1) Short answer

A) (3 pts) The Sun is a nuclear reactor that produces 4×10^{26} Watts of power as electromagnetic radiation. In one hour, how much does the mass of the Sun change to produce this power?

$E = mc^2$

in one second

$$\frac{4\times10^{26}\,J}{(3\times10^{8}\,m/s)^2} = m$$

$m = 4.4\times10^{9}\ kg/s$

$\times\ 60\ s/min$

$\times\ 60\ min/hour$

$$\boxed{m = 1.6\times10^{13}\ kg \text{ per hour}}$$

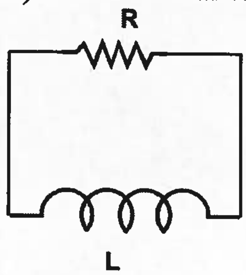
(about 16 billion tons per hour!)

B) (3 pts) In class we performed a demo where the image of a lightbulb was produced on the wall using a converging lens. When the top half of the lens was covered with opaque paper, what happened to the image?

It became dimmer

1) Short answer (continued)

C) (3 pts) At an intial time, $t=0$, the circuit below conducts I=3A. At t=2 seconds, the current reaches $1/e$ (which is 36.8%) of its initial value. When will the current reach $(1/e)^{3.5}$ of its initial value?



$I(t) = I_0 e^{-t/\tau}$

one factor of e is 2 secons

$\Rightarrow$ 3.5 factors of e = $\boxed{7 \text{ seconds}}$

D) (4 pts) You are given a piece of diamagnetic material. What will happen when…

i) …it is brought near a magnet?

It is repelled

ii) …instead, it is brought near unmagnetized iron?

No magnetic force

1) Short answer (continued)

E) (4 pts) A telescope operates on the top of Muana Kea in Hawaii. It has a 4 meter diameter primary mirror with a 25m focal length. It makes important observations using red light. For simplicity, you can assume the astronomers have removed atmospheric effects. What is its approximate angular resolution of this telescope? (You only need an answer here that is correct to within about a factor of two.)

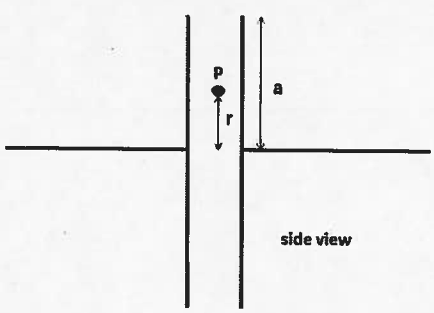
$\lambda_{red} \sim 600\,nm$

$D = 4m$

$$\delta\theta \simeq \frac{\lambda}{D} = \frac{600\times10^{-9}}{4} = \boxed{1.5\times10^{-7} \text{ radians}}$$

(also 8.6×10^{-6} degrees)

F) (3 pts) A parallel-plate capacitor initially has no charge on its circular plates with radius of a=4cm. At $t=0$ a steady current of 8 milliAmpere starts flowing onto one plate and off of the other. At a point P inside the capacitor at a distance $a/2$ from the center, what is the magnetic field at $t=3$ seconds?



i_D scales with area $\propto \left(\frac{r}{a}\right)^2$

$$i_D = \left(\frac{r}{a}\right)^2 (0.008)$$

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{(4\pi\times10^{-7})\left(\frac{1}{2}\right)^2(0.008)}{2\pi(0.02)}$$

$$= 100\times10^{-7} \times \frac{1}{4} \times 0.008$$

$$= \boxed{2\times10^{-8}\,T}$$

2) Work out problem

(15 pts) You are shipwrecked on a uncharted desert island near the equator. You want to build an AC generator to power your iPhone through its charger and call for help. You have no magnets but decide to make an electrical generator using the Earth's magnetic field. You need to be able to provide ~~an~~ an rms EMF of 120V (RMS). You decide you can make a square loop with 2000 turns of wire before you get tired of winding. The Earth's magnetic field is 5×10^{-5} T at your island's location and is pointing from South to North. You can turn the coil as fast as 10 revolutions per minute.

A) You want the axis of rotation to be parallel to the ground for ease-of-use. What direction is the axis of rotation aligned with?

