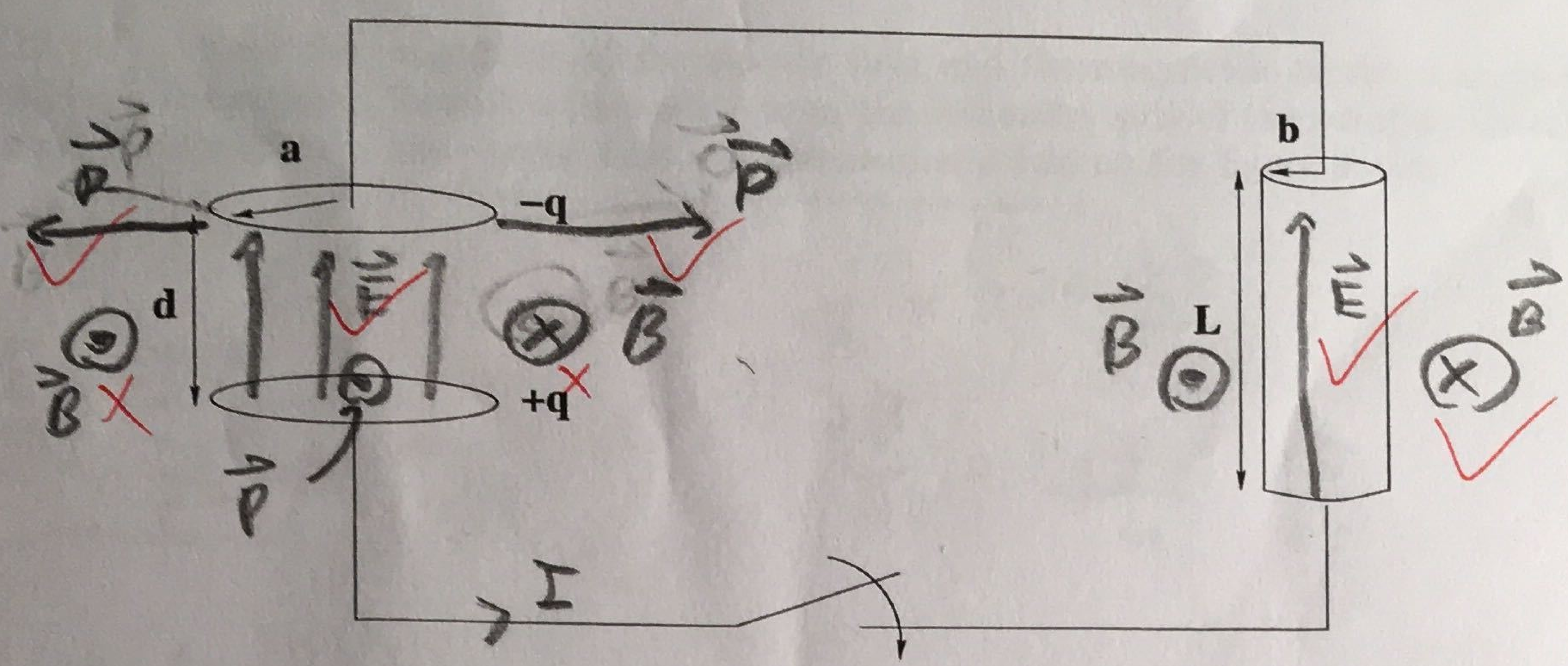


1) A circular parallel-plate capacitor of plate radius a and plate separation d is connected across a cylindrical resistor of radius b and length L At the instant under consideration, the capacitor has a charge q, with the polarity given in the figure.

You will be asked to draw vectors on the sketch above. When necessary, please use $\odot$ and $\otimes$ to unambiguously resolve the direction of any azimuthal fields.

6

- 1a) (10 pts) Find the magnitude of the magnetic field at points inside the capacitor, located a distance r from the symmetry axis of the capacitor. Clearly sketch and label the direction of the magnetic field, the electric field and the Poynting vector field (in the region around the capacitor) on the figure above. Discuss the consistency between the flow of energy as indicated by the Poynting vector field and the changing state of charge on the capacitor.

