Name: D.B.

Student ID #:

Physics 1C, Spring 2019, Lecture 2
Final Exam

Time allotted: 3 hours

No calculators or notes allowed.

No phones out during the exam.

All work must be your own.

Partial credit will be awarded for correct work.

Problem 1	/20
Problem 2	/20
Problem 3	/22
Problem 4	/18
Problem 5	/12
Problem 6	/28
Problem 7	/12
Problem 8	/12
Problem 9	/16
Total	/160

Inspirational quote:

The important thing is not to stop questioning. Curiosity has its own reason for existence. One cannot help but be in awe when he contemplates the mysteries of eternity, of life, of the marvelous structure of reality. It is enough if one tries merely to comprehend a little of this mystery each day.

—Albert Einstein, as quoted in *Life* magazine *(1955)*

Potentially useful formulas:

Vacuum magnetic permeability

$$\mu_0 = 4\pi \times 10^{-7}\,\frac{\mathrm{H}}{\mathrm{m}} = 4\pi \times 10^{-7}\,\frac{\Omega\,\mathrm{s}}{\mathrm{m}} \tag{1}$$

Vacuum speed of light

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \approx 3 \times 10^8\,\frac{\mathrm{m}}{\mathrm{s}} \tag{2}$$

The time averages

$$\langle \sin^2 \omega t\rangle_T = \langle \cos^2 \omega t\rangle_T = \frac{1}{2} \tag{3}$$

and

$$\langle \sin \omega t \cos \omega t\rangle_T = 0. \tag{4}$$

Spacetime interval (four dimensions)

$$\Delta s^2 = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \tag{5}$$

Lorentz factor for speed u

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \tag{6}$$

Lorentz boost by u along x direction

$$t' = \gamma\left(t - \frac{u}{c^2}x\right) \tag{7}$$

$$x' = \gamma\,(x - ut) \tag{8}$$

Flux rule for induction

$$\mathcal{E} = \int \mathbf{E}\cdot d\boldsymbol{\ell} = -\frac{d\Phi_B}{dt} \tag{9}$$

Intensity of two interfering sources with phase difference $\Delta\phi$

$$\mathcal{I} = \mathcal{I}_0 \cos^2\left(\frac{\Delta\phi}{2}\right) \tag{10}$$

Energy density of electric field $\mathbf{E}$

$$u_E = \frac{|\mathbf{E}|^2}{2\mu_0 c^2} \tag{11}$$

Imaging by plane reflecting surface

$$\frac{1}{s} + \frac{1}{s'} = 0 \tag{12}$$

Imaging by plane refracting surface

$$\frac{1}{s} + \frac{n}{s'} = 0 \tag{13}$$

AC Series RLC circuit phase

$$\tan\theta_{RLC} = \frac{\omega L - 1/(\omega C)}{R} \tag{14}$$

Problem 1. An object is distance $s = 4$ to the left of a converging lens of focal length $f = 1$. A distance $d = 2$ to the right of the lens is a planar interface with a large block of glass (index of refraction $n = 1.5$). The space outside the glass is vacuum.

(4) **(a)** Find the position of the image of the lens.

(6) **(b)** Treat the image of the lens as the object of the glass interface, and find the **magnitude of the distance from the lens** of the image produced by **refraction** from the interface.

(6) **(c)** Now find the **magnitude of the distance from the lens** of the image produced by **reflection** from the glass interface. Again, use the image of the lens as the object of the interface.

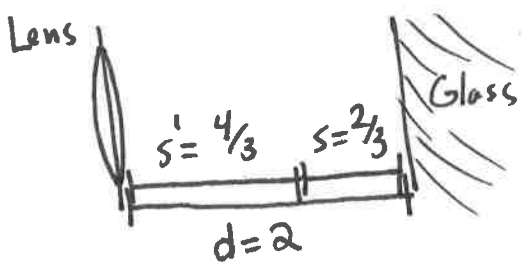
(2) **(d)** For each of the images in (a), (b), and (c), state whether the image is real or virtual.

(2) **(e)** How would you answers to (d) change if the lens were a diverging lens?

