0 = I_R$  $V_s(H) = V_L(H)$ Vocos (wt) = I w L cos (wt + 90°) Vocos (not) = - I w 1 sin( wt)  $I_{L} = \frac{V_{o} cos(\omega t)}{(\omega L sin(\omega t))}$ Vocatlwt  $I<sub>s</sub>$ 

$$
cos\phi = \frac{\rho}{2} \qquad cos\phi = \frac{\rho}{\sqrt{16}} \qquad 15 \qquad \Rightarrow \phi = 7 \text{ (b) } \frac{1}{\sqrt{16}}.
$$
\n
$$
V_{S} = V_{L} = L \frac{di}{dt}
$$
\n
$$
V_{0} cos(\omega t) = L \frac{di}{dt}
$$
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$$
\int \frac{di}{\rho dt} = \int \frac{V_{0}}{L} cos(\omega t) dt
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$$
i_{L}(t) = V_{0} \frac{di}{dt} sin(\omega t)
$$
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i_{L}(t) = V_{0} \frac{di}{dt} sin(\omega t)
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$$
i_{L}(t) = V_{0} \frac{di}{dt} sin(\omega t)
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$$
V_{S} = V_{R} = i_{R}R
$$
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$$
V_{0} cos(\omega t) = i_{R}(t) R
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$$
i_{R}(t) = \frac{V_{0}}{R} cos(\omega t)
$$
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$$
i_{R}(t) = i_{R}(t) - i_{R}(t)
$$
\n
$$
i_{R}(t) = \frac{V_{0}}{R} cos(\omega t)
$$
\n
$$
i_{R}(t) = \frac{V_{0}}{R} cos(\omega t
$$