PHYSICS 1C

Midterm 1

Fall, 2015

Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full credit, show all you work and reasoning. No credit will be given for answers that simply "appear". The exam is closed notes and closed book. You do not need calculators, so please put them, and all cell phones, away. If you need more space, use the backside of the page.

Sunnie So

Your Full Name - Printed

704430286

Your Student ID Number

Your Normal Signature

Problem	Score
1	20
2	18
3	29
4	20
Total	87

Ave = 55

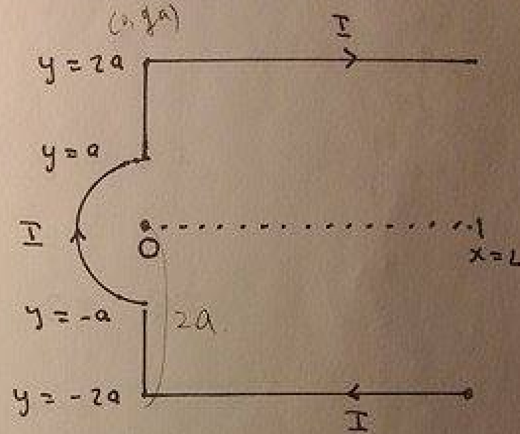
STD = 19

(25 Pts)

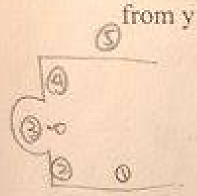
1. A thin wire lies in the x-y plane and carries a current I in the direction shown. The wire starts at $x = L$, $y = -2a$, runs parallel to the x-axis until reaching $x = 0$, $y = -2a$, then runs along the y-axis until $y = -a$. The wire then bends into a semi-circle of radius a so as to pass through the point $x = -a$, $y = 0$ before returning to the y-axis at the point $y = a$ and continuing to $y = 2a$. Finally, the wire runs from $x = 0$, $y = 2a$ to $x = L$, $y = 2a$ parallel to the x-axis.

(15) a. Determine the direction and magnitude of the magnetic field at the origin (O) that is produced by the two parallel wires. Take the limit $L >> a$, and explain why the result is physically reasonable.

(10) b. Determine the direction and magnitude of the magnetic field at the origin (O) that is produced by the wire that runs from $y = -2a$ to $y = 2a$.



(a)



$$B_1 = \frac{\mu_0 I}{4\pi}\int \frac{dl \cdot \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int \frac{dl \sin\theta}{r^2} = \frac{\mu_0 I}{4\pi}\int_0^L \frac{dl\, 2a}{((2a)^2+x^2)^{3/2}} = \frac{\mu_0 I (2a)}{4\pi} \frac{x}{(2a)^2((2a)^2+x^2)^{\frac{1}{2}}}\Big|_0^L = \frac{\mu_0 I L}{4\pi 2a((2a)^2+L^2)^{\frac{1}{2}}}$$

B_5 · same as B_1, same direction, into the page

$B_2 = B_4 = 0$ Why? -3 x -2

$$B_3 = \frac{\mu_0 I}{4\pi}\int \frac{dl \cdot \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{2\pi a}{a^2} = \frac{\mu_0 I}{2a}$$, into the page

$$B_{net} \text{ at O} = B_1 + B_3 + B_5 = \frac{\mu_0 I L}{4\pi a((2a)^2+L^2)^{\frac{1}{2}}} + \frac{\mu_0 I}{2a}$$

$$L >> a \quad \frac{\mu_0 I \cancel{L}}{4\pi a \cancel{L}} + \frac{\mu_0 I}{2a} = \boxed{\frac{\mu_0 I}{2a}\left(\frac{1}{2\pi}+1\right)}$$

$\frac{1}{2\pi}$ for cicular part
1 for the straight wires
make sence!