$\int \vec{E} \cdot d\vec{s} = V$

$CV = Q$

$E \cdot d = \frac{Q}{C}$

$E = \frac{Q}{Cd}$

$2\pi r \vec{B} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

$2\pi r B = \mu_0 \epsilon_0 \pi r^2 \frac{I}{Cd}$

$\boxed{\vec{B} = \frac{\mu_0 \epsilon_0 r I}{2\, Cd} \hat{\phi}}$

X

$\vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

$|P| = \frac{1}{\mu_0} \frac{Q}{Cd} \cdot \frac{\mu_0 \epsilon_0 r I}{2\, Cd}$

direction: $\hat{n} \times \hat{\phi}$ = out of the Capacitor

The flow of energy is out of the capacitor according to the Poynting vector. This is consistant with the fact that the cap. is discharging. (so it is losing Energy.

5

• 1b) (10 pts) Find the magnitude of the electric field and the magnitude of the magnetic field at points inside the resistor, located a distance r from the symmetry axis of the resistor. Clearly sketch and label the direction of the electric field and the magnetic field on the figure above.

$V = IR$

$EL = IR$

$\boxed{|\vec{E}| = \frac{IR}{L}}$

$\Phi_E = \pi r^2 \frac{IR}{L}$

$\frac{d\Phi_E}{dt} = \frac{\pi r^2 R}{L}\frac{dI}{dt}$

$2\pi r B = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

$'' = \mu_0 \epsilon_0 \pi r^2 \frac{\frac{dI}{dt} R}{L}$

$\boxed{B = \frac{\mu_0 \epsilon_0}{2} r R \frac{dI/dt}{L}}$

4

• 1c) (10 pts) Find the magnitude of the Poynting vector at the boundary of the resistor. Clearly sketch and label the direction of the Poynting vector field on the diagram above. Find the rate at which energy is entering or leaving the resistor, and discuss the result.

$|\vec{P}| = \frac{1}{\mu_0}|\vec{E} \times \vec{B}|$?

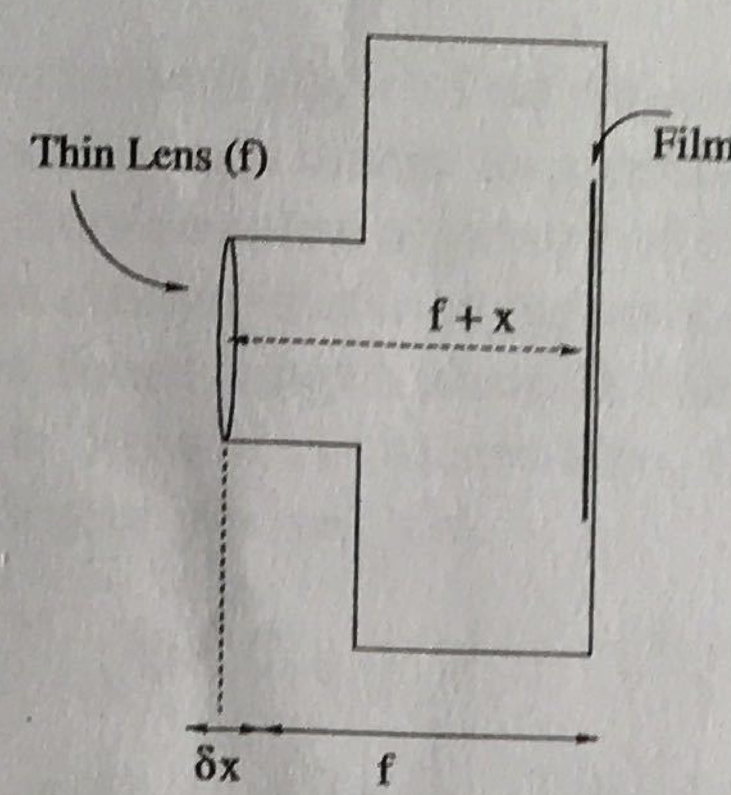
$= \frac{1}{\mu_0} \frac{\mu_0 \epsilon_0}{2} rR \frac{dI/dt}{L} \cdot \frac{IR}{L}$

