Name (clearly please!) ████████ ██████████

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## MIDTERM
## Version C
## Physics 1C, Prof. David Saltzberg
## November 7, 2019

Time: 90 minutes. Closed Notes. Closed Book. One 3"x5" sheet of notes allowed. Calculators (even graphing) are allowed. Show your work. *Please write your name on every page just in case.*

If a problem is confusing or ambiguous, notify the professor. Clarifications will be written on the blackboard. Check the board.

| Problem | Points |
|:-------:|:------:|
| 1 | /15 |
| 2 | /20 |
| 3 | /20 |
| 4 | /15 |
| 5 | /15 |
| 6 | /15 |
| ----- | ------ |
| TOTAL | /100 |

1) Short answers:

a) (5 pts) In class we performed a demonstration where a student pedaled a bicycle connected to a generator to power light bulbs in parallel. The reason he had to pedal harder as more bulbs were added was:

    1) due to increasing back-EMF (or "counter-EMF") of the generator
    (2) due to increasing back-torque (or "counter-torque") of the generator
    3) due to the presence of the Maxwell term $+\mu_0\varepsilon_0(d\Phi_E/dt)$ in the Ampère-Maxwell Law
    4) due to the presence of the conduction current term $\mu_0 I_C$ in the Ampère-Maxwell Law
    5) because aluminum is diamagnetic and like-poles (e.g., N and N) repel.

b) (5 pts) A block of material is placed near a bar magnet and is weakly attracted to it. When the bar magnet is removed, the block of material is found not to be magnetized. This is an example of:

    1) diamagnetism
    2) ferromagnetism
    (3) paramagnetism
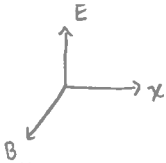    4) animal magnetism
    5) a magnetic monopole

c) (5 pts) A magnetic dipole $\mu$ is placed in a uniform magnetic field, $\mathbf{B}$. Draw a configuration where the magnitude of the torque on the dipole is maximal.



$$\tau = \mu \times B$$
$$= \mu B \sin\phi$$
$$\phi \to 90°$$
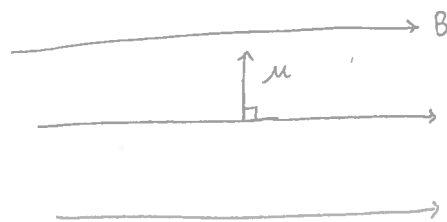
2) A sinusoidal electromagnetic wave with wavelength 2.0 m travels in vacuum in the +x direction with its electric field having an amplitude of 300 V/m that points along the y-axis. At x=0, t=0 the electric field is zero and decreasing in value (i.e., becoming more negative).

a) (10 pts) Write the equation for the magnetic field in terms of its amplitude, wavenumber $k$, and angular frequency $\omega$. [Hint: your answer should be a vector. It should have numerical values with units for $k$, $\omega$, and the field amplitude.]

E

B

$\lambda = 2.0\,m \qquad E_{max} = 300\frac{V}{m}$

$E(x,t) = \hat{j}\,E_{max}\cos(kx - \omega t)$

$B(x,t) = \hat{k}\,B_{max}\cos(kx - \omega t)$

$E_{max} = c\,B_{max}$

$B_{max} = \dfrac{E_{max}}{c} = \dfrac{300\,V/m}{8 \times 10^{8}\,m/s} = 10^{-6}\,T$

$\omega = ck \qquad k = \dfrac{2\pi}{\lambda}$

$\omega = c\,\dfrac{2\pi}{\lambda} \qquad \boxed{k = \dfrac{2\pi}{2} = \pi}$

$\omega = (3 \times 10^{8})\left(\dfrac{2\pi}{2}\right)$

$\boxed{\omega = 3\pi \times 10^{8}}$

$B(x,t) = \hat{k}\,10^{-6}\,T\,\cos\left(3.14\,x - (9.42 \times 10^{8})\,t\right)$

b) (10 pts) Suppose this wave is perfectly (absorbed) uniformly over an entire square-shaped sail with dimensions 2.0m × 2.0m.   The Poynting vector of the wave is parallel to the area vector of the sail.  What is the force on the sail in Newtons?  [Hint: this one may need your calculator.]