one straight wire

= [illegible] + 1

(b) same as $B_2 + B_3 + B_4 = B_3 = \frac{\mu_0 I}{2a}$, into the page

FORMULAE

Math

1. $$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$$

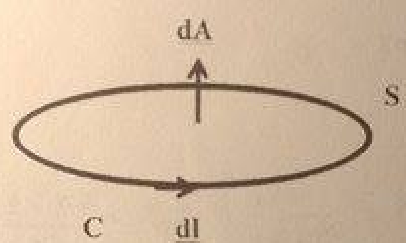
2. $$\frac{1}{(1+x)^2} \cong 1-2x, x << 1$$

1. Magnetism

Force on current element $\underline{\mathbf{dF}} = I\, \underline{\mathbf{dl}} \times \underline{\mathbf{B}}$

Biot-Savart Law $\underline{\mathbf{dB}} = \frac{\mu_0}{4\pi} \frac{I\mathbf{dl} \times \hat{\mathbf{r}}}{r^2}$

Ampere $\oint_C B \cdot dl = \mu_0 \int_S J \cdot dA = \mu_0 I_{enclosed}$



Right Hand Rules

$\underline{\mathbf{A}} \times \underline{\mathbf{B}}$ Palm along $\underline{\mathbf{A}}$, push $\underline{\mathbf{A}}$ into $\underline{\mathbf{B}}$, thumb along $\underline{\mathbf{A}} \times \underline{\mathbf{B}}$

Thumb along I - Fingers curl in direction of $\underline{\mathbf{B}}$

Fingers curl along $\underline{\mathbf{dl}}$ - Thumb is along $\underline{\mathbf{dA}}$

2. Faraday's Law

emf $\varepsilon = \oint_C [\underline{E} + \underline{v} x \underline{B}] \cdot \underline{dl} = -\frac{d\Phi_B}{dt}$ $\Phi_B = \int_S \underline{B} \cdot \underline{dA}$

Force on a charge $\underline{\mathbf{F}} = q\,[\underline{\mathbf{E}} + \underline{\mathbf{v}} \times \underline{\mathbf{B}}]$ $\varepsilon = dW/dq$

3. Inductance

Self-Inductance $L = \frac{N\Phi}{I}$ Mutual Inductance $M_{21} = \frac{N_2\Phi_{21}}{I_1}$ $M_{12} = \frac{N_1\Phi_{12}}{I_2}$

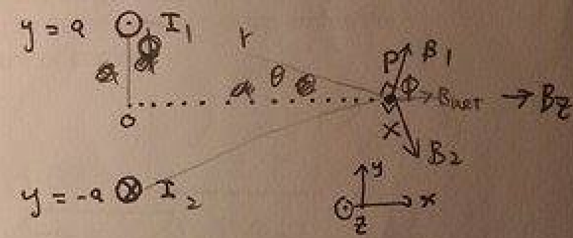
Φ_{21} = flux through coil 2 due to coil 1
Φ_{12} = flux through coil 1 due to coil 2

(20 Pts)

2. A pair of very long (effectively infinite) wires run parallel to the z-axis at x = 0 and y = a and y = - a , and carry a current I in opposite directions as shown. Consider the Point P at y = z = 0 at a distance x from the origin.

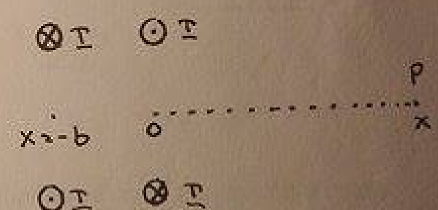
(12) a. Find the direction of the magnetic field at P, and show that its magnitude is (a 2D dipole field)

$$|\underline{B}| = \frac{\mu_0 I a}{\pi(x^2 + a^2)}$$



(8) b. A second pair of parallel wires is now placed at x = - b, y = a and y = - a, and carry a current I in the directions shown. If x >> a, x >> b, show that the total magnetic field at P is approximately given by (a 2D quadrupole field)

$$|\underline{B}| \approx \frac{2\mu_0 I a b}{\pi x^3}$$



ⓐ say upper wire I_1, B_1

lower I_2 B_2

5

long straight wire

$$B = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int_{-L}^{L}\frac{dl\sin\theta}{r^2} = \frac{\mu_0 I}{4\pi}\, 2\int_0^L \frac{dl\, a}{(a^2+x^2)^{3/2}} = \frac{\mu_0 I}{2\pi}\cdot\frac{a\,x}{a^2(a^2+x^2)^{1/2}} \quad x\to\infty \Rightarrow \frac{\mu_0 I}{2\pi a}$$

$$B_{z_1} = B_1\cos\phi = \frac{\mu_0 I}{2\pi r}\cdot\frac{a}{r} = \frac{\mu_0 I a}{2\pi r^2}$$

