Avg. 46
STD = 23

# PHYSICS 1C

## MIDTERM 1

Fall, 2017

Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full credit, show all you work and reasoning. No credit will be given for answers that simply "appear". The exam is closed notes and closed book. You do not need calculators, so please put them, and all cell phones, away. If you need more space, use the backside of the page.

[redacted]

| **Problem** | **Score** |
|---|---|
| 1 | 25 |
| 2 | 19 |
| 3 | 0 |
| 4 | 25 |
| **Total** | 69 |

# FORMULAE

## Math

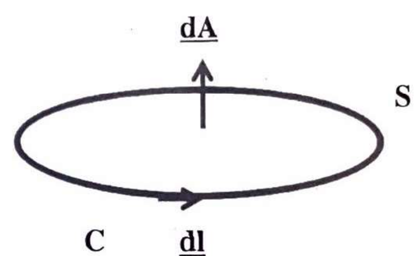
1. $\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$  2. $\frac{1}{1 \pm x} \approx 1 \mp x$  3. $\frac{1}{(1+x)^{1/2}} \approx 1 - x/2$

1. **Magnetism**

Force on current element $\underline{dF} = I\, \underline{dl} \times \underline{B}$

Biot-Savart Law $\underline{dB} = \frac{\mu_0}{4\pi} \frac{I\, \underline{dl} \times \hat{r}}{r^2}$

Ampere $\oint_C \underline{B} \cdot \underline{dl} = \mu_0 \int_S \underline{J} \cdot \underline{dA} = \mu_0 I_{enclosed}$




Right Hand Rules

**A** x **B** Palm along **A**, push **A** into **B**, thumb along **A** x **B**

Thumb along I - Fingers curl in direction of **B**

Fingers curl along **dl** - Thumb is along **dA**

2. **Faraday's Law**

emf $\varepsilon = \oint_C [\underline{E} + \underline{v} \times \underline{B}] \cdot \underline{dl} = -\frac{d\Phi_B}{dt}$  $\Phi_B = \int_S \underline{B} \cdot \underline{dA}$

Force on a charge $\underline{dF} = dq\, [\underline{E} + \underline{v} \times \underline{B}]$  $\varepsilon = dW/dq$

3. **Inductance**

Self-Inductance $L = \frac{N\Phi}{I}$  Mutual Inductance $M_{21} = \frac{N_2 \Phi_{21}}{I_1}$  $M_{12} = \frac{N_1 \Phi_{12}}{I_2}$

$\Phi_{21}$ = flux through coil 2 due to coil 1
$\Phi_{12}$ = flux through coil 1 due to coil 2

(25 Pts)

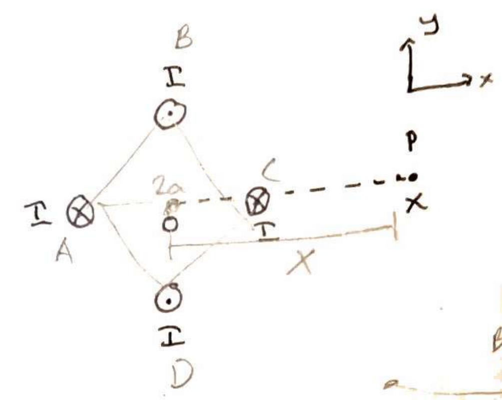
1. Four very long wires, each carrying a current of magnitude I , are arranged in a square pattern with the currents in the directions shown. The diagonal of the square is of length $2a$. Point P is a distance $x$ from the center of the square ($x = 0$, $y = 0$).

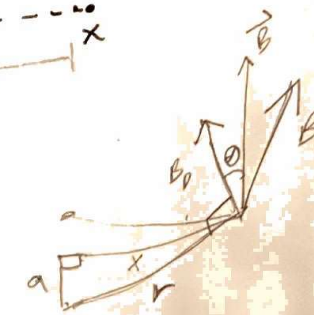
(10) a. Find the direction and magnitude of the magnetic field produced by the two wires located on the x-axis at $x = -a$ and $x = +a$.