radiation Pressure
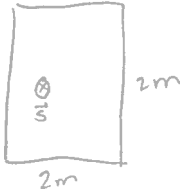
$$P_{rad} = \frac{2I}{c} \qquad\qquad I = \frac{E_{max}\, B_{max}}{2\mu_0}$$

$$P_{rad} = \frac{2\, E_{max} B_{max}}{2\mu_0 c} = \frac{e B^2}{\mu_0 e} = \frac{B^2}{\mu_0} = \frac{(10^{-6})^2}{\mu_0} = \frac{10^{-12}}{4\pi \times 10^{-7}}$$

$$[P_{rad}] = [P_a] = \frac{N}{m^2}$$
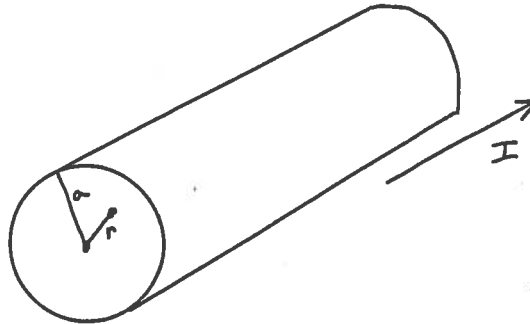
$$F = P_{rad} \cdot A = \frac{B^2}{\mu_0} \cdot A = \frac{10^{-5}}{4\pi} \times (4\, m^2) = \boxed{3.18 \times 10^{-6} \; N}$$

3) A long cylindrical wire of radius, $a$, carries a current density $J(r)$ along its axis that is proportional to the cube of the radius, $r$, that is:

$$J(r) = J_0(r/a)^3, r < a,$$
$$J(r) = 0, \text{ elsewhere}$$



a) (10 pts) Suppose $a=5.0$ m and the total current carried in the wire is $I_0=3.14$ A. What is $J_0$?
[Answer is a numerical value with units]
[extra space on next page if needed]

$$I_0 = \int J(r) \cdot dA$$

$$3.14 = \int_0^a J_0 (r/a)^3 (2\pi r) dr$$

$$1 = J_0 \frac{2}{a^3} \int_0^a r^4 dr$$

$$\frac{a^3}{2} = J_0 \left(\frac{1}{5}r^5\right)\Big|_0^a$$

$$\frac{a^3}{2} = J_0 \left(\frac{1}{5}a^5\right)$$

$$\frac{5}{2a^2} = J_0$$

$$J_0 = \frac{5}{2(5)^2} = \frac{1}{10} = \boxed{0.10}$$

b) (10 pts) Suppose the current is changed so that $J_0$=10 A/m². [Hint: this part does not depend on your answer for part a]. What is the value B at r=2.0m? [This is a numerical value with units.]

$$\int B \cdot d\ell = \mu_0 I_{encl}$$

$$I_{encl} = \int J_0 (r/a)^3 \cdot dA$$

$$B \cdot (2\pi r) = \mu_0 I_{encl}$$

$$= \int_0^2 J_0 \left(\frac{r^3}{a^3}\right) 2\pi r \, dr$$

$$B = \frac{\mu_0 I_{encl}}{2\pi r}$$

$$= J_0 \frac{2\pi}{a^3} \int_0^2 r^4 \, dr$$

$$= \frac{\mu_0 \cdot 64\pi \, (10)^{\overset{32}{\cancel{5}}}{\overset{1}{}}}{2\pi (2) \, 625/25}$$

$$= J_0 \frac{2\pi}{a^3} \left(\frac{1}{5} r^5\right)_0^2$$

$$= \frac{\mu_0 (32)}{125}$$

$$I_{enc} = J_0 \frac{64\pi}{625} = 10 \cdot \frac{64\pi}{625}$$

$$= \frac{(4\pi \times 10^{-7})(32)}{125}$$

$$\boxed{= 3.22 \times 10^{-7} \, T}$$

4) An AC wall socket in Europe provides a voltage that is sinusoidal in time with a frequency of 50 Hz and root-mean-square (R.M.S.) voltage of 240 V between its two conductors.