18

$$B_z = B_{z_1} + B_{z_2} = \frac{\mu_0 I a}{\pi r^2} = \boxed{\frac{\mu_0 I a}{\pi(x^2+a^2)}}$$

14

ⓑ new pair: same way of doing it, just that $|x_2| = x+b$

direction is opposite

$$B_{\text{new pair}} = \frac{-\mu_0 I a}{\pi((x+b)^2+a^2)}$$

why cancel

$$B_{\text{new net}} = \frac{\mu_0 I a}{\pi}\left(\frac{1}{x^2+a^2} - \frac{1}{(x+b)^2+a^2}\right) \xrightarrow[x\gg b]{x\gg a} \frac{(x+b)^2+\cancel{a^2}-x^2-\cancel{a^2}}{(x^2+a^2)((x+b)^2+a^2)} = \frac{\cancel{x^2}+2xb+b^2-\cancel{x^2}}{(x^2+a^2)((x+b)^2+a^2)}$$

$$\longrightarrow \frac{\mu_0 I a}{\pi}\left(\frac{b(2x+\cancel{b})}{x^2\cdot x^2}\right) = \frac{2\mu_0 I a b}{\pi x^3}$$

(30 Pts)

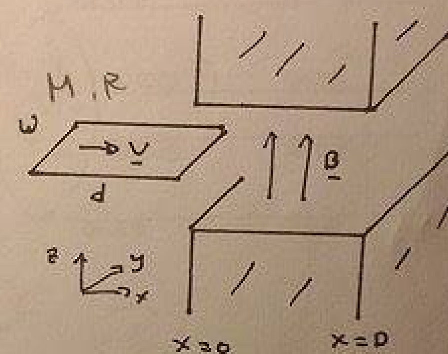
3. A rectangular conducting loop of mass M, width w, and length d moves horizontally in the x-y plane with velocity $\underline{v}$ (ignore gravity) into the region between the pole faces of a magnet that produces a spatially uniform magnetic field $\underline{B}$ in the z-direction ($|\underline{B}|$ = constant; ignore fringing fields). The magnetized region extends from x = 0 to x = D, and extends in the y-direction so the width of loop fits within the region. The loop has a total resistance R to the flow of current.

(10) a. Find the direction and magnitude of the induced emf when the front end of the loop has entered the magnetized region but the back end is still outside.

(7) b. Find the current that flows in the loop. Now find the magnitude and direction of the force that acts on the loop due to the induced current.

29 (5) c. Suppose that the length d << D so that the entire loop can enter the magnetized region before stopping. Find the induced emf when the whole loop is within the region.

(8) d. Now suppose that d > D. Show that if the initial speed v_0 of the loop as it enters the magnetized region satisfies the condition $v_0 < (D w^2 B^2)/MR$, the front end of the loop will stop moving before it reaches the end of the magnetized region (x = D). [Hint: Solve for v(t) and then x(t).]



ⓐ $\frac{d\Phi_B}{dt} = \frac{B\cdot dA}{dt} = B\cdot w\cdot v$

$|\varepsilon| = \left|-\frac{d\Phi_B}{dt}\right| = \boxed{Bwv, \text{ clockwise}}$, (resist the increasing $B\uparrow$) 10

ⓑ $\boxed{I = \frac{\varepsilon}{R} = \frac{Bwv}{R}}$

$F = I\,dl\times B = \frac{Bwv}{R}\cdot wB = \boxed{\frac{B^2w^2v}{R}} = F$, $\boxed{\text{left}}$ 7