$\perp$ to NS, parallel to ground

$\Rightarrow$ $\boxed{\text{East-West direction}}$

B) What area must the coil have?

$V_{rms} = 120V \Rightarrow V_0 = 120\times\sqrt{2} = 170\ V$

$\omega = 10 \frac{rev}{min} \times \frac{min}{60\ s} \cdot \frac{2\pi\ rad}{rev} = 1.05\ rad/sec$

$|\mathcal{E}| = \left| -\frac{d\Phi_B}{dt} \right| = \underbrace{\omega BAN}_{\uparrow\ \text{amplitude}} \sin \omega t$

$V_0 = \omega BAN$

$(170) = (1.05)(5\times10^{-5}) A\,(2000)$

$\boxed{A = 1.6\times10^{3}\,m^2}$

$S = \sqrt{A}$

side = 40 meters!

2) workout problem (continued)

C) Your generator provides 5W to the charger when being turned. Does this generator produce a counter-torque or a counter-EMF? Calculate its maximum value.

2 possible methods!

Generators produce counter-torque.

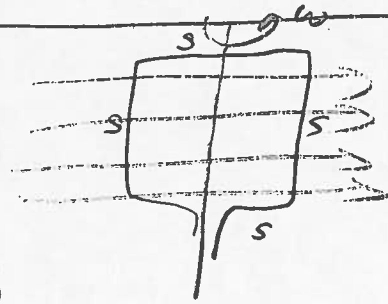
$$P = I_{rms} V_{rms} = \frac{1}{2} I_{max} V_{max}$$

$$5 = \left(\frac{1}{2}\right)(I_{max})(170)$$

$$I_{max} = 0.059\,A$$

method I

$$\tau_{max} = r F = \left(\frac{s}{2}\right) F$$



two sides produce force

$$F_{max} = I s B \quad \times 2000$$

$$\tau_{counter} = \left(\frac{40m}{2}\right)(2)(0.059)(40m)(5\times10^{-5}\,T)(2000)$$

$$\boxed{\tau_{counter} = 9.4\ N\text{-}m}$$

or method II

$$\vec{\tau} = \vec{\mu} \times \vec{B} \qquad |\mu| = IAN$$

$$\tau_{max} = |\mu||B|$$

$$= IANB = (0.059)(1.6\times10^{3})(2000)(5\times10^{-5})$$

$$= \boxed{9.4\ N\text{-}m}$$

3) Work-out problem

(10 pts) A long straight, solid cylinder of radius a is oriented with its axis along the positive z axis. It carries a current density J(r) which depends on r, the distance from the z axis:

$$J(r) = J_0 (r/a)^2 \quad \text{(for } r<a\text{) in the positive z direction.}$$

where the total current in the cylinder is 4.0 Amperes and a=3.0 meters.

A) What is the magnetic field at r = 6.0 m? Draw a picture showing its direction.

$$\oint B \cdot dl = \mu_0 I_{encl}$$

$$(B)(2\pi r) = \mu_0 I_{encl}$$

6m > 3m so all current is enclosed

$$B = \frac{(4\pi \times 10^{-7})(4)}{(2\pi)(6)}$$

$$= 1.33 \times 10^{-7} \text{ T}$$

$$\boxed{B = 1.3 \times 10^{-7} \text{ T}}$$

(Problem 3 continued)
B) What is the magnetic field at r = 2.0 m?

First find Jo

$$\int_A j\,dA = 4A$$

$$J_0 \int_0^{2\pi} d\theta \int_0^3 r\,dr \left(\frac{r}{a}\right)^2 = 4$$

$$J_0 (2\pi)\left(\frac{1}{4}\right)\left(\frac{r^4}{3^2}\right)\Bigg|_{r=0}^{r=3} = 4$$

$$J_0 (2\pi)\left(\frac{1}{4}\right)(9) = 4$$

$$J_0 = \frac{16}{18\pi} = \frac{8}{9\pi}$$

Then

$$(B)(2\pi r) = \mu_0 J_0 \int_0^{2\pi} d\theta \int_0^2 r\,dr \left(\frac{r}{a}\right)^2$$

