Problem 1

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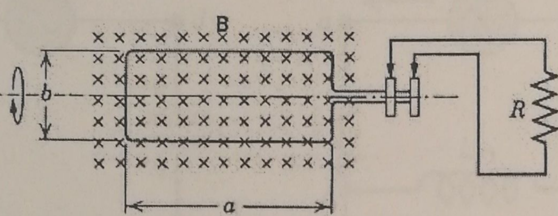
As a new electrical engineer for a given power company, you are assigned the project of designing a commercial alternating current generator based on a rectangular loop of N turns, each of length $a = 2\ m$ and width $b = 0.5\ m$, that is rotating in a uniform magnetic field $B = 1\ mT$ perpendicular to the axis of the rotation of the loop (see the figure).

A) What should be the number of turns in such a generator if it is to be used in the US, where it has to supply $\mathcal{E}_{max} = 170\ V$ at $f = 60\ Hz$. Here $\mathcal{E}_{max}$ is the maximal magnitude of the produced emf.(The well-known value of $\mathcal{E}_{rms} = 120\ V$ in a commercial power supply (in the US) is the so-called root-mean-square value of $\mathcal{E}$. It is connected to $\mathcal{E}_{max}$ by $\mathcal{E}_{max} = \sqrt{2}\,\mathcal{E}_{rms}$.)

B) Find N if you have to design such a generator for Europe, where it has to supply $\mathcal{E}_{max} = 340\ V$ at $f = 50\ Hz$.

C) How will the supplied emf $\mathcal{E}$ vary with time in the both cases? Make a sketch.

D) How does the flux Φ_B through the loop depend on the time? Make a sketch.



Solution of Problem 1

box you answers please

A) $V = NBA\omega$

$170 = N \cdot 0.001 \cdot 2 \cdot 0.5 \cdot 2\pi \cdot 60$

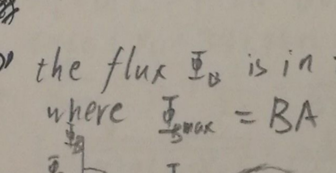
$N = 450.9$ turns $\approx \boxed{451 \text{ turns}}$

(B) $V = NBA\omega$

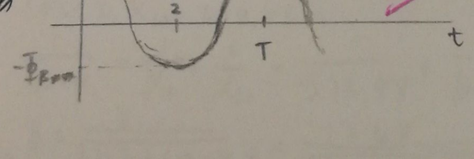
$340 = N \cdot 0.001 \cdot 2 \cdot 0.5 \cdot 2\pi \cdot 50$

$N = 1082.25$ turns $\approx \boxed{1082 \text{ turns}}$

(C) the supplied emf $\mathcal{E}$ makes a sinusoidal curve and varies according to the equation $\mathcal{E} = \mathcal{E}_{max} \sin(2\pi f t)$ where $\mathcal{E}_{max}$ is 340V in EU, 170V in US and f is 50 in EU 60 in US



(D) the flux Φ_B is in the form of cosine wave follows the equation $\Phi_B = \Phi_{max} \cos(2\pi f t)$ where $\Phi_{Bmax} = BA = B(ab)$



Problem 2

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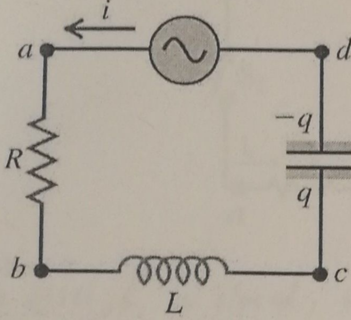
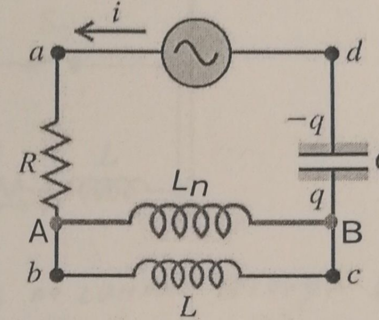
In the $L-R-C$ series circuit shown in the left figure it is known that $v = V\cos(\omega t)$, where $f = \omega/(2\pi) = 60$ Hz, the voltage amplitude of the source is $V = 120\ V$, $R = 80.0\ \Omega$, and the reactance of the capacitor is $X_C = 480\ \Omega$. The voltage amplitude across the capacitor is $V_C = 360\ V$.

For the circuit in the left figure: 1. What is the current amplitude I in the circuit? 2. What is the impedance Z of the circuit? 3. What two values can the reactance of the inductor have? For which of these two values is the angular frequency less than the resonance angular frequency? Explain your answer. 4. Determine the average power P dissipated in the circuit. 5. Determine the possible values of the phase angle Φ of the current $i = I\cos(\omega t - \Phi)$ in the circuit.