$\boxed{|P| = \frac{\epsilon_0 r R^2 I \, dI/dt}{2 L^2}}$

$\int \vec{P} \cdot dA = PA$

$= 2\pi b L \cdot \frac{\epsilon_0 a R^2 I \, dI/dt}{2 r L^2}$ IR

$\boxed{P = \pi b^2 \frac{\epsilon_0 R^2 I \, dI/dt}{L}}$



2) The important parts of the camera shown above are the film and the converging lens of focal length f. The distance between the lens and the film is given by $f + x$, where x can be adjusted to any value between 0 and $\delta x = f/10$ to bring the subject to be photographed into focus.

5

- 2a) (5 points) Find the near-point (the closest point at which you can take a focused picture of an object) and the far-point (the farthest point at which you can take a focused picture) for this camera.

near point: $x = f/10$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{f + \frac{f}{10}}$$

$$\frac{1}{p} = \frac{11}{11f} - \frac{1}{\frac{11f}{10}}$$

$$\frac{1}{p} = \frac{1}{11f}$$

$$\boxed{p = 11f}$$

far point $x = 0$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} = 0$$

$\boxed{\text{Infinity}}$

4

- 2b) (5 points) Suppose we're focused on an object and the lens is located at a distance $f + x$ from the film. What is the magnification of the object? Where would we have to place the object to get the greatest magnification and what would that magnification be?

$$M = \frac{-q}{p} = \frac{-(f+x)}{p} = -(f+x)\left(\frac{1}{f} - \frac{1}{f+x}\right)$$

$$\boxed{M = \frac{-(f+x)}{f} + 1}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{f+x}$$

$$p = \frac{1}{\frac{1}{f} - \frac{1}{f+x}}$$

$$\boxed{p = \frac{1}{\frac{1}{f} - \frac{1}{f+x}}}$$

for greatest magnification

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{q}$$

$$-\frac{q}{1} \cdot \frac{1}{p} \qquad -q\left(\frac{1}{f} - \frac{1}{q}\right)$$

$$-\frac{q}{f} + 1$$

6

- 2c) (10 points) In order to take close-up shots of an object, one may place a "*macro*" lens adjacent to the lens of the camera. In practice, the macro lens produces an image of the close object at the near-point of the camera and the camera takes a picture of that image. **What type of image must the macro lens produce? If the desired distance between the object and the two lenses is $\frac{1}{5}$ of the camera's original near-point, what focal length should the macro lens have? Is the macro lens converging or diverging?** [Hint: Sketch the macro lens, the object at the new near-point and the image at the old near-point. Mind your p's and q's].

Virtual image because the near point is in front of the lense.

q: np   p: $\frac{1}{5}$np   )(0

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$\frac{1}{f} = \frac{1}{np} - \frac{5}{np}$

$\frac{1}{f} = -\frac{4}{np}$

$f = -\frac{np}{4}$

$$\boxed{f_{macro} = -\frac{np}{4} = -\frac{11f}{4}}$$

diverging lense

X

2

- 2d) (5 points) With the macro lens in place and the object located at the new effective near-point ($\frac{1}{5}$ of the camera's original near-point distance), what is the total magnification of the object?