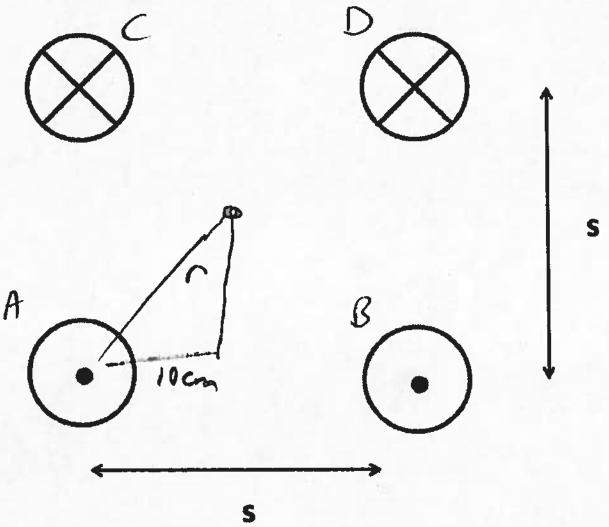
$$(B)(\cancel{2\pi}(2)) = \mu_0\left(\frac{8}{9\pi}\right)(\cancel{2\pi})\left(\frac{1}{4}\right)\left[r^4\right]_0^2\left(\frac{1}{3^2}\right)$$

$$B = \frac{(4\pi \times 10^{-7})}{2}\left(\frac{8}{9\pi}\right)\left(\frac{1}{4}\right) 2^4 \frac{1}{9}$$

$$\boxed{B = 7.9 \times 10^{-8}\ T}$$

4) Work-out problem

(10 pts) Four long parallel wires each carry 100A currents through the page as shown below. The wires are arranged so that each passes through the corner of a square with sides of length s=20cm. Calculate the magnetic field at the center of the square.



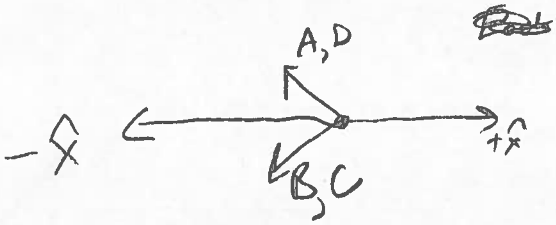
$r = 10\text{cm} \times \sqrt{2} = 14.1 \text{ cm}$

$|B|$ at center from each wire $= \dfrac{\mu_0 I}{2\pi r} = \dfrac{(4\pi\times10^{-7})(100)}{(2\pi)(0.141\text{ m})}$

$= 1.41 \times 10^{-4}\text{ T}$

Add vectorially

~~Both~~ Net $\vec{B}$ points along $-\hat{x}$ direction



$$|B| = 4\,|B_{each}| \cos 45° = (4)(1.41\times10^{-4})\left(\frac{1}{1.41}\right) = \boxed{4\times10^{-4}\text{ T}}$$

A+B+C+D

each

all projected 45°

$\boxed{\text{points in } -\hat{x} \text{ direction}}$

5) Work-out problem

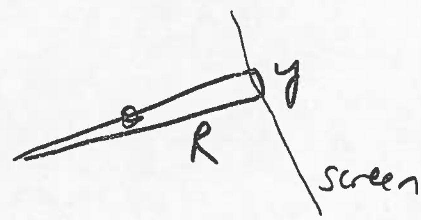
(12 pts) Two thin slits spaced 0.~~400~~ 0,300 mm apart are placed a distance of 2.0 m from a screen. They are illuminated with a laser of wavelength ~~[illegible]~~ 600 nm.

A) What is the distance between the third and fourth dark interference fringe?

$d \sin\theta = (m + 1/2)\lambda$

$d\,\theta \simeq (m+1/2)\lambda$

$d\left(\frac{y}{R}\right) \simeq (m+1/2)\lambda$



$\frac{y}{R} = \tan\theta \simeq \theta$

$\Delta m = 1$

$d\,\frac{\Delta y}{R} = \lambda$

$\Delta y = \frac{\lambda R}{d} = \frac{(600\times10^{-9})(2.0)}{(300\times10^{-6})} = 4\times10^{-3}\ m$

$= \boxed{4\ mm}$

B) If the slits are now 0.1 mm wide, how many interference ~~maxima~~ minima fit inside the ~~first~~ central diffraction peak?