$f = 50\,Hz$ $\quad V_{RMS} = 240$

a) (5 pts) What is the amplitude of the voltage?

$$V_{RMS} = \frac{V}{\sqrt{2}}$$

$$\sqrt{2}\, V_{RMS} = V$$

$$\boxed{V = 340\ V}$$

b) (10 pts) Now you connect an inductor and resistor in series to this European AC wall socket, as shown below:
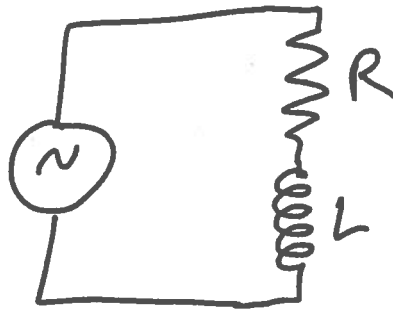


Where R=62.8 Ω and L= 100 mH.   The current is at its maximum positive value at *t=0*.  What is the RMS voltage across the inductor?  [Hint: you may need your calculator a little.]

[extra space on next page]

$R = 62.8\,\Omega$ $\quad L = 100\,mH$ $\quad V_L = ?$ $\quad \omega = 2\pi f$ $\quad f = 50\,Hz$

$$V_L = I\omega L \qquad\qquad V_{RMS} = IZ$$

$$V_{RMS} = I\sqrt{R^2 + (\omega L)^2}$$

$$V_L = \frac{V_{RMS}}{\sqrt{R^2+(\omega L)^2}}\,\omega L \qquad \frac{V_{RMS}}{\sqrt{R^2+(\omega L)^2}} = I$$

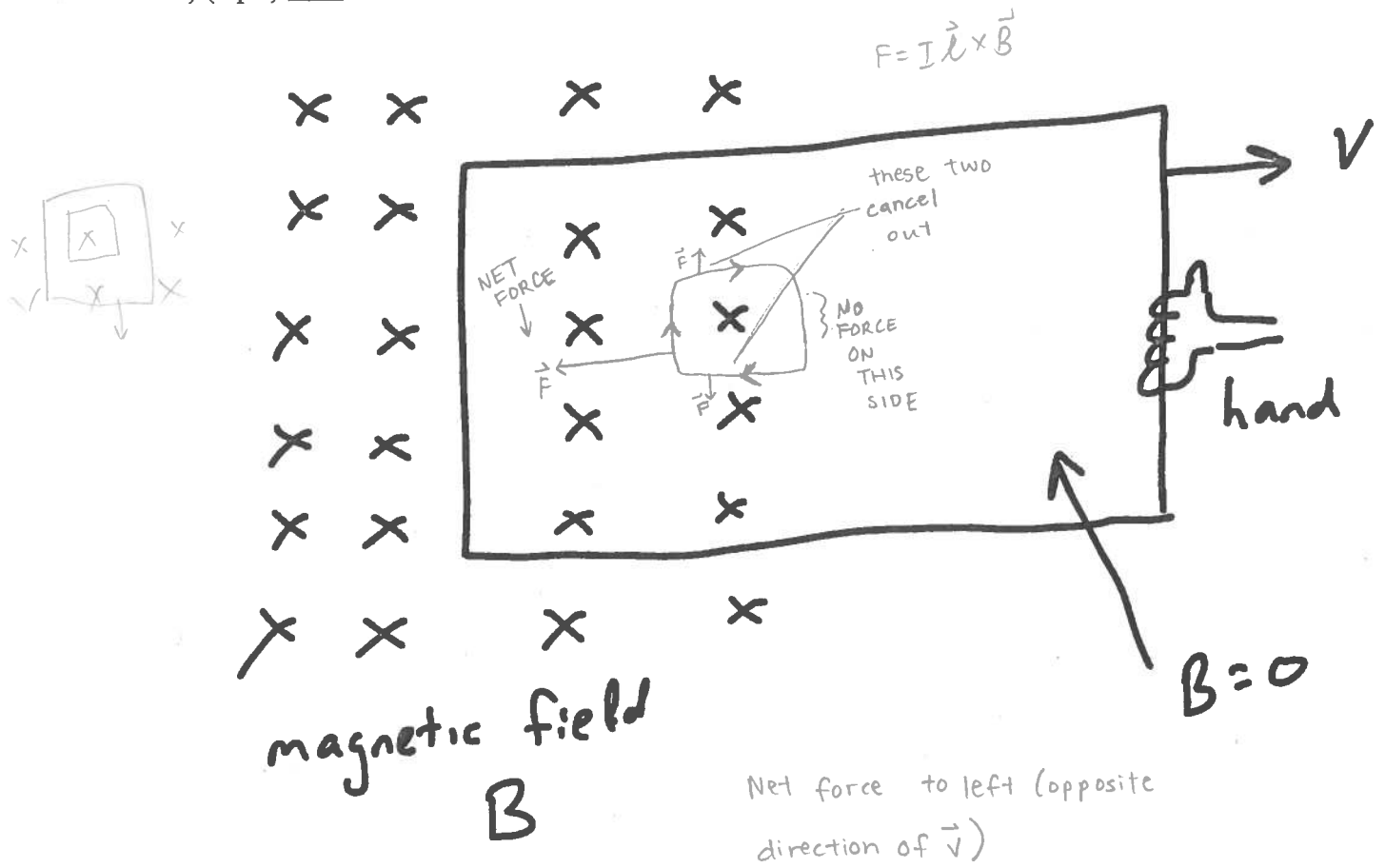$$V_L = \frac{(240)}{\sqrt{(62.8)^2+(2\pi(50)(100))^2}}(2\pi(50))(100)$$

