Name:

Studen

Discus:

Physics 1C, Spring 2019, Lecture 2
Midterm 2

Time allotted: 50 minutes
No calculators or notes allowed.
No phones out during the exam.
All work must be your own.
Partial credit will be awarded for correct work.

Problem 1	18	/18
Problem 2	22	/24
Problem 3	18	/18
Total	58	/60

Inspirational quote:
The time of day or the density of the light / Adhering to the face keeps it /
Lively and intact in a recurring wave / Of arrival. The soul establishes itself.

—John Ashbery, *Self-Portrait in a Convex Mirror (1974)*

Potentially useful formulas:

Vacuum magnetic permeability

$$\mu_0 = 4\pi \times 10^{-7} \, \mathrm{H\,m^{-1}} \tag{1}$$

Vacuum speed of light

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \, \mathrm{m\,s^{-1}} \tag{2}$$

The time averages

$$\langle \sin^2 \omega t \rangle_T = \langle \cos^2 \omega t \rangle_T = \frac{1}{2} \tag{3}$$

and

$$\langle \sin \omega t \cos \omega t \rangle_T = 0. \tag{4}$$

Radiation pressure (perfect reflection)

$$\mathcal{P}_{\text{ref}} = \frac{2I}{c} = \frac{2\langle |S| \rangle_T}{c}. \tag{5}$$

Problem 1. In the lab, you encounter a solenoid with unknown inductance. You measure the solenoid with an ohmmeter and find it has internal resistance R. To determine its inductance, you connect the solenoid in series with a known capacitor C and an AC signal generator (AC voltage source) set to frequency ω.

(a) If the current amplitude is I_0, find the voltage amplitude $|V_C|$ across the capacitor.

(b) With the same current, find the total voltage amplitude $|V_{tot}|$ across both the solenoid and capacitor in terms of the unknown inductance L and the known circuit parameters.

(c) Suppose you don't know the current amplitude, but you measure the two voltage amplitudes in (a) and (b). Find the inductance L of the solenoid using these measurements and the known parameters.

$L \checkmark$ has internal R

a) $\quad |V_C| = I_0 \cdot |Z_C|$

$|V_C| = I_0 \cdot \dfrac{1}{\omega C}$

$\boxed{|V_C| = \dfrac{I_0}{\omega C}}$ 6
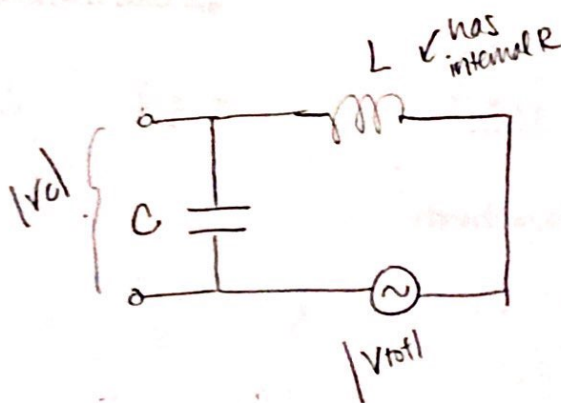
$Z_C = \dfrac{-i}{\omega C}$

$|Z_C| = \dfrac{1}{\omega C}$

b) $\quad |V_{tot}| = I_0 \cdot |Z_{RLC}|$

$|Z_{RLC}| = \sqrt{R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2}$

$\boxed{|V_{tot}| = I_0 \sqrt{R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2}}$ 6

c) $\quad \dfrac{|V_C|}{|V_{tot}|} = \dfrac{|Z_C|}{|Z_{RLC}|}$

$\dfrac{|V_C|}{|V_{tot}|} = \dfrac{\tfrac{1}{\omega C}}{\sqrt{R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2}} = \dfrac{1}{\omega C \sqrt{R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2}}$

$\omega L - \dfrac{1}{\omega C} = \sqrt{\left(\dfrac{|V_{tot}|}{|V_C|}\right)^2 - R^2}$

$|V_C| \, \omega C \sqrt{R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2} = |V_{tot}|$

$\sqrt{R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2} = \dfrac{|V_{tot}|}{|V_C|}$

$R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2 = \dfrac{|V_{tot}|^2}{|V_C|^2}$