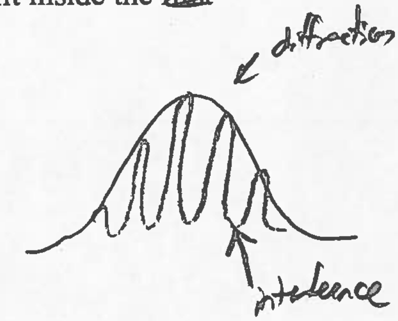
$a = 1\times10^{-4}\ m$

1st diffraction minima at

$a \sin\theta = \pm m\lambda \quad m=1$

$\theta = \sin^{-1}\left(\pm\frac{600\times10^{-9}}{1\times10^{-4}}\right)$

$\theta = 0.0060$ radians



interference minima at

$d\sin\theta = \pm(m+1/2)\lambda \quad (m = 0, \pm1, \pm2, \ldots$

$\frac{(300\times10^{-6})(0.0060)}{(600\times10^{-9})} > |m + 1/2|$

$3 > |m+1/2|$

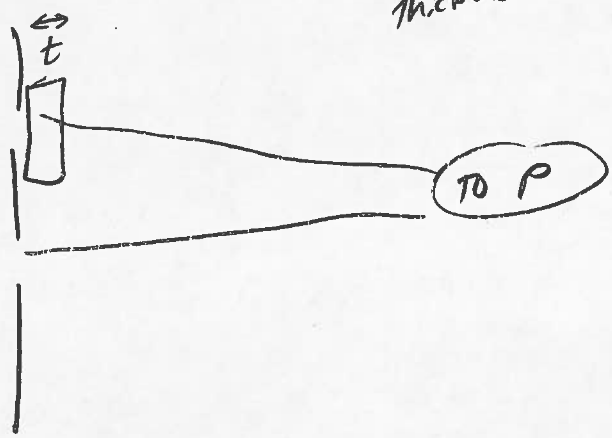
$m = 0, 1, -1, 2, -2, -3 \Rightarrow \boxed{6\ minima}$

(Problem 5 continued)

C) Normally the interference pattern has a bright line at its center. If you cover one of the slits with [redacted]f glass (n=[redacted]), what is the minimum thickness to change the bright line to a dark line?

1.4

thickness = t



Pathlengths are the same but time delay in glass. Normally constructive so if extra delay is 180° then destructive.

$$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{600\times10^{-9}} = 5\times10^{14}$$

$\Rightarrow$ period $= 2.0\times10^{-15}$ s

$\Rightarrow$ need extra delay of $\frac{\text{period}}{2} = 1.0\times10^{-15}$ s

$$\text{delay} = \frac{t}{(c/n)} - \frac{t}{c} = \frac{t}{c}(n-1) = 1.0\times10^{-15}\ \text{s}$$

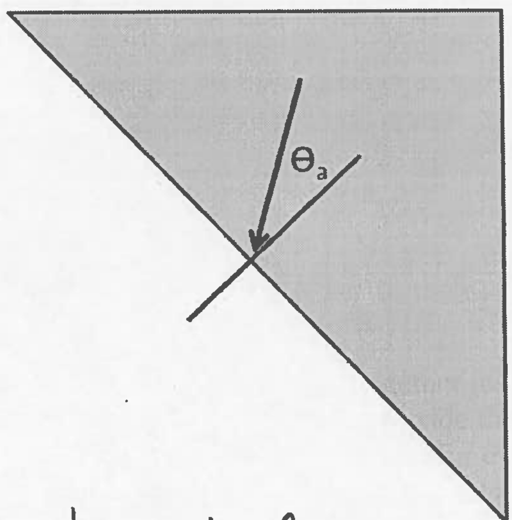
$$t = \frac{(1.0\times10^{-15}\ \text{s})(3\times10^{8}\ \text{m/s})}{0.4}$$

$$t = 7.5\times10^{-7}\ \text{m}$$

$$\boxed{t = 750\ \text{nm}}$$

6) Work-out problem

(8 pts) Rays of unpolarized light are traveling together inside a birefringent crystal in a direction perpendicular to the optical axis. Outside the crystal is just vacuum. The index of refraction for ordinary waves is n_o=1.5 and for extraordinary waves is n_E=1.4. The light strikes the far side of the crystal at an angle relative to the normal of Θ_a=44°. Sketch what happens to the rays of light. Label the angles of reflectance and/or transmission with their numerical values. If the rays end up travelling along different paths after striking the far side of the crystal, label which is ordinary and which is extraordinary.