(a) Thin lens eqn: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$

$s' = \frac{sf}{s-f} = \frac{4 \cdot 1}{4-1} = \boxed{\frac{4}{3}}$

(b)



obj. dist.: $s = 2 - 4/3 = 2/3$

Refraction: $\frac{1}{s} + \frac{n}{s'} = 0$

$\rightarrow s' = -ns = -\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = -1$

Image is $s' = -1$, so distance 1 to the left of glass surface

Distance to lens is

$2 - 1 = \boxed{1}$

(c) Reflection: $\frac{1}{s} + \frac{1}{s'} = 0 \rightarrow s' = -s = -2/3$

Same obj. dist. as in (b)

This is 2/3 to the right of the interface. So distance to lens is

$2 + 2/3 = \boxed{\frac{8}{3}}$

(d)

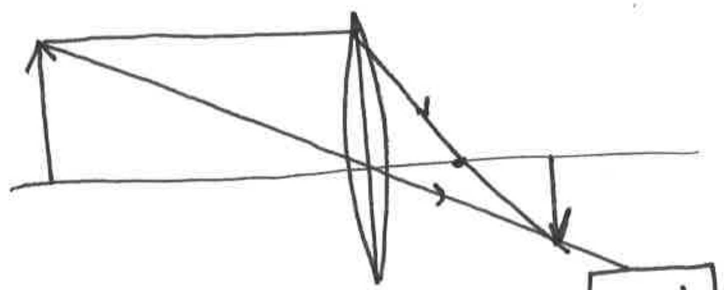
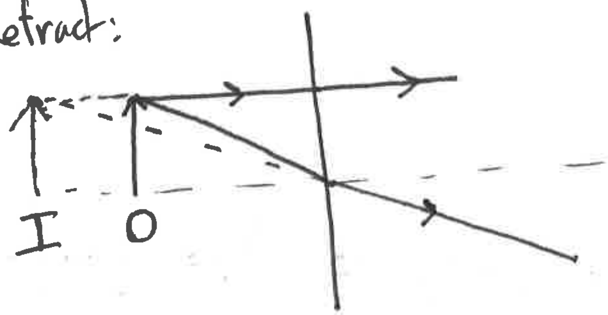


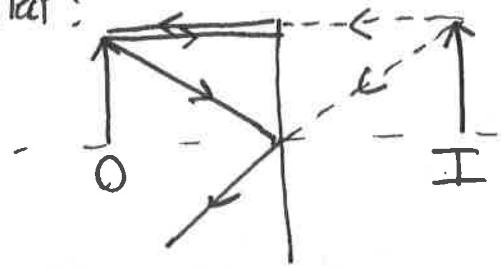
Image of lens is $\boxed{\text{real}}$

Refracted image & reflected image both $\boxed{\text{virtual}}$

Refract:



Reflect:



(e) Only change is now <u>image of lens</u> is $\boxed{\text{virtual}}$ (planar reflection & refraction always give virtual images)

Problem 2.

(6) **(a)** State Maxwell's equations in vacuum in either integral or differential form.

(8) **(b)** An infinitely long, solid cylinder of radius a centered at $r = 0$ lies parallel to the z-axis. The cylinder carries steady current I_0 in the $+z$ direction, and the current is distributed uniformly over the cross-section of the cylinder. The cylinder also carries charge per unit length λ_0 uniformly distributed over its volume. You may assume that

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{\partial \mathbf{B}}{\partial t} = 0. \tag{1}$$

State Maxwell's equations inside the cylinder in either integral or differential form. Your answer will be a set of differential or integral equations for $\mathbf{E}$ and $\mathbf{B}$ in terms of I_0, λ_0, a, and fundamental constants.

(6) **(c)** Use Ampere's law to find the magnetic field as a function of r inside the cylinder described in (b) (that is, for $r < a$).