(10) b. Find the direction and magnitude of the magnetic field produced by the two wires located on the y-axis at $y = -a$ and $y = +a$.

(5) c. Show that for $x >> a$, the magnitude of the total magnetic field at P is approximately given by

$$|\underline{\mathbf{B}}| \approx \frac{2\mu_0 I a^2}{\pi x^3}$$




25

a) wire A: $B = \frac{\mu_0 I}{2\pi(x+a)}$

wire C: $B = \frac{\mu_0 I}{2\pi(x-a)}$

$\vec{B} = \vec{B_A} + \vec{B_C}$

$|\vec{B}| = \frac{\mu_0 I}{2\pi}\left(\frac{1}{x+a} + \frac{1}{x-a}\right)$

$= \frac{\mu_0 I}{2\pi}\left(\frac{(x-a)+(x+a)}{(x^2-a^2)}\right)$

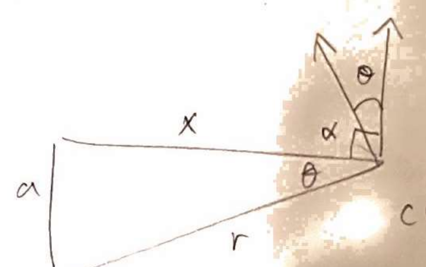
$= \frac{\mu_0 I}{2\pi}\left(\frac{2x}{x^2-a^2}\right)$

$|\vec{B}| = \frac{\mu_0 I x}{\pi(x^2-a^2)}$ +10

pointing downwards by RHS

b) $r = \sqrt{x^2+a^2}$

$\vec{B} = \vec{B_D}\cos\theta + \vec{B_B}\cos\theta$




$\cos\theta = \frac{x}{r}$

$|\vec{B}| = 2|\vec{B_D}|\cos\theta$ (symmetry)

$B_D = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi\sqrt{x^2+a^2}}$

$B_D \cos\theta = \frac{\mu_0 I x}{2\pi(x^2+a^2)}$

$|\vec{B}| = \frac{2\mu_0 I x}{2\pi(x^2+a^2)} = \frac{\mu_0 I x}{\pi(x^2+a^2)}$ + 10

pointing upwards by RHS and symmetry.

c)

$$|\vec{B}| = \frac{\mu_0 I x}{\pi(x^2+a^2)} - \frac{\mu_0 I x}{\pi(x^2-a^2)}$$

$$= \frac{\mu_0 I x}{\pi}\left(\frac{1}{x^2+a^2} - \frac{1}{x^2-a^2}\right)$$

$x >> a$

$$\|$$

$$\frac{1}{x^2\left(1+\frac{a^2}{x^2}\right)} - \frac{1}{x^2\left(1-\frac{a^2}{x^2}\right)}$$

$$|\vec{B}| = \frac{\mu_0 I x}{\pi x^2}\left(\frac{1}{1+\frac{a^2}{x^2}} - \frac{1}{1-\frac{a^2}{x^2}}\right)$$

$$\|$$

$$\left[1-\frac{a^2}{x^2} - \left(1+\frac{a^2}{x^2}\right)\right]$$

$$\|$$

$$-\frac{2a^2}{x^2}$$

$$|\vec{B}| \approx -\frac{2\mu_0 I x a^2}{\pi x^4}$$

(sign flip) $|\vec{B}| \approx \frac{2\mu_0 I a^2}{\pi x^3}$ ✓

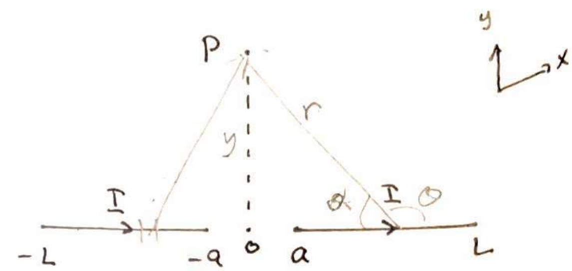
+5

(20 Pts)

2. A thin wire runs along the x-axis from x = a to x = L, and carries a current I in the positive x-direction. A second thin wire runs along the x-axis from x = - L to x = - a, and also carries a current I in the positive x-direction. Consider a Point P located at ( x = 0, y).

