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First Name: Jane
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Midterm #2
Physics 1C
Prof. David Saltzberg
February 29, 2012

Time: 50 minutes

Closed Notes. Closed Book. One 3"x5" index card with notes on both sides is allowed (plus the one from MT1). Calculators are allowed.

If a problem is confusing or ambiguous, notify the professor or Blake

Clarifications will be written on the blackboard. Check the board.

This exam is version **E**. Show your work.

There are 8 pages including this cover sheet. Make sure you have them all.

Problem	**Points**
1ab	/10
2abc	/ 30
3	/~~20~~ /30
4	/~~20~~ /30
~~5~~	~~/20~~
Extra Credit	
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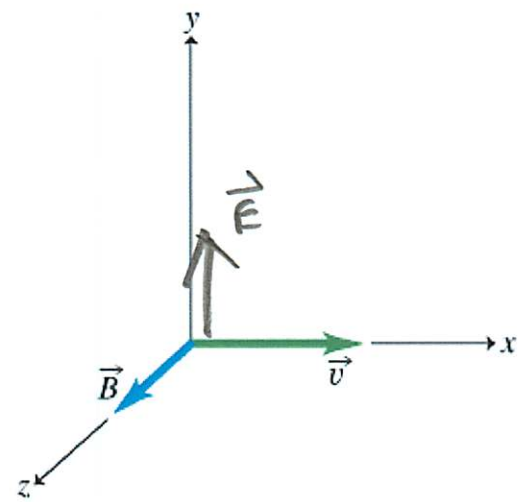
TOTAL	/100

Useful(?) constants etc.
$\mu_0=4\pi\times10^{-7}$ Wb/(A-m) $c=3.00\times10^{8}$ m/s
$\varepsilon_0=8.85\times10^{-12}$ $C^2/(N\text{-}m^2)$ $\sin 30° = ½$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$ $\cos$ ~~30~~ 60° = ½

Order of <1 SI prefixes by ×1000: yocto, zepto, atto, femto, pico, nano, micro, milli
Order of >1 SI prefixes by ×1000: kilo, Mega, Giga, Tera, Peta, Exa, Zetta, Yotta, Hella

1) <u>Short answer questions</u>
(Partial credit may be given based on your solution. Parts a & b are unrelated.)

1a) (5 pts) A linearly polarized electromagnetic wave is propagating in vacuum. At one instant, in the diagram below



$\vec{E} \times \vec{B}$ points along $\vec{v}$

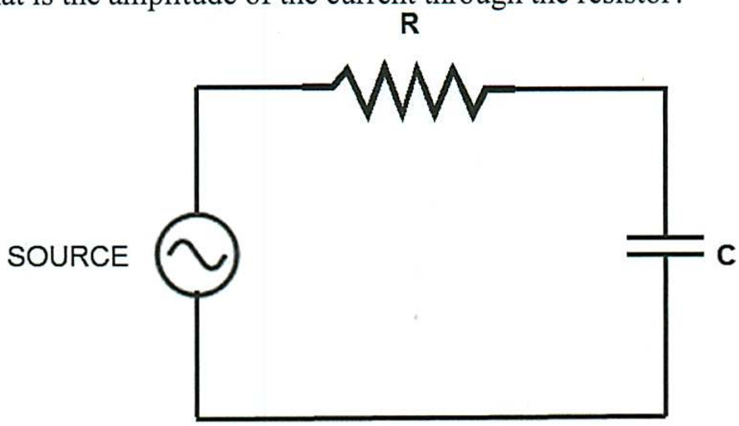
the direction of propagation is shown as **v**. The direction of the magnetic field in the is shown as **B**. On the figure above, draw the direction of the electric field **E** at the origin.

1b) (5 pts) An AC current of $i(t)=I_0 \cos(\omega t)$ passes through a resistor (100Ω) and capacitor (1µF) in a series circuit. The RMS current is 3 mA ~~and the period of the wave is 100 µs.~~ What is the average power dissipation by the resistor, if any? (Hint: you may not need all the information given.)

$$P = I_{rms}^2 R$$
$$= (0.003)^2 (100)$$
$$= (9 \times 10^{-6})(100)$$
$$P = 9 \times 10^{-4} \text{ W}$$
(or)
$$P = 900 \, \mu\text{W}$$

2. Medium-length questions (Partial credit may be given based on your solution.)
(Parts a,b and c are unrelated)

2a) (10 pts) The following circuit is driven with a sinusoidal voltage with amplitude 5V and angular frequency 1000 rad/sec (ν=159 Hz). The capacitor is 2.5 μF and the resistor is 300 Ω. What is the amplitude of the current through the resistor?