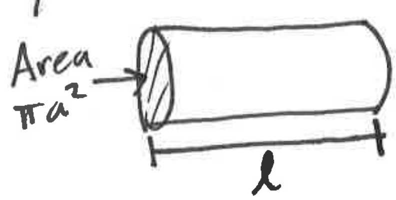
(a)
$\boxed{\nabla\cdot E = 0}$ $\boxed{\nabla\cdot B = 0}$
$\boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ $\boxed{\nabla\times\vec{B} = \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}}$

OR

$\boxed{\int\vec{E}\cdot d\vec{a} = 0}$ $\boxed{\int\vec{B}\cdot d\vec{a} = 0}$
$\boxed{\int\vec{E}\cdot d\vec{l} = -\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}}$
$\boxed{\int\vec{B}\cdot d\vec{l} = \frac{1}{c^2}\int\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}}$

(b) Now we have current density & charge density.
What are $\vec{J}$ & ρ?

$\vec{J} = \frac{I_0}{\pi a^2}\hat{z}$



For a segment of length l:
$\rho = \frac{Q}{V} = \frac{\lambda_0 l}{\pi a^2 l} = \frac{\lambda_0}{\pi a^2}$

If $\frac{\partial\vec{E}}{\partial t} = \frac{\partial\vec{B}}{\partial t} = 0$, Maxwell's eqns are:

$\nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho = \frac{\lambda_0}{\epsilon_0\pi a^2}$

$\nabla\cdot\vec{B} = 0$

$\nabla\times\vec{E} = 0$

$\nabla\times\vec{B} = \mu_0\vec{J} = \frac{\mu_0 I_0}{\pi a^2}\hat{z}$

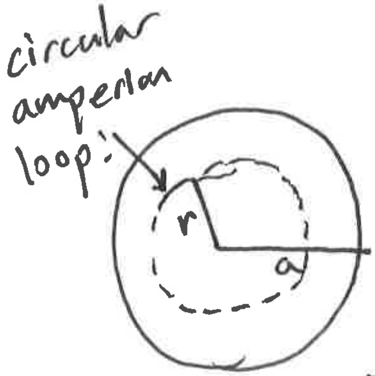
OR

$\int\vec{E}\cdot d\vec{a} = \frac{1}{\epsilon_0}\int\rho\, dV = \frac{1}{\epsilon_0}\int\left(\frac{\lambda_0}{\pi a^2}\right)dV$

$\int\vec{B}\cdot d\vec{a} = 0$

$\int\vec{E}\cdot d\vec{l} = 0$

$\int\vec{B}\cdot d\vec{l} = \mu_0\int\vec{J}\cdot d\vec{a} = \mu_0\int\left(\frac{I_0}{\pi a^2}\hat{z}\right)\cdot d\vec{a}$



$$\int \vec{B}\cdot d\vec{l} = \frac{\mu_0 I_0}{\pi a^2}\int da$$

$$B(2\pi r) = \frac{\mu_0 I_0}{\pi a^2}\pi r^3$$

$$\boxed{B = \frac{\mu_0 I_0}{2\pi a^2} r}$$

Problem 3. Consider a series RLC circuit connected to an AC voltage source with voltage amplitude V_0.

(6) **(a)** If the voltage source is set to the resonant frequency ω_0, find the current amplitude I_{res} of the source and the phase difference θ_{res} between the source current and voltage.

(6) **(b)** Now suppose that the source frequency ω is set very close to the resonant frequency ω_0. That is

$$\omega = \omega_0 + \delta\omega \tag{2}$$

with $\delta\omega \ll \omega_0$. Using the binomial approximation $(1+x)^p \approx 1+px$, calculate the quantity

$$|Z_{LC}| = \omega L - \frac{1}{\omega C} \tag{3}$$

to first order in $\delta\omega$ in terms of $\delta\omega$ and L only. Your answer should not contain C or ω_0.

(6) **(c)** Using your answer to (b), or otherwise, find the value of $\delta\omega$ at which the source current amplitude I_0 is

$$I_0 = \frac{I_{\text{res}}}{\sqrt{2}}. \tag{4}$$

You may continue to assume that $\delta\omega$ is small.