$\perp$ propagation to optical axis means one polarization is "ordinary" and the other is "extraordinary"

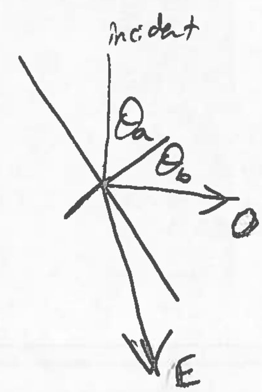
θ_{crit} for 1.4 $= \sin^{-1}\left(\frac{1}{1.4}\right) = 45.6°$ (extra ordin)

θ_{crit} for 1.5 $= \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°$ (ordinary)

$\theta_a = 44° \Rightarrow \theta_b = \theta_{refl} = 44°$ (but only ordinary wave totally internally reflects)

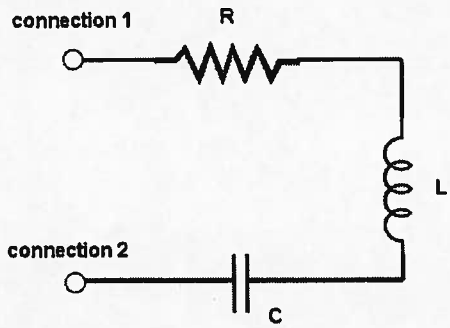
(1) $\sin\theta_{transmit} = 1.4 \sin\theta_a$

$\theta_{transmit} = \sin^{-1}(1.4 \sin(44°)) = 77°$



7) Work-out problem

(12 pts) Suppose you hook up the following series circuit to the two prongs of an AC plug and insert it into your wall AC socket.



$\omega = 2\pi\nu = 377 \frac{rad}{s}$

The wall voltage is 120 V_{RMS}, with a frequency of 60 Hz. The resistance is 10Ω, the inductance is [redacted]H 80mH and the capacitance is [redacted] 133μF

A) The circuit in your house is rated with a value stating the largest RMS current it can sustain before either catching on fire or throwing a circuit breaker. What RMS current must your house circuit be able to provide in order to drive this circuit successfully?

$R = 10\,\Omega$

$X_L = \omega L = (377)(0.08) = 30\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{(377)(133\mu F)} = 20\,\Omega$

$Z = \sqrt{10^2 + (30-20)^2} = \sqrt{100+100} = 14.1\,\Omega$

$V_{RMS} = I_{RMS} Z$

$I_{rms} = \frac{V_{RMS}}{Z} = \frac{120\,V}{14.1\,\Omega} = \boxed{8.5A}$

No problem! for typical home circuit

(problem 7 continued)

B) You measure the voltage V(t) between "connection 1" and "connection 2" and the currents. When the current into "connection 1" is at a maximum (which we call zero degrees) what is the phase of voltage?

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{30-20}{10} = 1$$

$$\phi = 45^\circ$$

$$V(t) = V_0 \cos(\omega t + 45^\circ)$$

Voltage leads the current by 45°

"leads" as expected for a circuit that is more inductive (30 Ω) than capacitive (20 Ω)

C) What value of the inductance would make the entire circuit purely resistive? (Do not change the resistor or capacitor.)

$$\omega L = \frac{1}{\omega C}$$

$$L = \frac{1}{\omega^2 C} = \frac{1}{(377)^2(133\,\mu F)}$$

$$\boxed{L = 53\,mH}$$

8) Work-out problem

(13 pts) Two lamps are attached to the ground. Each one flashes briefly. Stanley is standing on the ground in inertial frame S. He measures the distance between the lamps to be 100 meters. In his frame, the lights flash at the same time. Mavis flies by on a spaceship in inertial frame S' in a straight line that passes over both lamps. Her speed is 0.6c.

