Summary [A] 2015

Dr. Menachekanian

Physics 1C: Midterm 2

There are 180 points on the exam, the exam is 12 pages long (including the cover and formula pages), and you have 100 minutes. The exam is closed book and closed notes. The use of any form of electronics is prohibited, except for a basic scientific calculator. To receive full credit, show all your work and reasoning. No credit will be given for answers that simply "appear." *If you need extra space, use the backside of the page with a note to help the grader see that the work is continued elsewhere.*

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Problem	*Your Score*	*Max Score*
1	13	25
2	34	35
3	29	45
4	21	35
5	19	40
Total	116	180

Fundamental Constants

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \quad \& \quad \varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \times 10^8\ \mathrm{m/s}$$

Kinematics with Constant Acceleration ($s = x$ or y) & Centripetal Acceleration

$$s\,(t) = s_0 + v_{0s}t + \frac{1}{2}a_s t^2$$

$$v_s\,(t) = v_{0s} + a_s t$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Electric and Magnetic Force

$$\vec{F}_E = q\vec{E}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad \text{or} \quad \vec{F}_B = \int_c I d\vec{\ell} \times \vec{B}$$

Magnetic Torque

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \text{with} \quad \vec{\mu} = \int \hat{n} A dI$$

$$U_B = -\vec{\mu} \cdot \vec{B}$$

Gauss's Law

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \quad \text{or} \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_S \vec{B} \cdot d\vec{A} = 0 \quad \text{or} \quad \vec{\nabla} \cdot \vec{B} = 0$$

Biot-Savart Law

$$\vec{B} = \frac{\mu_0}{4\pi} I \int_c \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

Ampere-Maxwell Law

$$\oint_c \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{encl}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{or} \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Faraday's Law and Motional EMF

$$\mathcal{E} = \oint_c \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \quad \text{or} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\mathcal{E} = \int_c \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}$$

Mutual- and Self-Inductance

$$\Phi_{B1} = M_{12} I_2 \quad \& \quad \Phi_{B2} = M_{21} I_1$$
$$\Phi_B = LI$$
$$\mathcal{E}_L = -L\frac{dI}{dt} \quad \& \quad V_L = L\frac{dI}{dt}$$

Energy in an Inductor & Capacitor and Magnetic & Electric Energy Density

$$U_L = \frac{1}{2}LI^2 \quad \& \quad U_C = \frac{1}{2}\frac{Q^2}{C}$$
$$u_B = \frac{B^2}{2\mu_0} \quad \& \quad u_E = \frac{1}{2}\varepsilon_0 E^2$$

Damping Coefficient & Time Constant in RL Circuit, and Oscillation Frequency in LC Circuit

$$2\beta \equiv \frac{R}{L} \equiv \frac{1}{\tau_{\mathrm{RL}}}$$
$$\omega_0^2 \equiv \frac{1}{LC}$$

Complex Numbers

$$\tilde{z} = x + iy = re^{i\phi} \quad \& \quad \tilde{z}^* = x - iy = re^{-i\phi}$$
$$\tilde{z}\tilde{z}^* = |\tilde{z}|^2 = r^2 = x^2 + y^2$$
$$\tan\phi = \frac{\Im\{\tilde{z}\}}{\Re\{\tilde{z}\}} = \frac{y}{x}$$

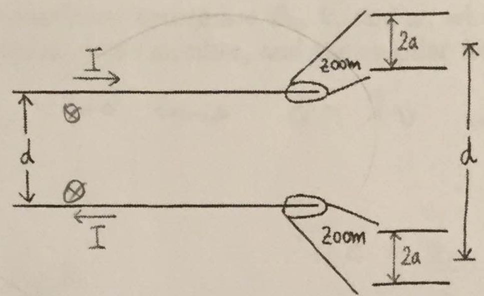
Complex Ohm's Law, Average Power, Complex Impedances, and Reactances

$$\tilde{V} = \tilde{I}\tilde{Z}$$
$$P = \frac{1}{2}\Re\left\{\tilde{V}\tilde{I}^*\right\} = \frac{1}{2}V_0 I_0 \cos\phi$$
$$\tilde{Z}_R = R \quad \& \quad X_R = R$$
$$\tilde{Z}_C = \frac{1}{i\omega C} \quad \& \quad X_C = \frac{1}{\omega C}$$
$$\tilde{Z}_L = i\omega L \quad \& \quad X_L = \omega L$$