(4) **(d)** Assume that $\delta\omega$ has the value you found in (c). Sketch a graph of the source current amplitude $I_0(\omega)$ as a function of ω. Indicate the following values on your graph: (i) I_{res}, (ii) $I_{\text{res}}/\sqrt{2}$, (iii) ω_0, (iv) $\omega_0 - \delta\omega$, (v) $\omega_0 + \delta\omega$.

(a) Series RLC current: $I_0 = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

at resonance: $\omega_0 L - \dfrac{1}{\omega_0 C} = 0$ so

$I_{res} = \dfrac{V_0}{\sqrt{R^2}} = \boxed{\dfrac{V_0}{R}}$ $\quad \tan\theta_{res} = \dfrac{\omega_0 L - \frac{1}{\omega_0 C}}{R} = \boxed{0}$

(b)

$$|Z_{LC}| = (\omega_0 + \delta\omega)L - \frac{1}{(\omega_0+\delta\omega)C} = \omega_0\left(1+\frac{\delta\omega}{\omega_0}\right)L - \frac{1}{\omega_0\left(1+\frac{\delta\omega}{\omega_0}\right)C}$$

$$\approx \omega_0\left(1+\frac{\delta\omega}{\omega_0}\right)L - \frac{1}{\omega_0 C}\left(1-\frac{\delta\omega}{\omega_0}\right)$$

since $\left(1+\frac{\delta\omega}{\omega_0}\right)^{-1} \approx 1 - \frac{\delta\omega}{\omega_0}$

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow \omega_0 C = \sqrt{\frac{C}{L}} \rightarrow \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} = \omega_0 L$$

So

$$|Z_{LC}| \approx \omega_0 L\left(1+\frac{\delta\omega}{\omega_0}\right) - \omega_0 L\left(1-\frac{\delta\omega}{\omega_0}\right) = 2\omega_0 L\frac{\delta\omega}{\omega_0}$$

$$= \boxed{2L\,\delta\omega}$$

(c)

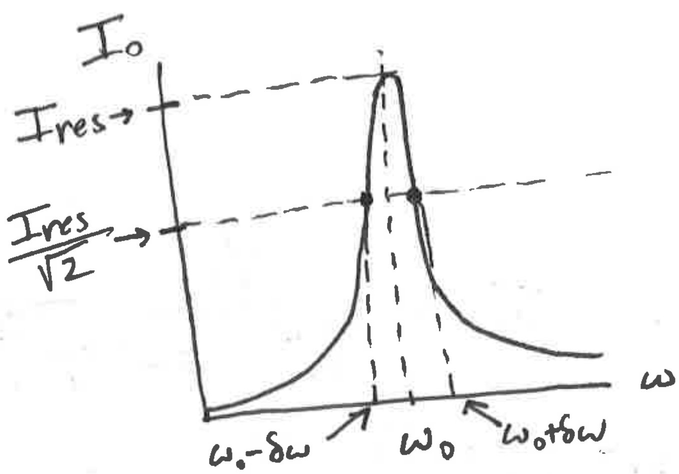
$$I_0 = \frac{V_0}{\sqrt{R^2 + |Z_{LC}|^2}}$$

want $I_0 = \frac{1}{\sqrt{2}} I_{res} = \frac{V_0}{\sqrt{2}R}$, which means

$$\sqrt{2}\,R = \sqrt{R^2 + |Z_{LC}|^2} \implies |Z_{LC}|^2 = R^2 \rightarrow |Z_{LC}| = R$$

so want $2L\,\delta\omega = R \rightarrow \boxed{\delta\omega = \frac{R}{2L}}$

(d)



Problem 4. A circular coil of wire of radius a with N turns is in the field of an electromagnet. The magnetic field is directed perpendicular to the plane of the coil and has uniform magnitude B_0 over the area of the coil. The coil is connected to an external resistor, making the coil into a closed circuit with total resistance R.

(8) **(a)** Suppose the electromagnet is suddenly switched off, reducing the field strength from B_0 to zero. Find the total charge

$$Q = \int I(t)\,dt \tag{5}$$

that passes through the resistor.

(2) **(b)** How does your answer to (a) change if the field strength of the electromagnet is instead reduced to zero gradually?