(15) a. Find the direction of the magnetic field at Point P, and show that its magnitude is given by

$$|\underline{\boldsymbol{B}}| = \frac{\mu_0 I}{2\pi y}\left[\frac{L}{(L^2+y^2)^{\frac{1}{2}}} - \frac{a}{(a^2+y^2)^{\frac{1}{2}}}\right]$$




19

(5) b. If $L \to \infty$ and y << a, show that

$$|\underline{\boldsymbol{B}}| \approx \frac{\mu_0 I y}{4\pi a^2}$$

Why should the $|\underline{\boldsymbol{B}}|$ vanish at the origin?

a) By RHS, direction is out of the plane.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\ell}\times\vec{r}}{r^2} \qquad r = \sqrt{y^2+x^2}$$

$$d\vec{\ell}\times\vec{r} = dx \sin\theta$$

$$\sin\theta = \sin\alpha$$

$$\sin\alpha = \frac{y}{r}$$

$$|\vec{B}| = \frac{\mu_0 I}{4\pi}\int_a^L \frac{y}{(x^2+y^2)^{3/2}}\,dx$$

$$= \frac{\mu_0 I y}{4\pi}\left[\frac{x}{y^2(x^2+y^2)^{1/2}}\right]_a^L$$

$$|\vec{B}| = \frac{\mu_0 I}{4\pi y}\left[\frac{L}{\sqrt{L^2+y^2}} - \frac{a}{\sqrt{a^2+y^2}}\right]$$

By symmetry, multiply the magnitude by 2.

15

Thus, $$|\vec{B}| = \frac{\mu_0 I}{2\pi y}\left[\frac{L}{\sqrt{L^2+y^2}} - \frac{a}{\sqrt{a^2+y^2}}\right]$$

b) $|\vec{B}| = \frac{\mu_0 I}{2\pi y}\left[\frac{L}{\sqrt{L^2+y^2}} - \frac{a}{\sqrt{a^2+y^2}}\right]$

$L \to \infty$

$y << a$

$$\frac{\cancel{L}}{\cancel{L}\sqrt{1+\frac{y^2}{L^2}}} - \frac{\cancel{a}}{\cancel{a}\sqrt{1+\frac{y^2}{a^2}}}$$

$$1 - \frac{y^2}{2L^2} - \cancel{1} + \frac{y^2}{2a^2}$$

$$|\vec{B}| = \frac{\mu_0 I y}{4\pi a^2}$$

y

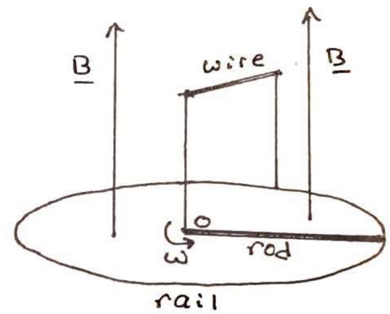
(30 Pts)

3. A thin conducting rod of length L and mass m rotates in the x-y plane about the z-axis through one end of the rod (the origin) with an angular frequency $\omega$ as shown. A uniform and temporally constant magnetic field $\underline{\mathbf{B}}$ is parallel to the z-axis. The rod slides without friction along a conducting, circular, fixed rail that is connected by a fixed wire to the origin as shown, thereby allowing a current to flow along the rod and the rail, and to close back to the rod through the wire. The system has a total resistance R to the flow of current.

(10) a. Find the direction and magnitude of the emf that is induced by the rotating rod.

(5) b. Find the current that flows in the circuit.