Poynting's Vector, Intensity, and Radiation Pressure

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$
$$\mathcal{I} = \left\langle \left| \vec{S} \cdot \hat{n} \right| \right\rangle$$
$$\mathcal{P}_{\mathrm{rad}} = \eta\frac{\mathcal{I}}{c} \quad \text{with} \quad \eta = 1 \text{ or } 2$$

13 1. Two long, parallel wires, each of radius, a, have their centers a distance, d, apart and carry equal currents in opposite directions, as shown below. Assuming that $d \gg a \neq 0$, calculate the inductance per unit length of such a pair of wires. **[25]**



From ampère's Law we know:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

∴ $B_{long\,wire} = \frac{\mu_0 I}{2\pi(2a)} = \frac{\mu_0 I}{4\pi a}$

we see that the magnetic field strength adds together on the inside

∴ $B = \frac{\mu_0 I}{4\pi a} \times 2 = \cancel{\frac{\mu_0 I}{2\pi a}}$ -3

$\Phi_B = \int_S \vec{B}\cdot dA$; $A = lr$ ∴ $\frac{dA}{dr} = l \Rightarrow dA = l\,dr$

$\Phi_B = \int_{\cancel{a}}^{d} \frac{\mu_0 I}{2\pi \cancel{a}} l\,dr = \cancel{\frac{\mu_0 I d l}{2\pi a}}$ -5

-1

∴ $L = \frac{\Phi_B}{I} \Rightarrow \frac{\mu_0 I d l}{2\pi a I} = \frac{\mu_0 d L}{2\pi a}$

∴ $\frac{L}{l} = \frac{\mu_0 d}{2\pi a}$

approx? -3

2. A linearly polarized microwave (i.e., a microwave with the electric-field vector pointing along a line), of wavelength, λ, is moving along the $+x$-axis. The electric-field vector has a maximum value of E_0 and vibrates in the yz-plane.

4

(a) In terms of the given quantities, determine B_0, k, and ω, which are the maximum value of the magnetic field, the (angular) wave number, and the angular frequency, respectively. **[4]**

$B_0 = \frac{E_0}{c}$ ✓ ; we know $c = \lambda\nu$ and $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$ ✓

$\therefore \lambda = \frac{2\pi}{k}$, $\nu = \frac{\omega}{2\pi}$

$\therefore B_0 = \frac{E_0}{(\frac{\omega}{k})} = \frac{E_0 k}{\omega}$ $\therefore c = \frac{2\pi}{k} \times \frac{\omega}{2\pi} = \frac{\omega}{k}$ ✓

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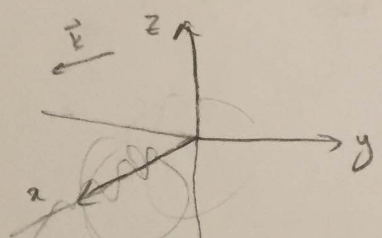
(b) The line defined by the electric-field vector, at any time, t, and position, x, makes an angle $\theta > 0$ relative to the $+y$-axis (with positive values of θ measured counterclockwise from the $+y$-axis, as viewed with the $+x$-axis pointing straight at you). The fields vary as sinusoids. Assuming that the electric field attains its maximum value at $x = t = 0$ AND resides in the yz-quadrant with $y < 0$ and $z < 0$, determine the full electric- and magnetic-field vectors, $\vec{E}(x,t)$ and $\vec{B}(x,t)$, respectively, in terms of the unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ and the given quantities. **[13]**



$\tilde{\vec{E}} = \tilde{E}_0 e^{i(\vec{k}\cdot\vec{r} \pm \omega t)}$ where $\vec{k} = k\hat{x}$

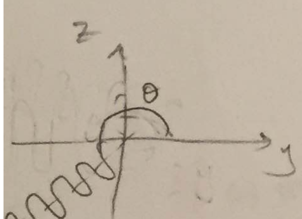
$\tilde{\vec{B}} = \tilde{\vec{B}}_0 e^{i(\vec{k}\cdot\vec{r} \pm \omega t)}$

we know that $\vec{E}$ and $\vec{B}$ are $\perp$ to each other and to $\vec{k}$.

we take the real parts because we are told max amplitude is attained at $x = t = 0$ which deems a cosine wave.