(4) **(c)** Does the induced magnetic field of the coil point in the same direction or opposite the initial field of the electromagnet?

(4) **(d)** If the electromagnet is switched back on suddenly, increasing the field strength from zero to B_0, the coil will experience a force. Will the force on the coil be directed toward or away from the electromagnet?

(a) $\mathcal{E} = -\frac{d\Phi}{dt}$, $I = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}$

$Q = -\frac{1}{R}\int \frac{d\Phi}{dt}\,dt = -\frac{\Delta\Phi}{R}$

initial flux: $\Phi_i = NB\pi a^2$, final flux $\Phi_f = 0$

$\Delta\Phi = \Phi_f - \Phi_i = -NB\pi a^2$

$\boxed{Q = \frac{NB\pi a^2}{R}}$

(b) $\boxed{\text{No change.}}$ Answer to (a) does not depend on Δt at all.

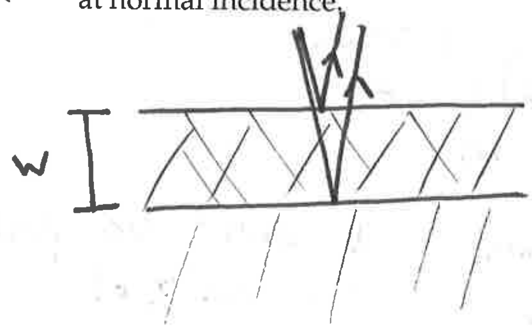
(c) Lenz's law: coil wants to maintain field of magnet, so $\boxed{\text{same direction}}$

(d) Lenz's law: initally coil has no flux through. wants to maintain this state ~~of~~ so moves away from flux. i.e, ~~f~~orce is $\boxed{\text{away from magnet}}$

Problem 5. A thin film of oil (index of refraction n_1 = 1.5) sits atop a body of water (index of refraction n_2 = 1.3). The region above the oil film is vacuum.

(6) **(a)** Find the minimum thickness in nanometers of the oil film such that infrared light of wavelength 1020 nm interferes **constructively** when reflecting from the film at normal incidence.

(6) **(b)** For the same thickness of film you found in (a), find all possible wavelengths (in nanometers) of *visible* light that will interfere **destructively** when reflecting from the film at normal incidence.



Phase shift of π at vacuum/oil interface.

(a) Phase diff $\Delta\varphi = k_m \Delta r + \pi = k n_{oil}(2w) + \pi$

Constructive: $\Delta\varphi = 2m\pi$

$\rightarrow \frac{2\pi}{\lambda} n_{oil}(2w) + \pi = 2m\pi$

$\rightarrow \frac{2n_{oil}}{\lambda}(2w) = 2m - 1$

$2w = \frac{\lambda}{2n_{oil}}(2m-1) \rightarrow w = \frac{\lambda}{2n_{oil}}\left(m - \frac{1}{2}\right)$

Min. thickness for $m = +1$ (m=0 gives unphysical negative thickness)

$w = \frac{\lambda}{2n_{oil}}\left(\frac{1}{2}\right) = \frac{\lambda}{4n_{oil}}$

$= \frac{1020\text{ nm}}{4\cdot(3/2)} = \boxed{\frac{1020}{6}\text{ nm}} = 170\text{ nm}$

(b) $W = \frac{1020}{6}$ nm

~~[illegible]~~ now destructive: $\Delta\varphi = (2m+1)\pi$

$$\frac{2\pi}{\lambda} n_{oil}(2w) + \pi = 2m\pi + \pi$$

$$\rightarrow \quad \frac{n_{oil}}{\lambda}(2w) = m \quad \rightarrow$$

$$\lambda = \frac{2 n_{oil} w}{m} = \frac{2 \cdot (3/2)\left(\frac{1020\text{ nm}}{6}\right)}{m}$$

~~must be positive~~

$$= \frac{1020\text{ nm}}{2m}$$

λ must be positive, so $m \geq +1$

For $m=1$, $\lambda_1 = \frac{1020\text{ nm}}{2} = \boxed{510\text{ nm}}$

If $m=2$, $\lambda_2 = \frac{1020\text{ nm}}{4} = 255\text{ nm}$ NOT visible

All $m \geq 2$ give non-visible wavelengths.