$$V_s = I_0 Z$$

$$Z = \sqrt{R^2 + X_L^2}$$

$R = 300$

$$X_L = \frac{1}{(1000)(2.5\times10^{-6})} = 400$$

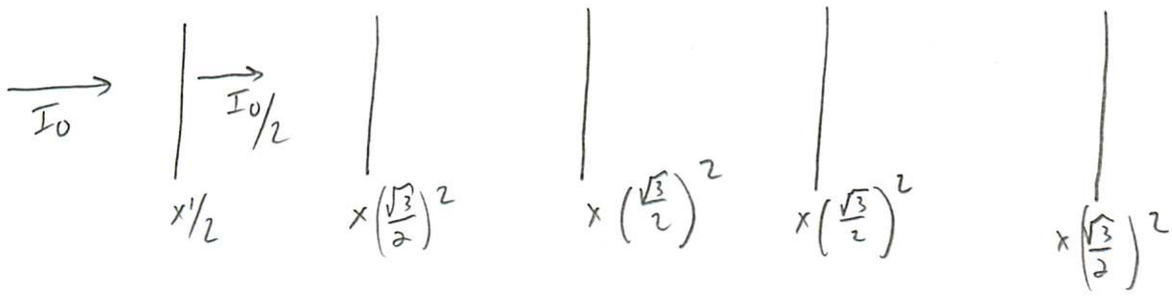
$Z = 500\,\Omega$ ("3-4-5 triangle")

$$I_0 = \frac{5V}{500\Omega}$$

$I_0 = 0.01A$
or
$I_0 = 10mA$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

2b) (10 pts) Unpolarized sunlight with intensity ~~1024~~ 512 W/m^2 impinges on a five-layer stack of polarizing filters. The first is oriented vertically and each subsequent filter is rotated 30 degrees clockwise (as seen from the light source) past the previous one. What is the intensity and direction of polarization of the light that emerges from the last filter?



$$I = 512 \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)^4$$

$$= 512 \times \frac{81}{512}$$

$$= 81$$

$$\boxed{I = 81 \text{ W/m}^2}$$

$0^\circ \rightarrow 30^\circ \rightarrow 60^\circ \rightarrow 90^\circ \rightarrow 120^\circ$

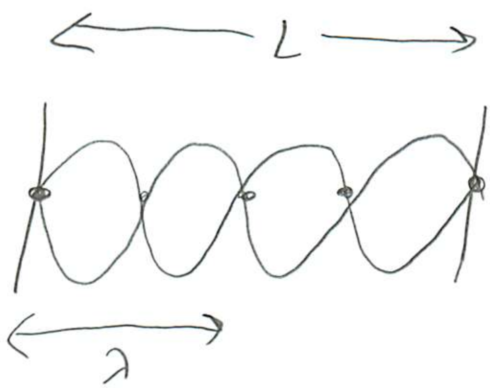
120° from vertical

or

30° from horizontal

2c) (10 pts) The waves in a microwave oven have a frequency of 2.5 GHz. The walls of the oven are perfect conductors and a standing wave is set up. What width must the oven be in order to have five nodal planes of electric field (ie, planes with $E(t)=0$) across it? (Hint: each wall counts as one of the nodal planes if $E=0$ on it.)

Walls are conductive $\Rightarrow E=0 \Rightarrow$ nodal



$L = 2\lambda$

$\lambda\nu = c$

$\lambda = \dfrac{3\times10^8\ m/s}{2.5\times10^9/s} = 0.12m$

$L = 0.24\,m$

or

$L = 24\,cm$

3) <u>Work-out problem</u> (30 pts)
An object is placed on one side of a lens, 4 cm from the vertex (center of lens). An image is found to be formed on the same side as the object, 3 cm from the lens.

a) What is the focal length of the lens? Is it a converging or diverging lens?

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$\frac{1}{f} = \frac{1}{4} + \frac{1}{-3}$$

$$\frac{1}{f} = \frac{3}{12} - \frac{4}{12}$$

$$\frac{1}{f} = -1/12 \Rightarrow \boxed{f = -12\text{ cm}}$$

minus sign ⇒ **diverging**

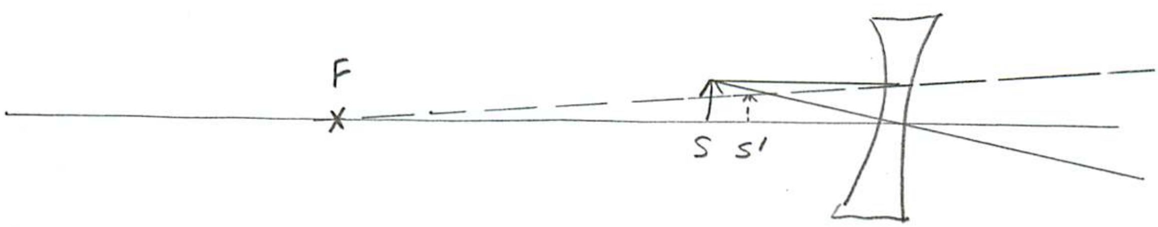
b) The object is 1 cm tall, how tall is the image? Is it erect or inverted?

$$m = -\frac{s'}{s} = -\frac{-3}{4} = 3/4$$

positive ⇒ **erect**

$$3/4 \times 1\text{cm} = \boxed{0.75\text{cm}}$$

c) Sketch the situation, showing the object, the image and at least two principle rays. Note if the image is real or virtual.



Not all rays pass thru image

⇒ **Virtual**

4 5) Work-out problem (2 pages!)