$\therefore \vec{E} = \vec{E}_0 \cos(kx - \omega t)$

$\vec{B} = \vec{B}_0 \cos(kx - \omega t)$



$\therefore \vec{E} = E_0 \cos(kx - \omega t)\,(\cos\theta(-\hat{y}) + \sin\theta(-\hat{z}))$

$\vec{B} = B_0 \cos(kx - \omega t)\,(-\cos\theta(\cancel{\hat{x}}\ \hat{z}) + \sin\theta(\hat{y}))$

-1

k—)

3 (c) Determine the full Poynting vector, $\vec{S}(x,t)$. **[3]**

$\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E}\times\vec{B}$; since $\vec{E}$ and $\vec{B}$ are $\perp$ $\therefore$ $\vec{E}\times\vec{B} = EB$

$\therefore$ $\vec{S}(x,t) = \frac{1}{\mu_0} E_0 B_0 \cos^2(kx-\omega t)\,\hat{x}$ ✓

11 (d) *Showing all your work*, determine the radiation pressure this wave would exert if it hits a reflecting sheet at normal incidence in terms of the given quantities. **[11]**

Reflecting surface $\therefore$ $\eta = 2$

$P_{rad} = 2\frac{I}{c}$ where $I = \langle|\vec{S}\cdot\hat{n}|\rangle$ and recall that $\cos^2(\sim)$ is $\frac{1}{2}$ when averaged over a period T.

$P_{rad} = \frac{E_0 B_0}{\mu_0 c} = \frac{E_0 B_0 k}{\omega \mu_0} = \frac{E_0^2 k^2}{\omega^2 \mu_0}$ OK

$c = \frac{\omega}{k}$

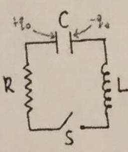
4 (e) What acceleration would be imparted to the sheet in Part (d) if it is a rectangle of dimensions, 1.00 m × 2.00 m, with a mass of 0.50 kg? Assume the microwave's wavelength is 1.5 cm, and that the electric-field amplitude is 200 V/m. **[4]**

$\frac{F_{net}}{A} = \frac{E_0^2 k^2}{\omega^2 \mu_0}$ ✓

$ma = \frac{A E_0^2 k^2}{\omega^2 \mu_0}$

$a = \frac{A E_0^2 k^2}{m\omega^2 \mu_0} = \underline{1.41 \times 10^{-6}\ m/s^2}$

3. Consider the RLC series circuit shown below, which contains a switch, S, that has been open for a very long time, and a capacitor that is already charged to a value q_0. The inductor has inductance, L, the resistor has resistance, R, and the capacitor has capacitance, C. The switch is then closed at $t = 0$.



(a) Using Kirchhoff's loop rule, write down the appropriate differential equation that the charge, $q(t)$, on the capacitor obeys at any time $t \geq 0$. *Be sure to explain why the terms have the signs that they do when applying the loop rule element by element.* **[8]**

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when the switch is closed, we see the conventional current flow left to right.

explanation?

$$\therefore \frac{q(t)}{C} - I(t)R - L\frac{dI(t)}{dt} = 0$$

Note $I(t) = -\frac{dq(t)}{dt}$ (discharging capacitor)

however $\frac{dI(t)}{dt} < 0$ as current decreases

$$\Rightarrow L\frac{d^2q(t)}{dt} + \frac{dq(t)}{dt}R + \frac{q(t)}{C} = 0$$

$$\Rightarrow \frac{d^2q(t)}{dt} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q(t) = 0$$

(b) *Upon showing your work step by step,* properly solve this differential equation *assuming the system is underdamped.* Be sure to use the appropriate initial condition to get THE solution for $q(t)$, instead of a general solution. **[17]**

14/17

We assume the solution $q(t) = q_0 e^{\gamma t}$

$$\frac{dq(t)}{dt} = q_0 \gamma e^{\gamma t} \quad \text{and} \quad \frac{d^2q(t)}{dt} = q_0\gamma^2 e^{\gamma t}$$