So only answer is $\boxed{\lambda_1 = 510\text{ nm}}$

Problem 6. Two twins, Abby and Gabby, celebrate their 20th birthday on earth. As a birthday gift, Abby receives a relativistic Bird scooter. She immediately jumps on the scooter and travels to Star X at speed $4c/5$. Gabby stays home. When Abby gets to Star X, she immediately makes a U-turn and returns to earth, again at speed $4c/5$. She arrives on her 38th birthday, as determined by her watch.

(6) **(a)** How old is Gabby at the moment Abby arrives back on earth?

(6) **(b)** According to Gabby, how far away is Star X, in lightyears? (1 lightyear = 1 year $\times\, c$)

Label the earth frame $\mathcal{C}$, the frame of the outgoing scooter $\mathcal{C}'$, and the frame of the returning scooter $\mathcal{C}''$. Coordinates for each frame are chosen such that Abby's departure is at $x = x' = x'' = 0$ and $t = t' = t'' = 0$ and Star X is along the positive x axis.

(2) **(c)** What are the coordinates t, x of the U-turn in $\mathcal{C}$?

(4) **(d)** What are the coordinates t', x' of the U-turn in $\mathcal{C}'$?

(6) **(e)** What are the coordinates t'', x'' of the U-turn in $\mathcal{C}''$?

(4) **(f)** On one set of spacetime axes, sketch a spacetime diagram showing the event of the U-turn in each of the three frames $\mathcal{C}$, $\mathcal{C}'$, $\mathcal{C}''$.

(a) Gabby's time is dilated : $\Delta t_G = \gamma\, \Delta t_A$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{4}{5}\right)^2}} = \frac{1}{\sqrt{1-\frac{16}{25}}} = \frac{1}{\sqrt{\frac{9}{25}}} = \frac{5}{3}$$

$$\Delta t_G = \frac{5}{3}\Delta t_A = \frac{5}{3}(18\text{ yr}) = 30\text{ yr}, \quad \boxed{\text{Gabby is } 50}$$

(b) In Gabby's frame, takes Abby 15 yr to get to Star X at $u = \frac{4c}{5}$. $\Delta x = ut = \left(\frac{4c}{5}\right)(15\text{ yr}) = \boxed{12\text{ ly}}$

(ly = lightyear)

(c) $\boxed{t = 15\text{ yr},\ x = 12\text{ ly}}$

(d) In outgoing scooter frame, $x' = 0$ b/c scooter travels to U-turn. $\boxed{t' = 9\text{ yr},\ x' = 0}$

(e) Lorentz boost:

$$t'' = \gamma\left(t - \frac{u}{c^2}x\right)$$

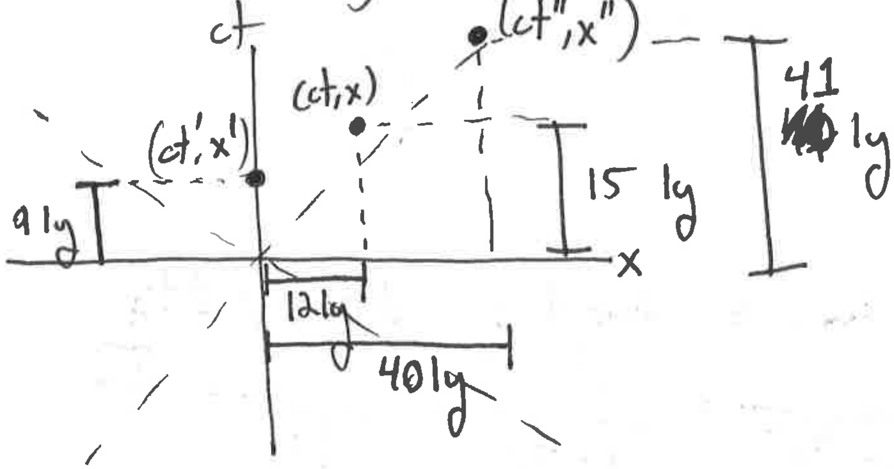
$$x'' = \gamma\left(x - ut\right)$$

~~scooter~~ returning scooter moves in $-x$ direction, so $u = -\frac{4c}{5}$

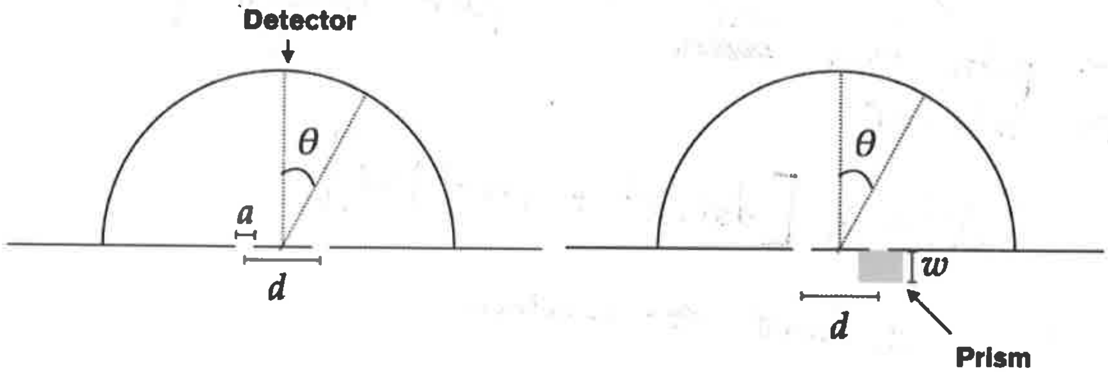
$$t'' = \frac{5}{3}\left(15\,\text{yr} + \frac{4}{5}\frac{12\,\text{yr}\cdot c}{c}\right) = 25\,\text{yr} + \frac{4}{3}\cdot 12\,\text{yr} = \boxed{41\,\text{yr}}$$

$$x'' = \frac{5}{3}\left(12\,\text{yr}\cdot c + \frac{4}{5}\cdot 15\,\text{yr}\right) = 20\,\text{ly} + 20\,\text{ly} = \boxed{40\,\text{ly}}$$

(f)



Problem 7. Coherent light with a wavelength of λ is sent through two parallel slits in a flat, opaque wall. Each slit has width $a = 3\lambda$, and the centers of the slits are a distance $d = 6\lambda$ apart. The light falls on a semi-cylindrical detector, with its axis at the midline between the slits. This setup is pictured below on the left.



(6) **(a)** How many bright fringes are observed on the detector?

(6) **(b)** Now suppose a rectangular prism of thickness w and index of refraction n is placed behind one of the slits, as in the figure above on the right. Find an expression for the angles θ at which double-slit interference maxima are observed. For this part of the problem, you can ignore effects due to single-slit diffraction. Assume the light is incident normal to the opaque wall, so the light passes straight through the prism with no deflection.

(a) Double slit maxima at $\sin\theta = \frac{m\lambda}{d} = \frac{m}{6}$

Single slit minima at $\sin\theta = \frac{m\lambda}{a} = \frac{m}{3}$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \rightarrow -1 \le \sin\theta \le 1 \rightarrow$

Double slit: $-1 \le \frac{m}{6} \le 1 \rightarrow -6 \le m \le 6$

without diffraction would be $6+6+1 = 13$ maxima

With single-slit diffraction, $\sin\theta = \frac{m}{3}$ dark. $(m = \ldots, -2, -1, 1, 2, \ldots)$

$-1 \le \sin\theta \le 1 \rightarrow -1 \le \frac{m}{3} \le 1 \rightarrow -3 \le m \le 3,\ m \ne 0$

so $3+3=6$ dark fringes. Total $\boxed{13-6=7 \text{ bright fringes}}$

(b)

Phase diff. btw slits:

I don't care about this sign. i.e., could have $(k-k_{prism})w$ here.

$$\Delta\varphi = k\,d\sin\theta + (k_{prism} - k)w$$

From path from wall to detector

path through prism

$$\Delta\varphi = k\left[d\sin\theta + (n-1)w\right]$$

$$= 2m\pi \leftarrow \text{maxima}$$

$$\frac{2\pi}{\lambda}\left(d\sin\theta + (n-1)w\right) = 2\pi m$$

$$d\sin\theta + (n-1)w = m\lambda$$

$$\boxed{\sin\theta = \frac{1}{d}\left[m\lambda - (n-1)w\right]}$$

this sign could be + or −

Problem 8.