$M_{macro} = \frac{-q}{p} = \frac{-n}{\frac{1}{5}np} = 5$

$M_{regu} = \frac{-q}{p} = \frac{-(f+x)}{11f/4} = -\frac{4(f+x)}{11f}$

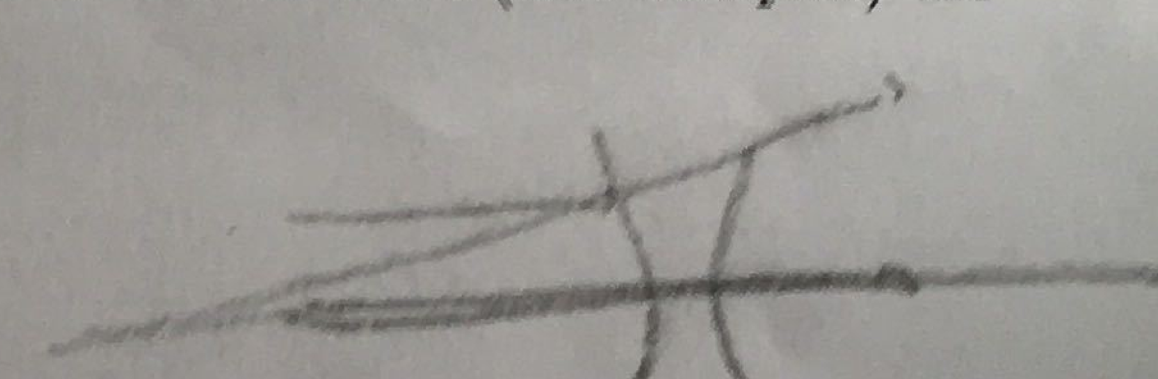
magnification total $= \boxed{-20\frac{f+x}{11f}}$

X

2

- 2e) (5 points) The ability to take large close-up photos comes at a cost. Find (and interpret) the new far-point for the camera when the macro lens is attached.