$\left(\omega L - \tfrac{1}{\omega C}\right)^2 = \dfrac{|V_{tot}|^2}{|V_C|^2} - R^2$

$\omega L = \sqrt{\left(\dfrac{|V_{tot}|}{|V_C|}\right)^2 - R^2} + \dfrac{1}{\omega C}$

$L = \dfrac{\sqrt{\left(\dfrac{|V_{tot}|}{|V_C|}\right)^2 - R^2} + \dfrac{1}{\omega C}}{\omega}$ 6

$\boxed{L = \dfrac{1}{\omega} \sqrt{\left(\dfrac{|V_{tot}|}{|V_C|}\right)^2 - R^2} + \dfrac{1}{\omega^2 C}}$

**Problem 2.** The magnetic field of a monochromatic, plane EM wave in vacuum is

$$\mathbf{B}_{inc}(x, t) = B_0 \,\hat{y} \cos(kx - \omega t) \tag{1}$$

(a) Find the electric field $\mathbf{E}_{inc}$ of this wave.

(b) Find the Poynting vector $\mathbf{S}_{inc}$ for this wave.

(c) Suppose this wave is totally reflected by a thin sheet of a perfect conductor occupying the yz plane at $x = 0$. What is the force per unit area on this sheet due to the EM field?

(d) When the wave reflects from the conductor at $x = 0$, a reflected plane wave traveling opposite the direction of the incident wave is formed. The total electric field $\mathbf{E}_{tot}$ is a superposition of the incident and reflected fields, $\mathbf{E}_{tot} = \mathbf{E}_{inc} + \mathbf{E}_{ref}$. Take for granted that the total electric field parallel to the surface of a perfect conductor is zero, and find the reflected field $\mathbf{E}_{ref}$.

a) $\hat{k} = \hat{x}$
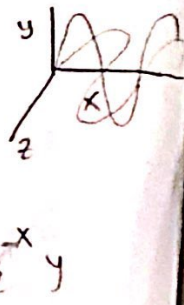
$\dfrac{\hat{k} \times \vec{E_0}}{c} = \vec{B_0}$

direction of $\vec{E_0}$ is

$\hat{x} \times ? = \hat{y}$

$\left(-\hat{z} \right)$

$\dfrac{|E_0|}{c} = |B_0|$

$|E_0| = c|B_0|$

magnitude of $\vec{E_0}$ is cB_0

$E_{inc}(x,t) = -cB_0 \,\hat{z} \cos(kx - \omega t)$ **6**

b) $\vec{S}_{inc} = \dfrac{1}{\mu_0} \vec{E}_{inc} \times \vec{B}_{inc}$

$\vec{S}_{inc} = \dfrac{1}{\mu_0} \left(-cB_0\cos(kx-\omega t) \right)\left(B_0 \cos(kx-\omega t) \right)\left(\hat{z} \times \hat{y} \right)$

$\downarrow$

$-\hat{x}$

$\vec{S}_{inc} = \dfrac{1}{\mu_0} cB_0^2 \cos^2(kx-\omega t) \,\hat{x}$ **6**

d)

$E_{tot} = E_{inc} + E_{ref}$

$0 = -cB_0\hat{z}\cos(kx-\omega t) + E_{ref}$

$E_{ref}(x,t) = cB_0\hat{z}\cos(kx+\omega t)$

4

c) $P = \dfrac{2I}{c}$

pressure $\dfrac{F}{A}$ ↗   total reflection ↓

$P = \dfrac{2\langle|\vec{S}|\rangle_T}{c}$

$\langle|\vec{S}|\rangle_T = \dfrac{1}{\mu_0} cB_0^2 \cdot \dfrac{1}{2} = \dfrac{1}{2}\dfrac{cB_0^2}{\mu_0}$

$P = \dfrac{2 \cdot \frac{1}{2}\frac{cB_0^2}{\mu_0}}{c} = \dfrac{B_0^2}{\mu_0}$ **6**