(30 pts) An electomagnetic wave travels in vacuum with an electric field given with a magnitude $\mathbf{E}(x,t) = (10V/m)\sin(kx + \omega t)\,\hat{\mathbf{j}}$, where $k = 10^7\ m^{-1}$, x is distance, t is time and ω is angular frequency. (Hint: note the plus sign.)

a) What is the period of the wave?

$\lambda\nu = c \qquad k = \frac{2\pi}{\lambda} \qquad \nu = \frac{1}{T}$

$\left(\frac{2\pi}{k}\right)\left(\frac{1}{T}\right) = c$

$T = \frac{2\pi}{kc} = \frac{2\pi}{10^7/m \;\; 3\times10^8 m/s} = 2.1\times10^{-15}\ s$ or $2.1\ fs$

b) Write down an expression for the wave corresponding to the magnetic field. (For simplicity just use kx and ωt in your answer.) Remember it is a vector with units.

in phase!

$B_0 = \frac{E_0}{c} = \frac{10 V/m}{3\times10^8 m/s} = 3.3\times10^{-8}\ T$

$\vec{E}$ is along $\hat{j}$, $\vec{v}$ is along $-\hat{i}$ $\Rightarrow$ $\vec{B}$ along $-\hat{k}$

$$\vec{B}(x,t) = -(3.3\times10^{-8}\ T)\sin(kx+\omega t)\,\hat{k}$$

Problem ~~5~~ 4 (continued)

c) This plane wave reflects from a 2m ×2m square mirror and is reflected backward. (The wave is larger in extent than the mirror.) What is the average force on the ~~paper~~ mirror?

reflected wave pressure = $\frac{2I}{c}$

$$I = \frac{1}{\mu_0} E_{rms} B_{rms}$$

$$= \frac{1}{2\mu_0} E_0 B_0$$

$$= \frac{1}{(2)(4\pi \times 10^{-7})} (10\,V/m)(3.3\times10^{-8}\,T)$$

$$I = 0.13\ W/m^2$$

$$\text{pressure} = \frac{(2)(0.13)}{3\times10^8} = 8.8\times10^{-10}\ N/m^2$$

$$F = \text{pressure} \times \text{area} = (8.8\times10^{-10})(4) = \boxed{3.5\times10^{-9}\,N}$$

d) Write down an expression for the Poynting vector for this wave using *kx* and ω*t*. (The answer should be a vector with units.)

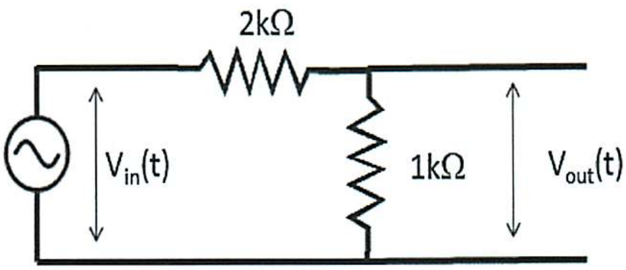
$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\boxed{\vec{S} = -\frac{1}{\mu_0}(10)(3.3\times10^{-8}) \sin^2(kx+\omega t)\ \hat{i}}$$

↑ propagates along x direction

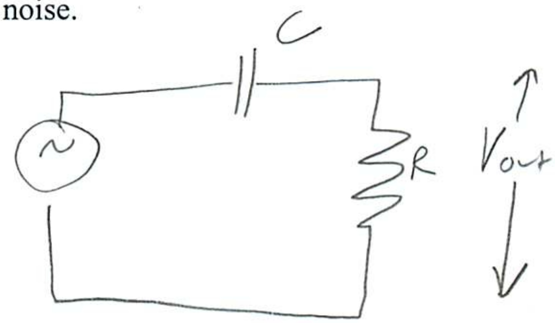
Extra Credit

The circuit below is an example of a circuit that passes the signal $V_{in}(t)$ to $V_{out}(t)$, but with 33% of the voltage, regardless of the frequency.



Design a simple circuit that instead that "blocks" signals with frequencies 60 Hz or less by making the amplitude of $V_{out}(t)$ only 1% of the amplitude of $V_{in}(t)$. Your circuit should still pass signals with very high frequencies (for example >1 kHz or more) with at least 95% of the input amplitude. This is an example of a very common circuit used to reduce noise.

$\omega = 60Hz\,(2\pi) = 377$ rad/s



$$\frac{V_{out}}{V_{in}} = \frac{R}{Z_{tot}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = 0.01$$

$$\frac{R^2}{R^2 + \left(\frac{1}{\omega C}\right)^2} = 0.1$$

$$R^2 = 0.1R^2 + 0.1\left(\frac{1}{\omega C}\right)^2$$

$$0.9R^2 = 0.1\left(\frac{1}{\omega C}\right)^2$$

Using $R = 1k\Omega \Rightarrow$

$$(0.9)(10^6) = (0.1)\left(\frac{1}{377\,C}\right)^2$$

$$\boxed{C = 0.9\mu F}$$

as $\omega \to \infty$

$$\frac{V_{out}}{V_{in}} \to 1$$