$\frac{1}{p_{mac}} + \frac{1}{q_{mac}} = \frac{1}{f_{mac}}$

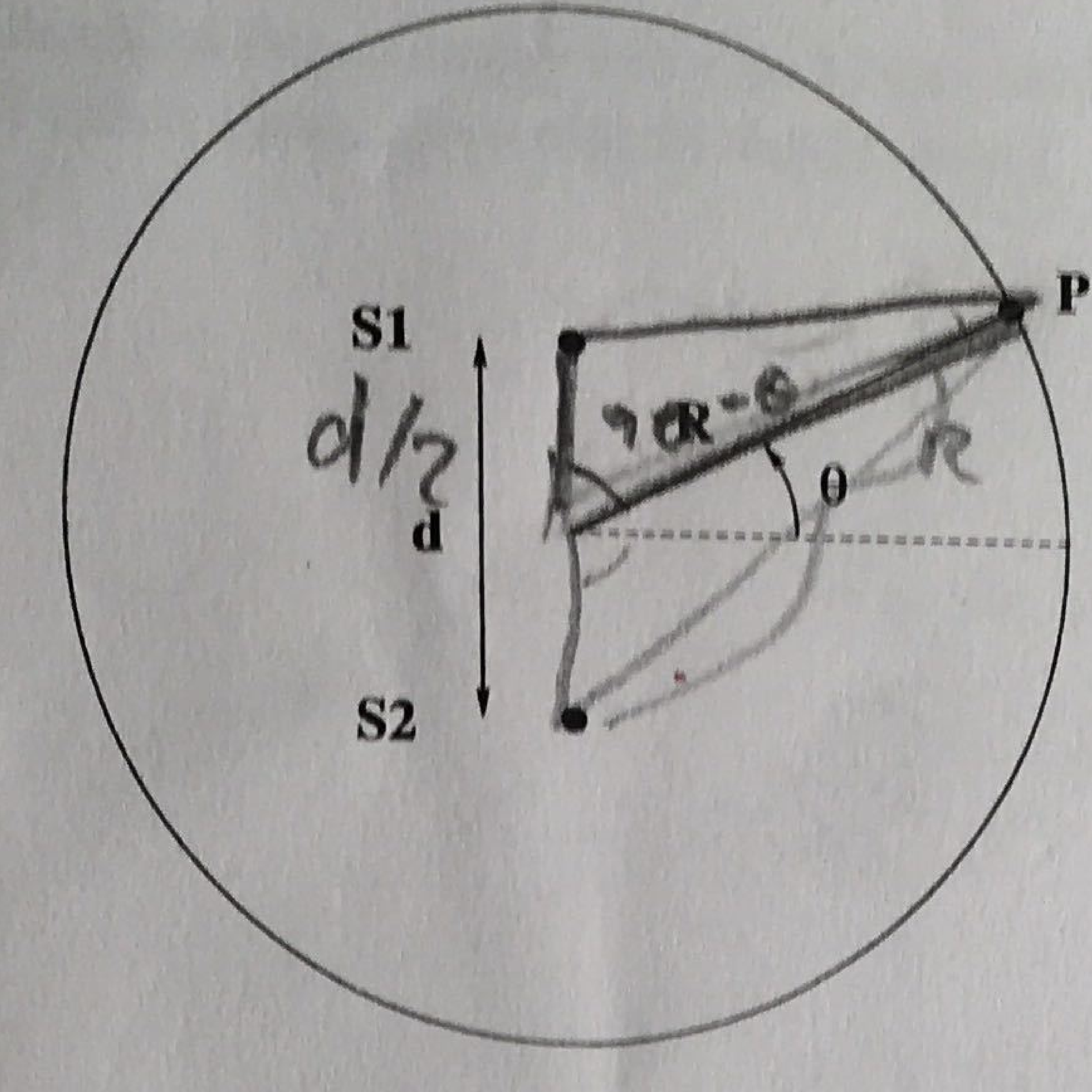


$\frac{1}{q_{mac}} = \frac{-4}{np} - \frac{1}{P_m}$

$q_m = \frac{1}{\frac{-4}{np} - \frac{1}{P_m}} < np$

$1 > -4 - \frac{np}{P}$

$5 > -\frac{np}{P}$



3) It is possible to use interference to create antenna arrays that preferentially radiate radio-frequency energy along preferred directions ("lobes") in the azimuth.

Let's analyze the simplest case - take a pair of vertically-oriented antennas separated by a horizontal distance d, driven in phase by a radio signal that has a wavelength λ in vacuum. The illustration shows a top-down view of the arrangement.

10

- 3a) (10 points) Find the exact phase difference between the waves that originate at sources S_1 and S_2 when they arrive at point P, located a distance R from the midpoint between the sources, in the azimuthal direction θ.