We use the fact $2\beta = \frac{R}{L}$ and $\omega_0^2 = \frac{1}{LC}$

$\beta = \frac{R}{2L^?} = \frac{R^2}{4L}$

$$\rightarrow q_0\gamma^2 e^{\gamma t} + 2\beta q_0 \gamma e^{\gamma t} + \omega_0^2 q_0 e^{\gamma t} = 0$$

$$\Rightarrow q_0 e^{\gamma t}(\gamma^2 + 2\beta + \omega_0^2) = 0$$

this cannot be zero as $q_0 \neq 0$ and $e^{\gamma t} \neq 0$

$$\therefore \gamma^2 + 2\beta + \omega_0^2 = 0$$

$$\therefore \gamma = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega_0^2}}{2} = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

we assume underdamped $\therefore \beta^2 - \omega_0^2 < 0$ ← This means we get an imaginary solution

$$\therefore \gamma = \beta \pm i\sqrt{\omega_0^2 - \beta^2}$$

$$\therefore q(t) = q_0 e^{\gamma t} \Rightarrow q_0 \cos(\omega' t \pm \varphi)$$

$$q(t) = q_0 e^{-\beta t}\cos(\omega t)$$

we take the real part and $\varphi = 0$ as there is no phase.

$$\therefore q(t) = q_0 \cos(\omega' t)$$

where $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

(c) Determine the period of oscillation, T, of this system AND sketch $q(t)$. **[6]**

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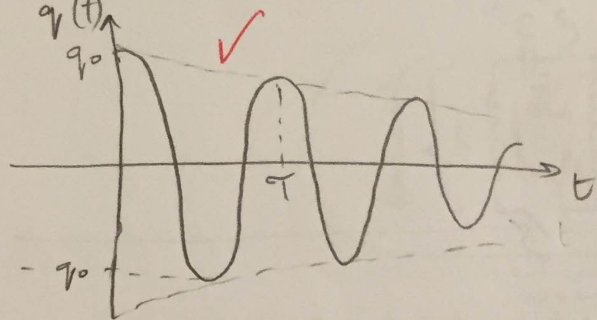
$\omega' = \frac{2\pi}{T} \therefore T = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$ ✓ ✓



(d) The quality factor of this underdamped oscillator (which describes the extent of damping) can be defined as,

$$Q \equiv 2\pi \frac{\text{Total Energy Stored}}{\text{Total Energy Lost In A Cycle}}.$$

Assuming the system is VERY underdamped, prove, *with appropriate approximations*, that

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

for this circuit. **[14]**

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Total energy stored in capacitor $= \frac{q_0^2}{2C}$ ✓

Very underdamped suggests almost no damping $\therefore \omega = \frac{1}{\sqrt{LC}}$

$\therefore P = I^2 R \Rightarrow u = q(t)^2 R = q_0^2 \cos^2(\omega t) R = \frac{1}{2} q_0^2 R$

? ×

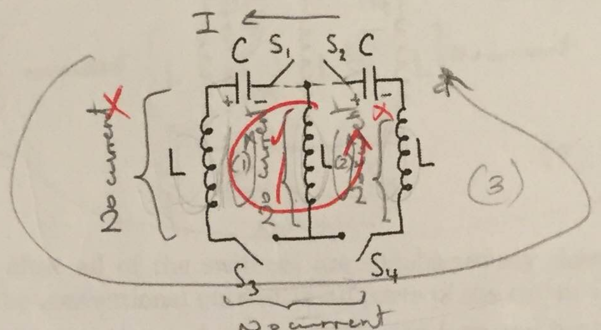
$Q \equiv \pi \frac{q_0^2}{2C} \times \frac{2}{R q_0^2} = 2\pi \frac{1}{R} \frac{1}{C}$

where $2\pi \approx \frac{1}{\omega}$

$\therefore Q \equiv \frac{1}{R} \sqrt{LC} \frac{1}{C} \Rightarrow \frac{1}{R}\sqrt{\frac{L}{C}}$

4. Three identical inductors, with inductances, L, and two identical capacitors, with capacitances, C, are connected in a two-loop circuit, as shown below. The switches, S_1, S_2, S_3, and S_4, are initially open.

(a) Suppose that the capacitors are charged identically and independently, with the in-between plates having opposite sign, as shown below. All switches are then closed at $t = 0$.