A) In Mavis's frame, how far apart are the lamps?

$$\gamma = \frac{1}{\sqrt{1-(6/10)^2}} = \frac{1}{\sqrt{1-\frac{36}{100}}} = \frac{1}{\sqrt{64/100}} = \frac{10}{8} = 5/4$$

$$l = \frac{l_0}{\gamma} = \frac{100}{5/4} = \boxed{80\text{m}}$$

B) Mavis shines a red light (~~652~~ 600 nm) at Stanley when she is approaching him. What wavelength does he see?

approaching ⇒ blueshifted

$$\nu_0 = \frac{c}{\lambda_0} = \frac{3\times10^8 \text{ m/s}}{600\times10^{-9}\text{m}}$$

$$\nu_0 = 5.0\times10^{14}\text{ Hz}$$

$$\nu = \sqrt{\frac{c+0.6c}{c-0.6c}}\,\nu_0$$

$$\nu = \sqrt{\frac{1.6}{0.4}}\,(5.0\times10^{14})$$

$$\nu = \sqrt{4}\,(5.0\times10^{+14})$$

$$\nu = 1\times10^{+15}\text{ Hz}$$

$$\lambda = c/\nu = \frac{3\times10^8}{1\times10^{15}} = 3\times10^{-7} = \boxed{300\text{nm}}$$

(Problem 8 continued)
C) If Mavis defines $t=0$ in her frame as the time when the first lamp flashes, when does the second lamp flash? (in her frame)

Same time in S $\Rightarrow \Delta t = 0$

in S, $\Delta x = 100\text{m}$

$$\Delta t' = \gamma\left(\Delta t - \frac{v\,\Delta x}{c^2}\right)$$

$$= \frac{5}{4}\left(0 - \frac{(0.6)(100)}{(3\times10^8)}\right)$$

$$= -\frac{5}{4}\cdot\frac{6}{10}\cdot\frac{100}{3\times10^8}$$

$$= \boxed{-2.5\times10^{-7}\,\text{s}} \quad (= -250\,\text{nsec})$$

Note, mavis is <u>not</u> at second light when it flashes

$\Rightarrow \Delta x' \neq 0$ so cannot use simple proper time method.

Extra Credit (continues from Problem 8)

We have seen that time intervals and distance intervals are measured differently by different inertial observers. However, all observers in all inertial reference frames, regardless of their speeds will measure the same value for a quantity called "*the spacetime interval*" between two events. The spacetime interval, *I*, is defined as:

$$I = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2$$

This is an example of an "invariant" quantity and will be equal in all inertial frames, regardless of their relative speeds.

A) Show that the value of *I* between the light flashing events is the same in both intertial frames of problem 8.

$x_1 = 0$ $t_1 = 0$

$x_2 = 100$ $t_2 = 0$

$x_1' = 0$ $t_1' = 0$

$t_2' = -2.5\times10^{-7}$

$x_2' = \gamma(x_2 - vt_2)$
$= \frac{5}{4}(100 - (0.6c)(0))$
$= 125\,m$

careful, not a length!

$I \text{ (in } S) = 100^2 + 0 + 0 - 0 = 100^2 = 10000\,m$

$I \text{ (in } S') = 125^2 - \left[(3\times10^8)(2.5\times10^{-7}\,s)\right]^2$
$= 125^2 - 75^2$
$= 10000\,m^2$

B) Use the invariance of the spacetime interval to explain in words why if two events could have caused each other in one frame, then they *could* have caused each other in any other frame. Likewise, if they *could not* have caused each other in one frame, they could not have caused each other in any other frame. (Thus the "causality" of events is also agreed upon in all inertial reference frames, even if though distances and times between them may not.)

$I < 0$ if light could have travelled the distance $\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$ in time Δt $\Rightarrow$ hence could be causal

$I > 0$ if light could not have travelled distance $\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$ in time Δt $\Rightarrow$ hence not causal

---

Final note: better to measure time as a distance (ct)?