$c^2 = a^2 + b^2 - 2ab\cos C$

$c_2 = \sqrt{R^2 + \frac{d^2}{4} - dR\cos(90+\theta)}$

$c_1 = \sqrt{R^2 + \frac{d^2}{4} - dR\cos(90-\theta)}$

$\boxed{\Delta\theta = \frac{2\pi}{\lambda}(c_1 - c_2)}$

$\Delta\phi = k\,\Delta p$

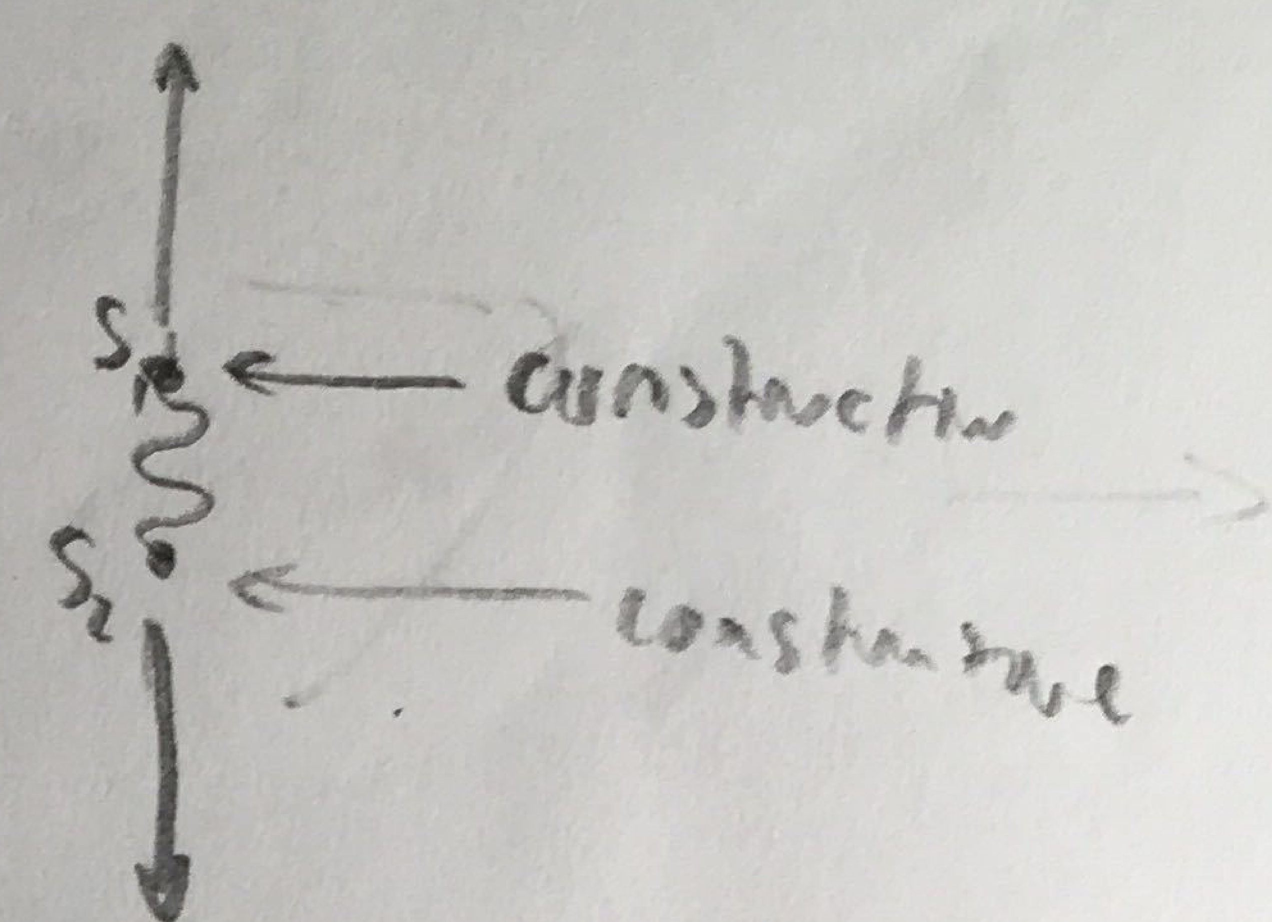
2

- 3b) (5 points) Show that, in the limit that $R \gg d$, that total phase difference is no longer a function of R and θ, but now, simply a function of θ only (remember, the values of d and λ are fixed). For the rest of the problem, we will assume that we are in this so-called *far-field* limit.

$\lim_{R \gg d} \quad \Delta\theta = \sqrt{R^2 - dR\cos(90-\theta)} - \sqrt{R^2 + dR\cos(90+\theta)}$

X

- 3c) (5 points) In what azimuthal direction(s) will the lobes point if $d = 2\lambda$? How many azimuthal lobes are there? (*Careful!* A quick qualitative sketch might help.)



- 3d) (5 points) In what azimuthal direction(s) will the lobes point if $d < \lambda$? How many azimuthal lobes are there? (Again: *Careful!*)

- 3e) (5 points) Large trucks often have antennas strapped to the side mirrors on either side of the truck - $d \approx 2$ m, $f \approx 30$ MHz. Where is most of the energy going? How might this be advantageous to the truck driver? Much like biological evolution, technological solutions that work well tend to persist. Police cars usually have a single radiator that emits uniformly in all azimuthal directions. How did they evolve differently?