A student makes a connection between the points A nd B (right figure) through an inductor $L = 1\ H$ and measures the current i. He establishes that the current i differs from the one on the left figure, i.e., he measures different I and Φ. Can you find 6) what are the possible values of I and 7) of Φ he measures?





Solution of Problem 2

1. $I = \frac{V_C}{X_C} = \frac{360V}{480\Omega} = \boxed{0.75A}$

2. $Z = \frac{V}{I} = \frac{120V}{0.75A} = \boxed{160\Omega}$

3. $Z = \sqrt{R^2 + (X_L - X_C)^2} \rightarrow Z^2 - R^2 = (X_L - X_C)^2 \rightarrow X_L = X_C \pm \sqrt{Z^2 - R^2}$

$X_L = 480 \pm \sqrt{160^2 - 80^2} = 480\Omega \pm 138.56\Omega$ $X_{L1} = 480 - 138.56 = \boxed{341.44\Omega}$ $X_{L2} = 480 + 138.56 = \boxed{618.56\Omega}$

since the resonance frequency is when $X_L = X_C$, and $X_L = \omega L$, $X_C = \frac{1}{\omega C}$. the frequence is less than resonance frequency when $X_L < X_C$. which in this case is $X_{L1} = 341.44\Omega$

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4. $Power = \frac{1}{2} IV\cos\phi = \frac{1}{2} IV\frac{R}{Z} = \frac{1}{2} \cdot 120 \cdot 0.75 \cdot \frac{80}{160} = \boxed{22.5W}$

5. $\tan\phi = \frac{X_L - X_C}{R} = \frac{\pm 138.56}{80}$ $\phi = \tan^{-1}\left(\frac{\pm 138.56}{80}\right) = \boxed{\pm 60°}$

6. $X_L = \omega L = 1 \cdot 2\pi \cdot 60 = 376.99\Omega$

case 1 $X_{L1} = 341.44\Omega$

$\frac{1}{X_{total}} = \frac{1}{X_L} + \frac{1}{X_{L1}} = \frac{1}{376.99} + \frac{1}{341.44}$ $X_{Ltotal} = 179.17\Omega$

$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{120V}{\sqrt{80^2 + (179.17 - 480)^2}} = \boxed{0.3855A}$

case 2 $X_{L2} = 618.56\Omega$

$\frac{1}{X_{Ltotal}} = \frac{1}{X_L} + \frac{1}{X_{L1}} = \frac{1}{376.99} + \frac{1}{618.56}$ $X_{Ltotal} = 234.23\Omega$

$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{120V}{\sqrt{80^2 + (234.23 - 480)^2}} = \boxed{0.4643A}$

Sign (-1)

7. $\tan \frac{X_L - X_C}{R}$

case 1 $X_{Ltotal} = 179.17\Omega$

$\phi = \tan^{-1}\frac{179.17 - 480}{80} = \boxed{-75.11°}$

case 2 $X_{Ltotal} = 234.23\Omega$

$\phi = \tan^{-1}\frac{234.23 - 480}{80} = \boxed{-71.97°}$

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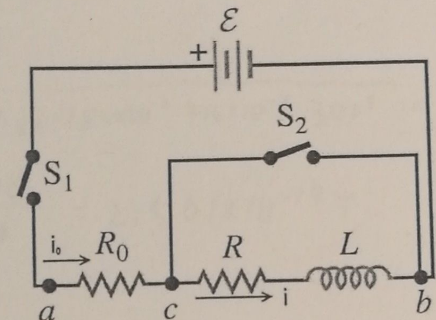
Problem 3

In the circuit shown in the figure $\mathcal{E} = 36.0\ V$, $R_0 = 50.0\ \Omega$, $R = 150\ \Omega$ and $L = 4.00\ H$.
Initially, both the switch S_1 and the switch S_2 are left open. a) Find the values of the currents i_0 through R_0 and i through R, as well as the potential differences v_{ab} and v_{cb}, immediately after the switch S_1 is closed. b) Find the expressions for $i_0(t)$, $i(t)$, $v_{ac}(t)$ and $v_{cb}(t)$ as a function of time t. c) With the switch S_2 still open what will be the values of i_0, i, v_{ab} and v_{cb} after S_1 has been closed for a very long time, so that the currents have reached their steady final values?

After the currents have reached their steady values with S_1 closed and S_2 open, the switch S_2 has been closed too. d) What are i_0, i, v_{ab} and v_{cb} just after S_2 is closed. e) What are i_0, i, v_{ab} and v_{cb} a long time after S_2 is closed.