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i. Immediately after all of the switches are simultaneously closed (i.e., at $t \gtrsim 0$), draw the direction of the conventional current in all parts of the circuit *in the diagram above.* **[4]**

ii. *Showing all of your steps and reasoning,* use the loop and junction rules to find the angular frequency at which the current oscillates in this circuit. **[13]**

$I = I_1 + I_2$ 9/13

$\frac{1}{C_{eq}} = \frac{2}{C} \quad \therefore \quad C_{eq} = \frac{C}{2}$ ✓

$\frac{1}{L_{eq}} = \frac{3}{L} \quad \therefore \quad L_{eq} = \frac{L}{3}$ ✗

(3) Looking at outer loop: $\frac{Q}{C_{eq}} - 2L\frac{dI}{dt} = 0 \Rightarrow \frac{2Q}{C} - 2L\frac{dI}{dt} = 0$ ✓

$\Rightarrow \frac{Q}{C} - L\frac{dI}{dt} = 0$

But $\frac{dI}{dt} < 0$ as charge on C reduces $\therefore \frac{Q}{C} + L\frac{dI}{dt} = 0$

✓

$\therefore \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0 \quad \therefore \frac{d^2Q}{dt^2} + \frac{Q}{LC} = 0$

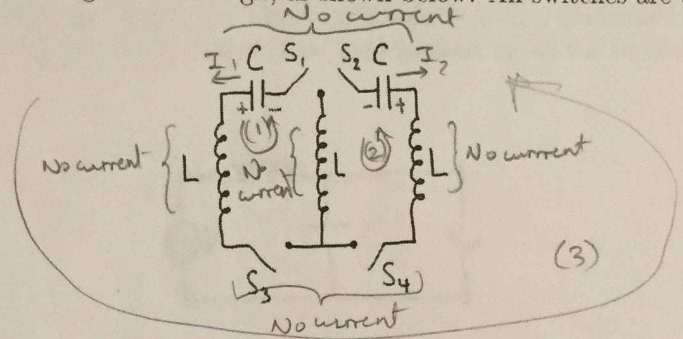
$\Rightarrow Q(t) = q_0 \cos(\omega_0 t)$

$\Rightarrow I(t) = -q_0 \omega_0 \sin(\omega_0 t) \rightarrow \omega_0 = ?$

(1) $\frac{Q}{C} - L\frac{dI}{dt} - L\frac{dI_1}{dt} = 0$

(2) $\frac{Q}{C} - 2L\frac{dI_2}{dt} = 0$

(b) Now, suppose that the capacitors are again identically and independently charged, but with the in-between plates having the same sign, as shown below. All switches are then closed at $t = 0$.



4/4

i. Immediately after all of the switches are simultaneously closed (i.e., at $t \gtrsim 0$), draw the direction of the conventional current in all parts of the circuit *in the diagram above.* **[4]**

ii. *Showing all of your steps and reasoning,* use the loop and junction rules to find the angular frequency at which the current oscillates in this case. **[14]**

3 We now see that as t progresses the currents coming from each capacitor will add in the middle L to give 2I effectively.

where $I = I_1 + I_2$.

$\therefore$ (1) $\frac{Q}{C} - 2L\frac{dI_1}{dt} = 0 \Rightarrow I_1 = -q_0\omega_0 \sin(\omega_0 t) \rightarrow \omega_0 = \frac{1}{\sqrt{2LC}}$

(2) $\frac{Q}{C} - 2L\frac{dI_2}{dt} = 0 \Rightarrow I_2 = -q_0\omega_0 \sin(\omega_0 t) \rightarrow \omega_0 = \frac{1}{\sqrt{2LC}}$

(3) $\frac{2Q}{C} - L\frac{dI}{dt} = 0$ 2

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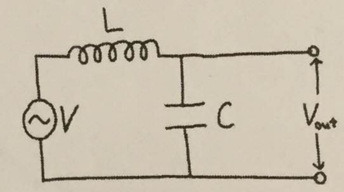
5. Consider the AC circuit shown below. The AC source outputs a voltage, $V(t) = V_0 \cos(\omega t)$, which is, of course, the real part of its complex counterpart: $\tilde{V}(t) = V_0 e^{i\omega t}$. The inductor has inductance, L, and the capacitor has capacitance, C. This circuit's output, V_{out} (indicated by the terminals), will be fed to another circuit. Currently, there is no load hooked up to the output, so that the load has infinite resistance.



