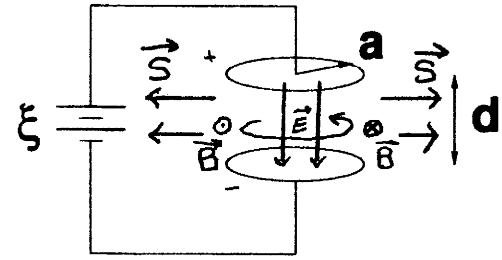


A parallel-plate capacitor made up of circular conducting plates of radius a separated by a distance d is connected across a battery of potential difference ξ

- 1a) (5 points) Find the magnitude and direction of the electric field produced by the capacitor. Sketch it in on the diagram.

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 \pi a^2}$$

$$q = C\,\Delta V$$

$$q = \frac{\epsilon_0 \pi a^2}{d}\mathcal{E}$$

$$\boxed{E = \mathcal{E}/d \text{ directed downward}}$$

- 1b) (10 points) Suppose we increase the separation between the plates at a rate $\frac{dd}{dt}$. Find the magnitude and direction of the magnetic field that results. Sketch it on the diagram.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\left(I + \epsilon_0 \frac{d\Phi_E}{dt}\right)$$

$$B2\pi r = \mu_0\epsilon_0 \frac{d}{dt}\left(\frac{\mathcal{E}}{d}\pi r^2\right)$$

$$B2\pi r = -\frac{\mu_0\epsilon_0\mathcal{E}\pi r^2}{d^2}\frac{dd}{dt}$$

Take the Amperian loop to be a circle who's plane is perpendicular to $\vec{E}$, centered on the longitudinal symmetry axis of the capacitor. Let $d\vec{s}$ run around the loop in the right-hand direction with respect to $\vec{E}$

$$B = -\frac{1}{2}\frac{\mu_0\epsilon_0\mathcal{E}r}{d^2}\frac{dd}{dt}$$

$$\frac{dd}{dt} > 0$$

$$\Rightarrow B < 0$$

B loops "left-handed" around $\vec{E}$

$$\boxed{B = \frac{1}{2}\frac{\mu_0\epsilon_0\mathcal{E}r}{d^2}\frac{dd}{dt}}$$

$\vec{B}$ runs along the azimuthal circle opposite the right-hand direction relative to $\vec{E}$

- 1c) (10 points) **Find the magnitude and direction of the Poynting vector at the boundary of the capacitor. Sketch it on the diagram.**

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$: $\vec{S}$ points radially outward from the longitudinal symmetry axis

$S = \frac{1}{\mu_0} EB$ (at $r=a$)

$S = \frac{1}{\mu_0} \left(\mathcal{E}/d\right)\left(\frac{1}{2} \frac{\mu_0 \epsilon_0 a \mathcal{E}}{d^2} \frac{dd}{dt}\right)$

$$\boxed{S = \frac{1}{2} \frac{\epsilon_0 a}{d^3} \mathcal{E}^2 \frac{dd}{dt}}$$

$\vec{S}$ points radially away from the longitudinal symmetry axis

- 1d) (5 points) **Show that electromagnetic waves are delivering (or removing) energy at the same rate that the energy stored in the capacitor is changing.**

Waves:

$P = \int \vec{S} \cdot d\vec{A}$

$P = SA$

$P = \frac{1}{2} \frac{\epsilon_0 a}{d^3} \mathcal{E}^2 \frac{dd}{dt} \cdot 2\pi a d$

$P = \frac{\epsilon_0 \pi a^2}{d^2} \mathcal{E}^2 \frac{dd}{dt}$

Capacitor:

$U = \frac{1}{2} C \Delta V^2$

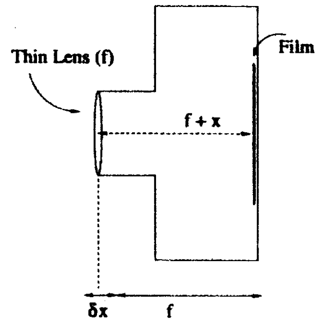
$U = \frac{1}{2} \frac{\epsilon_0 \pi a^2}{d} \mathcal{E}^2$

$P = \frac{dU}{dt} = \frac{dU}{dd} \frac{dd}{dt}$

$P = -\frac{1}{2} \frac{\epsilon_0 \pi a^2}{d^2} \mathcal{E}^2 \frac{dd}{dt}$

As I mentioned in the exam, this doesn't actually work - what happened? The Poynting vector describes the rate at which the capacitor sends energy to the battery; it is blind to the work the capacitor does (mechanically) on the agent that separates the plates...