ⓒ $\boxed{\varepsilon = 0}$ $\left(\because \frac{d\Phi}{dt} = 0\right)$ 5

ⓓ $F = ma = m\frac{dv}{dt} = \frac{B^2w^2v}{R}$

$\int_{v_0}^{v} \frac{1}{v}\,dv = \int_0^t \frac{B^2w^2}{mR}\,dt$

$\ln\left(\frac{v}{v_0}\right) = \frac{B^2w^2}{mR}t$

$v = v_0 e^{\frac{B^2w^2}{mR}t}$

$v(t) = v_0 e^{\frac{B^2w^2}{mR}t} = \frac{dx}{dt}$

7

$\int_0^t v_0 e^{\frac{B^2w^2}{mR}t} = \int_0^x dx$

$x(t) = v_0\frac{mR}{B^2w^2}\left(1 - e^{-\frac{B^2w^2}{mR}t}\right)$

$v(0)?\ x = D \rightarrow e^{\frac{B^2w^2}{mR}t} = \frac{DB^2w^2}{v_0 mR}$

$t = \ln\left(\frac{DB^2w^2}{v_0 mR}\right)\cdot\frac{mR}{B^2w^2}$

$v(t) = v_0 e^{\frac{B^2w^2}{mR}(\ldots)}$

$0 = v_0 e^{\ln\frac{DB^2w^2}{v_0 mR}} = v_0\frac{DB^2w^2}{v_0 mR} \rightarrow v_0 < \frac{Dw^2B^2}{mR}$

if $v = 0$ then $v_0 < \frac{Dw^2B^2}{mR}$

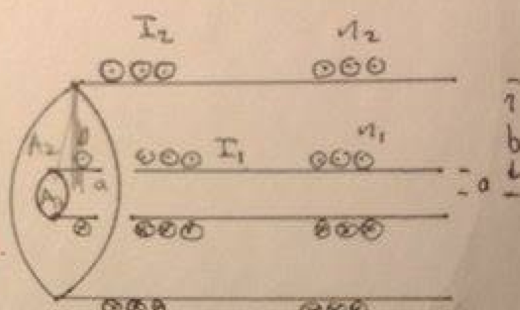
$\frac{DB^2w^2}{v_0 mR}$

(25 Pts)

4. A long ($l \gg a$) cylindrical solenoid with radius a (cross-sectional area A_1) has n_1 turns/unit length (dN_1/dz), and carries a current I_1 through each turn. A second concentric long ($l \gg b$) cylindrical solenoid with radius $b > a$ (cross-sectional area A_2) has n_2 turns/unit length, and carries a current I_2 through each turn.

(10) a. Calculate the self-inductance L_1 and L_2 of each solenoid.

(15) b. Calculate the mutual inductance M_{12} and M_{21} of each solenoid, and explicitly show that $M_{12} = M_{21}$. Explain why $M^2 = M_{12} M_{21} \neq L_1 L_2$.



solenoid  $B = \mu_0 n I$

proof? (-3).

(a) $L_1 = \frac{N\Phi}{I} = \frac{N_1 l \, \mu_0 n_1 I_1 \cdot A_1}{I_1 \cdot l} = \cancel{N_1 \mu_0 n_1 A_1}$ $n_1^2 \mu_0 A_1 \, l$

(-2).

$L_2 = \cancel{N_2 \mu_0 n_2 A_2}$ $n_2^2 \mu_0 A_2 \, l$

(b) $M_{21} = \frac{N_{21}\Phi_{21}}{I_1 l} = \frac{n_2 l \cdot \mu_0 n_1 I_1 A_1}{I_1 l} = n_1 n_2 \mu_0 A_1$ → $M_{12} = M_{21}$

$M_{12} = \frac{N_{12}\Phi_{12}}{I_2 l} = \frac{n_1 l \cdot \mu_0 n_2 I_2 A_1}{I_2 l} = n_1 n_2 \mu_0 A_1$

✓

$M^2 = n_1^2 n_2^2 \mu_0^2 A_1^2$

$L_1 L_2 = n_1^2 n_2^2 \mu_0^2 A_1 A_2$

→ they are different because the area ~~covered~~ enclosed by each ~~coil~~ solenoid is different.