(a) Find the current (magnitude and phase) AND the charge (magnitude and phase) across the capacitor. *Quote the phases relative to the source voltage.* **[24]**

13/24

$$\tilde{Z}_{eq} = \tilde{Z}_L + \tilde{Z}_C = i\omega L - \frac{i}{\omega C} = i\left(\omega L - \frac{1}{\omega C}\right)$$ 3

$$Z_{eq} = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2} = \omega L - \frac{1}{\omega C} \Rightarrow \frac{\omega^2 LC - 1}{\omega C}$$

$$\varphi = \tan^{-1}\left(\cancel{\frac{\omega^2 LC - 1}{\omega C}}\right) = \frac{\pi}{2} = \tan^{-1}(+\infty)$$

$$\therefore \tilde{I} = \frac{\tilde{V}}{\tilde{Z}_{eq}} = \frac{V_0 e^{i\omega t}}{Z_{eq} e^{i\varphi}} \Rightarrow \frac{V_0}{Z_{eq}} e^{i(\omega t - \varphi)}$$

4 I_0 (magnitude) $= \dfrac{V_0 \omega C}{\omega^2 LC - 1}$, φ (phase) $= \tan^{-1}\left(\dfrac{\omega^2 LC - 1}{\omega C}\right)$

$\therefore$ I lags behind voltage by φ.

$$\tilde{Q}(t) = \int_0^t I_0 e^{i(\omega t - \varphi)}\, dt \Rightarrow I_0 \int_0^t e^{i(\omega t - \varphi)}\, dt$$

$$\Rightarrow \frac{I_0}{i\omega}\left(e^{i(\omega t - \varphi)} - e^{-i\varphi}\right)$$

$$\Rightarrow -\frac{I_0 i}{\omega}\left(e^{i(\omega t - \varphi)} - e^{-i\varphi}\right)$$

$$\Rightarrow \frac{I_0}{\omega}\left(e^{i(\omega t - \varphi - \frac{\pi}{2})} - e^{-i(\varphi + \frac{\pi}{2})}\right)$$

4 Magnitude of charge $= \dfrac{V_0 C}{\omega^2 LC - 1}$

phase of chare $-(\varphi + \frac{\pi}{2}) = -\left(\tan^{-1}\left(\dfrac{\omega^2 LC - 1}{\omega C}\right) + \dfrac{\pi}{2}\right)$

Q lags behind voltage by $\varphi + \frac{\pi}{2}$ 2

(b) Suppose, now, that a finite load is attached to the leads placed across the capacitor.

i. Noting that the amplitude of a sinusoidal function is the factor multiplying the sinusoidal function, *accurately* sketch the amplitude of the output voltage as a function of the angular frequency, ω. **[7]**

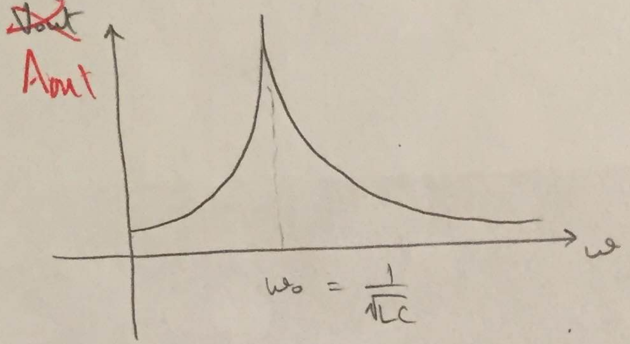
$$\tilde{V}_{out} = \tilde{V}_C = \tilde{I}\tilde{Z}_c$$

$$= \frac{I_0 - i}{\omega C} e^{i(\omega t - \varphi)}$$

$$= \frac{I_0}{\omega C} e^{i(\omega t - \varphi - \frac{\pi}{2})}$$

$$= \frac{I_0}{\omega C} \cos\left(\omega t - \varphi - \frac{\pi}{2}\right)$$

$$= \frac{V_0 \cancel{\cos\left(\omega t - \varphi - \frac{\pi}{2}\right)}}{\omega^2 LC - 1}$$



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ii. Suppose that the input voltage is modified into the following form:

$$V'(t) = V_{DC} + V_0 \cos(\omega t),$$

where V_{DC} is some background, constant DC voltage, and the remaining term is what we have already analyzed. Discuss how you would construct this circuit in order to ensure that the attached load gets minimal AC voltage delivered to it. **[9]**

for the load to get minimal AC voltage we have to reduce $V_0 \cos(\omega t - \frac{\pi}{2} - \varphi)$ and we can do this by changing the frequency of the source to reach the highest value possible so that the only term remaining is V_{DC}.

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