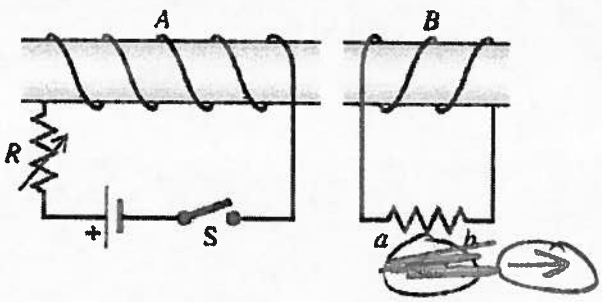
Short Answer questions

Partial credit may be given based on your explanation.

1a) (6 pts) What is the direction of the current in resistor *ab* of the (familiar) figure below when switch S is opened after having been closed for several minutes.

careful. Note these are wound opposite directions →



← Because this was trickier than intended, everyone will get 6 pts for this

By Lenz's law:

Before switch is opened, $\vec{B}$ points to right in tube B. After switch is opened current moves to try to keep Φ_B the same

a→b through resistor

1b) (6 pts) A magnet is brought close to a diamagnetic material. What happens?

It is repelled

1c) (6 pts) A magnet that produces 1×10^{-5} T at point P. Some unmagnetized, room-temperature iron placed at that point. Approximately what magnetic field is produced in the iron ? (Hint: a wide range of values is acceptable here.)

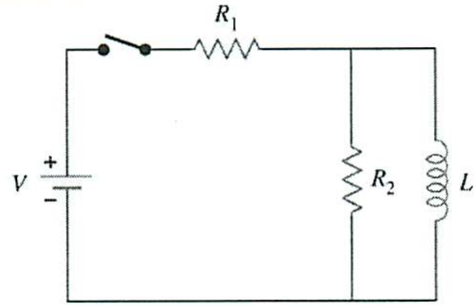
$\mu_0 \rightarrow K_m\mu_0$ in ferromagnetic material

$K_m \sim 1000$ to $100{,}000$

so $B \sim 0.01$ to 1 Tesla

Short-answer (continued)

1d) (6 pts) Consider the circuit below.



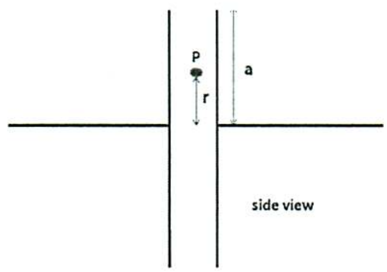
Suppose the battery has a 1.5 Volt EMF and that resistor R_1=1 kΩ and R_2=2 kΩ. A long time after the switch is closed, what is the current through resistor R_2 ?

No voltage drop across L because $\frac{di}{dt} = 0$

⇒ Inductor behaves like a wire

$0 = iR_2 \Rightarrow$ $i = 0$ through R_2

1e) (6 pts) A battery is connected to a parallel-plate capacitor with circular plates of radius a=3cm, shown below. There is only air or vacuum between the two plates.



A constant current of 3A flows onto one plate of the capacitor and off of its other plate from wires connected to the center of the plates. At a Point P, r=2 cm from the centerline of the capacitor, what is the magnetic field? (For simplicity, assume the electric field does not fringe outside the capacitor.)

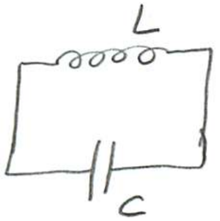
$$i_{encl} = 3A\left(\frac{2}{3}\right)^2 = (3A)\left(\frac{4}{9}\right) = \frac{4}{3}A = 1.33\,A$$

↑ displacement current

By Ampere's Law: $(B)(2\pi r) = \mu_0 I_{encl}$

$$B = \frac{(4\pi \times 10^{-7})(1.33A)}{(2\pi)(0.02)} = 1.33 \times 10^{-5}\,T$$

1f) (6 points) You have an inductor with inductance 8mH. Design a circuit that would allow a current to oscillate at1 kHz. You may choose any resistor or capacitor.



$$\omega = \frac{1}{\sqrt{LC}}$$

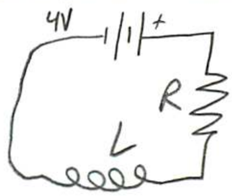
$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

$$C = \frac{1}{L}\left(\frac{1}{2\pi\nu}\right)^2$$

$$C = \frac{1}{0.008}\left(\frac{1}{(2\pi)(1000)}\right)^2 \Rightarrow \boxed{C = 3.2\mu F}$$

(Next two problems) An 8H inductor is hooked up to a 4V battery .The inductor's wire has a resistance of 2Ω and the battery has negligible resistance. After 20 minutes,…

1g) (6 pts) How much energy is stored in the magnetic field of the inductor ?



long time $\Rightarrow \frac{di}{dt} = 0 \Rightarrow L\frac{di}{dt} = 0$

$$V = IR \Rightarrow I = \frac{4V}{2\Omega} = 2A$$

$$U = \frac{1}{2}LI^2 = \left(\frac{1}{2}\right)(8)(2)^2 = \boxed{16J}$$

1h) (6 pts) The battery suddenly runs out (i.e., its EMF becomes zero but it still conducts current), what fraction of the current is still circulating 12 seconds later?

$$I(t) = I_0 e^{-t/\tau} \qquad \tau = L/R \text{ (time constant)}$$

$$= 8/2 = 4 \text{sec}$$

fraction of current remaining $= \frac{I(t=6\text{sec})}{I_0} = \frac{I_0 e^{-12/4}}{I_0} = e^{-3} = \boxed{5\%}$

1i) (6 points) Express the units of magnetic flux in terms of the basic SI units.