(6) **(a)** When a radioactive nucleus of astantine 215 decays at rest, the whole atom is torn in two in the reaction

$$^{215}\text{At} \longrightarrow {}^{211}\text{Bi} + {}^{4}\text{He}. \qquad (6)$$

If the masses of the three atoms are $m_{\text{At}} = 200271\,\text{MeV}/c^2$, $m_{\text{Bi}} = 196534\,\text{MeV}/c^2$, $m_{\text{He}} = 3728\,\text{MeV}/c^2$, find the kinetic energy of the two outgoing atoms in MeV.

(6) **(b)** A mad physicist claims to have observed the decay of a particle of mass M into two identical particles of mass m, with $M < 2m$. In response to the objection that this violates conservation of energy, he replies that if M was traveling fast enough it could have total energy greater than $2mc^2$ and hence could decay into two particles of mass m. Show that he is wrong.

(a) energy cons.

$$m_{At} c^2 = m_{Bi} c^2 + m_{He} c^2 + E_{kin}$$

$$E_{kin} = (m_{At} - m_{Bi} - m_{He}) c^2$$

$$= (200271 - 196534 - 3728)\ \text{MeV}$$

$$= (3737 - 3728)\ \text{MeV}$$

$$= \boxed{9\ \text{MeV}}$$

$$\begin{array}{r} 2002 \\ -1965 \\ \hline 37 \end{array} \qquad \begin{array}{r} 71 \\ -34 \\ \hline 37 \end{array}$$

(b) In rest frame of mass M, total energy is always Mc^2. Energy cons. (in rest frame of M) says $Mc^2 = 2mc^2 + E_{kin}$ w/ $E_{kin} \geq 0$, but this is impossible since $2m > M$.

Problem 9.
(a) Describe an experiment demonstrating that electromagnetic waves carry energy.

(b) Of all the electromagnetic energy in the universe, by far the largest amount is contained in the *cosmic microwave background* (CMB) radiation. It apparently fills all space, including the vast space between galaxies, with an average energy density u_{CMB}. Model the CMB radiation as a monochromatic plane wave, and find the magnetic field amplitude (maximum magnetic field strength) B_0 of this wave, in terms of u_{CMB} and fundamental constants.

(c) Which of the following EM field configurations, 1 or 2, could be a solution to Maxwell's equations in vacuum? Explain how you know. (Choose *either* 1 or 2. The answer is not "both" or "neither.")

$$\mathbf{E}_1 = E_0\,\hat{\mathbf{z}}\cos\left(k_x x + k_y y - \omega t\right) \tag{7}$$
$$\mathbf{B}_1 = B_0\,\hat{\mathbf{y}}\cos\left(k_x x + k_y y - \omega t\right) \tag{8}$$
$$\mathbf{E}_2 = E_0\,\hat{\mathbf{z}}\,e^{-k^2(x-ct)^2} \tag{9}$$
$$\mathbf{B}_2 = B_0\,\hat{\mathbf{y}}\,e^{-k^2(x-ct)^2} \tag{10}$$

(a) Many ways, but one example is: shine light on a conductor. It heats up, so the light carried energy.

(b) Avg energy density of plane wave:

$$\langle u \rangle = \frac{1}{4}\epsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

$$\boxed{B_0 = \sqrt{2\mu_0 u_{CMB}}}$$

(c) EM waves in vacuum must be transverse, so 1 cannot be a solution (since $\vec{k}$ is not orthogonal to $\vec{B}$).

$\boxed{\text{2 can be a solution}}$ (it's transverse and clearly satisfies the wave eqn.)