0




Recall that the moment of inertia of the rod about the z- axis is $m\,L^2/3$, and that the differential torque is $\underline{\boldsymbol{d\tau}} = \underline{\boldsymbol{r}}\ \boldsymbol{x}\ \underline{\boldsymbol{dF}}$.

(10) c. At time t = 0, the angular frequency of the rod is $\omega_0$. Show that $\omega\ (t)$ is

$$\omega\ (t) = \ \omega_0 \exp\left[-\ t/\tau\right]$$

where you must determine the characteristic damping time $\tau$.

(5) d. By integrating $d\phi/dt = \ \omega\ (t)$, show that the rod will not make a complete rotation if $\omega_0 < 2\pi/\tau$.

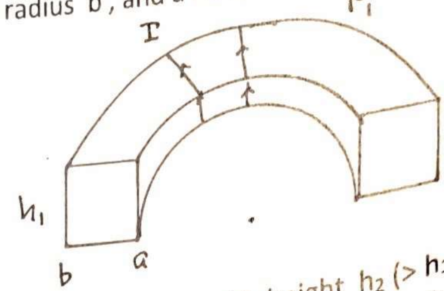
a) $\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$

$\omega = 2\pi f$

0

(25 Pts)

4. A toroidal solenoid with an inner radius $a$, an outer radius $b$, and a height $h_1$ has $N_1$ turns that carry a current $I_1$.

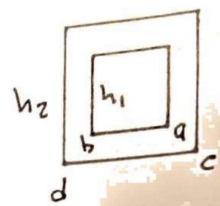



(12) a. Use Ampere's Law to find the magnetic field inside the solenoid, and show that the self-inductance is

$$L_1 = \frac{\mu_0 N_1^2 h_1}{2\pi} \ln\left(^b/_a\right)$$

25

A second toroidal solenoid with inner radius $c$ (< $a$), outer radius $d$ (> $b$), height $h_2$ (> $h_1$), and $N_2$ turns that carry a current $I_2$ concentrically surrounds the first solenoid as shown in cross-section.




(13) b. By explicit calculation, show that the mutual inductances $M_{21}$ and $M_{12}$ are equal. Why are $M_{21}$ and $M_{21}$ not equal to $(L_1 L_2)^{1/2}$ ?

a) $\oint \vec{B} \cdot d\vec{\ell} = N_1 \mu_0 I_1$

12

$$B = \frac{N_1 \mu_0 I_1}{2\pi r} \checkmark$$

$$\cancel{\Phi_B = \frac{N_1 \mu_0 I_1}{2\pi} \int_a^b \frac{dr}{r}}$$

$$B = \frac{N_1 \mu_0 I_1}{2\pi} \ln\left(\frac{b}{a}\right) \qquad \Phi_1 = BA$$

$$L = \frac{N_1 \Phi_1}{I_1} = \frac{N_1 (N_1 \mu_0 I_1 \ln(\frac{b}{a}))h_1}{2\pi I_1}$$

$$L = \frac{\mu_0 N_1^2 h_1}{2\pi} \ln\left(\frac{b}{a}\right) \checkmark$$

b) $M_{21} = \frac{N_2 \Phi_{21}}{I_1} \qquad M_{12} = \frac{N_1 \Phi_{12}}{I_2}$

$\Phi_{21} = B_1 A_2$

13

$$= \frac{N_1 \mu_0 I_1 h_2 \ln(\frac{b}{a})}{2\pi I_1}$$

$\Phi_{12} = \Phi_{21}$ due to the field entering the area of first solenoid

$$M_{21} = \frac{N_1 N_2 \mu_0 h_1 \ln(\frac{b}{a})}{2\pi}$$

$$M_{12} = \frac{N_1 N_2 \mu_0 h_1 \ln(\frac{b}{a})}{2\pi}$$

because the area is smaller for $h_1$, s flux is the same.

$M_{12}$ and $M_{21}$ are not equal to $\sqrt{L_1 L_2}$ because $L_2$ is determined by its own inner and outer radius.