$[\Phi_B] = T \cdot m^2$

use any equation for units on B: $F = I\vec{L} \times \vec{B}$

$\frac{kg \cdot m}{s^2} = A \cdot m \cdot T$

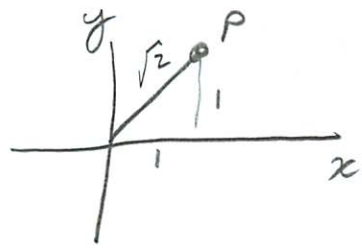
$T = \frac{kg}{A \cdot s^2}$

$[\Phi_B] : \boxed{\frac{kg \cdot m^2}{A \cdot s^2}}$

Medium-length problem

2) (15 pts) A charge of 5μC is passing along the *x*-axis in the positive direction with speed 4×10^6 m/s. At the moment the particle passes through the origin, what is the magnitude of the magnetic field it creates at the point (*x*,*y*,*z*)=(1.0, 1.0, 0) meters?

Biot Savart $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$



$\vec{v}$ and $\hat{r}$ are in plane of page

so $\vec{B} = B_z \hat{z}$ or $|B| = |B_z|$

$$|B| = \frac{(4\pi \times 10^{-7})}{4\pi} \frac{(5 \times 10^{-6})(4 \times 10^6)(1) \sin 45^\circ}{(\sqrt{1^2 + 1^2})^2}$$

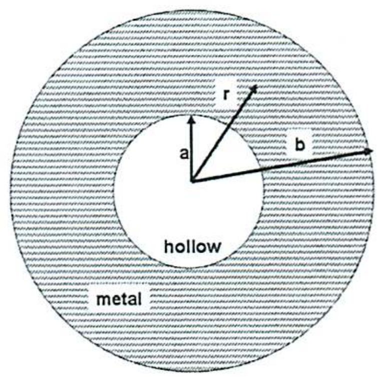
$$= \boxed{7.1 \times 10^{-7}\ T}$$

3) Short Essay

(15 pts) In class we performed a demo where a battery caused the armature of a motor to turn. It is often said that "Every motor is also a generator and every generator is also a motor". Was that the case here? Explain using a diagram why or why not. (For simplicity, don't worry about whether it was a DC or AC motor.)

Work-out problem:

4) (16 pts) The figure below shows the cross section of an infinitely long hollow cylindrical conductor of inner radius a=1m and outer radius b=3m with current flowing into the page. The current density $j(r)$ is given by $j_0*(k/r)^2$, where j_0=3A/m^2 and k=2 meters.



What is the magnetic field at r=2 meters?

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$$

$$(B)(2\pi r) = \mu_0 \int_a^r j_0\left(\frac{k^2}{r'^2}\right) r'dr' \int_0^{2\pi} d\theta$$

$$= \mu_0 j_0 k^2 (2\pi) \int_a^r \frac{dr'}{r'}$$

$$B = \frac{\mu_0 j_0 k^2 (2\pi) \ln(r/a)}{2\pi r}$$

$$= \frac{(4\pi\times 10^{-7})(3)(2)^2 \ln(2/1)}{2}$$

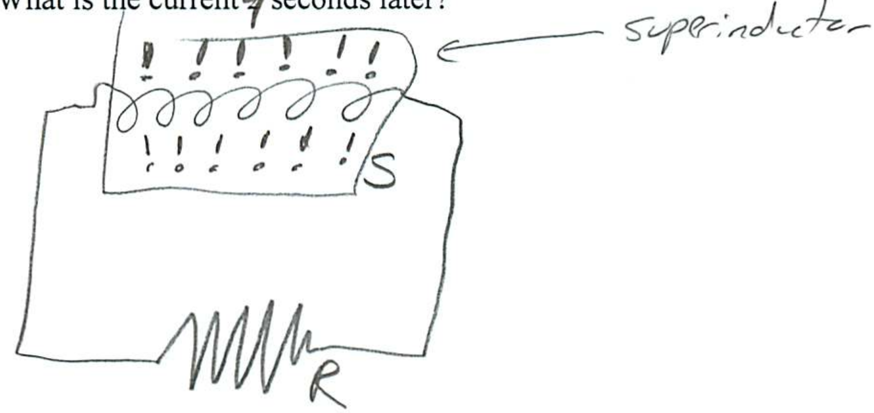
$$\boxed{B = 5.22\times 10^{-6}\,T}$$

Extra Credit problem

EC) (10 pts) Scientists at a top-secret government laboratory recently invented a new *super-inductor* that produces a counter-EMF ("back EMF") proportional to $d^3i(t)/dt^3$ that opposes the change in current. I.e., for a constant S,

$$EMF=-S\, d^3i(t)/dt^3.$$ 6400

Suppose each end of a *super-inductor* with $S=\sout{5}$ (in SI units) is connected to the opposite sides of a resistor, with resistance $R=100\Omega$. At time $t=0$, there is a current of 3 A through the circuit. What is the current 4 seconds later?



$$-iR - S\frac{d^3i}{dt^3} = 0$$

$$\frac{d^3i}{dt^3} = -\frac{R}{S}\, i(t)$$

Exponentials are their own derivative so try

$$i(t) = I_0 e^{-t/\tau}$$

$$-\frac{I_0}{\tau^3} e^{-t/\tau} = -I_0 \frac{R}{S} e^{-t/\tau}$$

$$\frac{1}{\tau^3} = \frac{R}{S}$$

$$\tau = \sqrt[3]{\frac{S}{R}} = \sqrt[3]{\frac{6400}{100}} = \sqrt[3]{64} = 4$$

$$i(t) = 3e^{-4/4} = \frac{3}{e} = \boxed{1.1A}$$