Solution of Problem 3

a) Immediately after S_1 is closed, ther is no current through L, thus

$\boxed{i_0 = 0 \quad i = 0}$

Since $V_{ab} = V_{ac} + V_{cb}$, $V_{ac} = 0$, $\boxed{V_{ab} = 36V \quad V_{cb} = 36V}$ (+5)

b) $V_{total} = V_{R_0} + V_R + V_L$

$i_0 = i = \frac{\mathcal{E}}{R}(1-e^{-\frac{R}{L}t}) = \frac{\mathcal{E}}{R_0+R}(1-e^{-\frac{R_0+R}{L}t}) = 0.18(1-e^{-50t})$

$V_{ac} = i_0 R_0 = 50 \cdot 0.18(1-e^{-50t}) = 9(1-e^{-50t})$ (+5)

$V_{cb} = V - V_{ac} = 36 - 9(1-e^{-50t}) = 27 + 9e^{-50t} = 9(3+e^{-50t})$

c) $t \to \infty$

$i_0 = i = 0.18 \cdot (1-0) = 0.18A$

$V_{cb} = 9 \cdot (3-0) = 27V$ (+5)

$V_{ab} = 36V$

d) Since there is a inductor, just after S_2 is closed i won't change

$i = 0.18A$

$i_0 = \frac{\mathcal{E}}{R} = \frac{36}{50} = 0.72A$ (+5)

$V_{ab} = 70 \cdot 0.72 = 36V$

$V_{cd} = 0V$

e) Long time after S_2 closed

$i = 0$

$i_0 = \frac{\mathcal{E}}{R_0} = \frac{36}{50} = 0.72A$ (+5)

$V_{ab} = 36V$ $V_{cd} = 0V$

Problem 4

20/20

An airplane flying at a distance of 11.3 km from a radio transmitter receives a signal of intensity 7.83 $\mu W/m^2$. Calculate

a) the amplitude of the electric field E_m at the airplane due to this signal.
b) the maximum value of the magnetic field B_m at the airplane.
c) the total power radiated by the transmitter, assuming the transmitter to radiate uniformly in all directions.
d) what is the average value of the Poynting vector S and its direction at the airplane? Make a sketch.

Solution of Problem 4

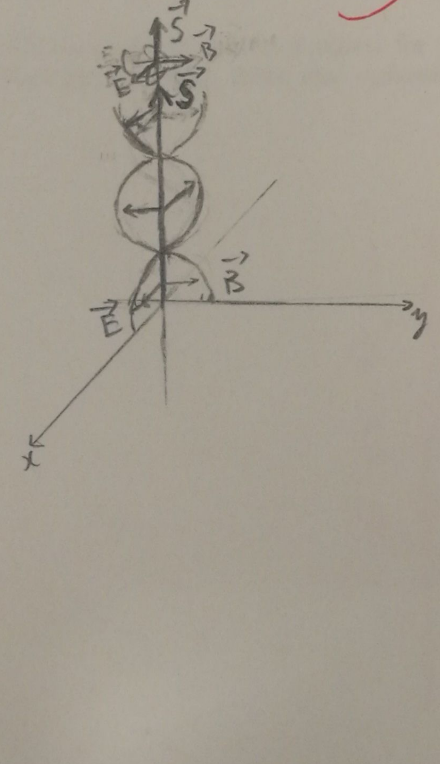
a) $I = \frac{1}{2\mu_0} E_m B_m = \frac{E_m^2}{2\mu_0 c}$ (+5)

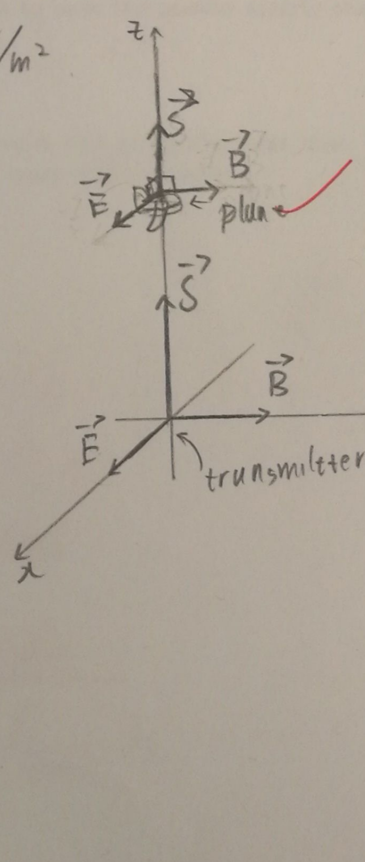
$E_m = \sqrt{2I\mu_0 c} = \sqrt{2\cdot 7.83\times10^{-6}\cdot 4\pi\times10^{-7}\cdot 3\times10^{8}} = 0.0768\ V/m$

b) $B_m = \frac{E_m}{c} = \frac{0.0768}{3\times10^{8}} = 2.561\times10^{-10}\ T$ (+5)

c) $I = \frac{P}{A}$ $\quad P = IA = I\,4\pi r^2 = 7.83\times10^{-6}\cdot 4\pi\cdot 11300^2 = 12564\ W$ (+5)

d) $S_{aver} = \frac{1}{2\mu_0} E B = I = 7.83\ \mu W/m^2$





(+5)