$$\boxed{= 240\ V}$$

[extra space]

5) You are pulling a rectangular plate of copper out of a region with uniform magnetic field B with a constant speed v.
a) (10 pts) On the figure below draw one of the eddy current loops with non-zero current. Show the direction of current flow.
b) (5 pts) <u>Also</u> indicate the direction of the net force on the copper due to eddy currents



$$F = I \vec{\ell} \times \vec{B}$$

these two cancel out

NET FORCE

$\vec{F}$↑

$\vec{F}$

$\vec{P}$

NO FORCE ON THIS SIDE

V

hand

B = 0

magnetic field B

Net force to left (opposite direction of $\vec{v}$)

6) (15 pts) Scientists at a top-secret government laboratory recently invented a *super-inductor*. The voltage drop across a super-inductor is given by $V(t) = S\frac{d^3}{dt^3}I(t)$, where $S$ is a constant known as the *super-inductance*, and $I(t)$ and $V(t)$ are the current and voltage. In the circuit below, a super-inductor with super-inductance $S=800,000$ (in the appropriate S.I. units) is in series with a resistor with resistance $R=100\ \Omega$ and the initial current at time $t=0$ is $I_0$.
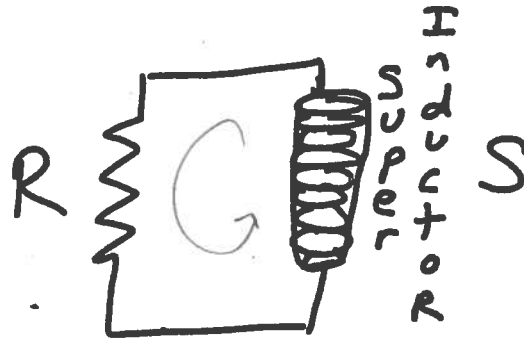


At what time does the current become $(1/e)I_0$?
   [extra page follows if needed]

$V(t) = S\frac{d^3}{dt^3}I(t)$    $S = 8\times10^5\ H$    $R = 100\ \Omega$    $t = ?$ when    $I = \frac{1}{e}I_0$

$I(t) = I_0 e^{-R/L\, t}$

$\frac{1}{e}I_0 = I_0 e^{-R/L\, t}$

$\frac{1}{e} = e^{-R/L\, t}$

$\ln\left(\frac{1}{e}\right) = -\frac{R}{L}t$

$-\frac{L}{R}\left(\ln\left(\frac{1}{e}\right)\right)^{=-1} = t$

$t = +\frac{L}{R} : \frac{2000}{100} = \boxed{20\ s}$

$\mathcal{E} = -L\frac{dI}{dt}$

$S\frac{d^3}{dt^3}I(t) = -L\frac{dI}{dt}$

$S\frac{d^3}{dt^3}\left(I_0 e^{-R/L\, t}\right) = -L\frac{d}{dt}\left(I_0 e^{-R/L\, t}\right)$

$S\left(\left(-\frac{R}{L}\right)^3 I_0 e^{-R/L\, t}\right) = -L\left(-\frac{R}{L}I_0 e^{-R/L\, t}\right)$

@ $t=0$

$S\left(-\frac{R}{L}\right)^3 I_0 = +L\left(+\frac{R}{L}I_0\right)$

$L = -S\left(\frac{R^2}{L^2}\right)$

$L^3 = -S\left(R^2\right)$

$L = -\sqrt[3]{SR^2}$

$L = -\sqrt[3]{(800,000)(100)^2}$

$L = -2000\ H$