Problem 3. Two monochromatic light waves of different wavelengths λ_1 and λ_2 in vacuum are both incident at angle θ_0 on rectangular prism of Material X. Material X is dispersive, and its index of refraction depends on the vacuum wavelength λ as

$$n(\lambda) = \frac{3}{2} + \frac{X}{\lambda^2}, \qquad (2)$$

with $X > 0$ an *unknown* positive constant. Because n depends on λ, the two waves are refracted at different angles θ_1 and θ_2 in Material X, with $\theta_2 > \theta_1$.
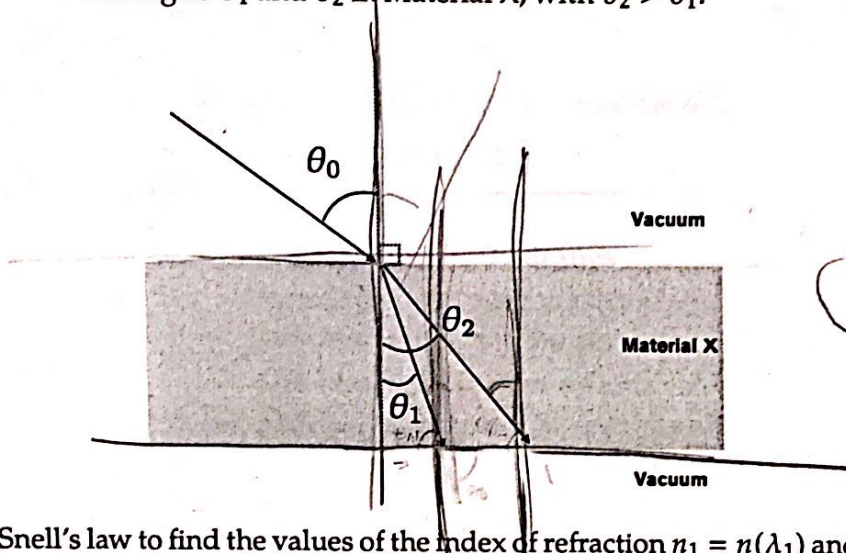
(a) Use Snell's law to find the values of the index of refraction $n_1 = n(\lambda_1)$ and $n_2 = n(\lambda_2)$ in terms of the incident and refracted angles.

For the two wavelengths, λ_1 and λ_2:

(b) Which wavelength is larger?

(c) Which wavelength of light travels faster in Material X?

(d) In the figure above, which of the wavelengths, if any, will undergo total internal reflection at the bottom surface of the prism?

Both could potentially total undergo internal refraction, as long as $n_1 \geq 1$ $n_2 \geq 1$

$\frac{3}{2} + \frac{X}{\lambda_1} \geq 1$

$\frac{X}{\lambda_1^2} = -\frac{1}{2}$

$\lambda_1^2 = -2X$

$\cancel{\lambda_1 = \sqrt{-2X}}$

But, None of them will undergo T.I.R. because this inequality proves it false.

a) $n_A \sin\theta_A = n_B \sin\theta_B$

$\sin\theta_0 = n_1 \sin\theta_1$

$\boxed{n_1 = \dfrac{\sin\theta_0}{\sin\theta_1}}$ $\dfrac{\sin 30}{\sin 30} = 1$

$\sin\theta_0 = n_2 \sin\theta_2$ ✓

$\boxed{n_2 = \dfrac{\sin\theta_0}{\sin\theta_2}}$ $\dfrac{\sin 30}{\sin 60}$ $\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{\sqrt{3}}{3}$ $\dfrac{\sqrt{3}}{3} < 1$

b) $n_2 < n_1$

$n(\lambda_2) < n(\lambda_1)$

$\cancel{\frac{3}{2}} + \dfrac{X}{\lambda_2^2} < \cancel{\frac{3}{2}} + \dfrac{X}{\lambda_1^2}$

$\dfrac{X}{\lambda_2^2} < \dfrac{X}{\lambda_1^2}$ $\lambda_1^2 < \lambda_2^2$

$\boxed{\lambda_1 < \lambda_2}$

second one is larger

c) $n = \dfrac{c}{v}$ $v = \lambda f$ w is constant, f is constant

$2\pi f = w$ $f = \dfrac{w}{2\pi}$

$\lambda_1 < \lambda_2$

$\lambda_1 f < \lambda_2 f$ ✓

$\boxed{v_1 < v_2}$ Second one is faster

d) T.I.R. $\sin\theta_A = \dfrac{n_B}{n_A} \to$ vacuum

$\sin\theta_{A1} = \dfrac{1}{n_1}$ $\sin\theta_{A2} = \dfrac{1}{n_2}$

18/18