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$q=f+x$$
$$0\le x\le \delta x$$



2) The important parts of the camera shown above are the film and the converging lens of focal length f. The distance between the lens and the film is given by $f+x$, where x can be adjusted to any value between 0 and δx to bring the subject to be photographed into focus.

- 2a) (5 points) Find the near-point (the closest point at which you can take a focused picture of an object) and the far-point (the farthest point at which you can take a focused picture) for this camera.

$$\frac{1}{p}+\frac{1}{f+x}=\frac{1}{f}$$
$$\frac{1}{p}=\frac{x}{f(f+x)}$$
$$p=f\left[1+f/x\right]$$
where
$$0\le x\le \delta x$$

$$P(0)=f(1+\infty)$$
$$P(\delta x)=f(1+f/\delta x)$$

$$\boxed{\text{Near Point: } P=f\left[1+f/\delta x\right] \quad \text{Far Point: } P=\infty}$$

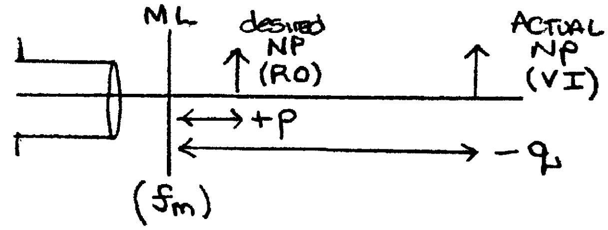
- 2b) (5 points) Suppose we're focused on an object and the lens is located at a distance $f+x$ from the film. What is the magnification of the object? Where would we have to place the object to get the greatest magnification and what would that magnification be?

$$M=\frac{-q}{p} \qquad q=f+x \qquad \frac{1}{p}=\frac{x}{f(f+x)} \quad \text{(part a)} \qquad 0\le x\le \delta x$$

$$\boxed{M(x)=\frac{-x}{f}}$$

By inspection, the greatest magnification occurs when we focus on an object located at the near point: $M(\delta x)=-\frac{\delta x}{f}$

- 2c) (10 points) In order to take close-up shots of an object, one may place a "*macro*" lens adjacent to the lens of the camera. In practice, the macro lens produces an image of the close object at the near-point of the camera and the camera takes a picture of that image. **What type of image must the macro lens produce?** If the desired distance between the object and the two lenses is $\frac{1}{4}$ of the camera's original near-point, **what focal length should the macro lens have? Is the macro lens converging or diverging?** [Hint: Sketch the macro lens, the object at the new near-point and the image at the old near-point. Mind your p's and q's].



$\boxed{\text{The macro lens must produce a } \underline{\text{Virtual}} \text{ image}}$

$P = -\frac{1}{4}q \quad (q<0)$

$-\frac{4}{q} + \frac{1}{q} = \frac{1}{f_m} \Rightarrow f_m = -\frac{q}{3} \; (>0)$

$\boxed{\text{The macro lens is a Converging lens}}$

$$\boxed{f_m = \frac{1}{3} f \left(1 + f/\delta x\right)}$$

The nearpoint distance from part a was:

$P = f\left(1 + f/\delta x\right) \rightarrow$ we need to drop a $\underline{\text{Virtual}}$ $(q<0)$ image there.

$q = -f\left(1 + f/\delta x\right)$

- 2d) (5 points) With the macro lens in place and the object located at the new effective near-point ($\frac{1}{4}$ of the camera's original near-point distance), what is the total magnification of the object?

For the macro lens: $p = -\frac{1}{4}q$: $M_{macro} = -q/p = 4$

For the Camera (part b) : $M_{camera} = -\frac{\delta x}{f}$

$M = M_{macro} \cdot M_{camera}$

$$\boxed{M = -4\frac{\delta x}{f}}$$

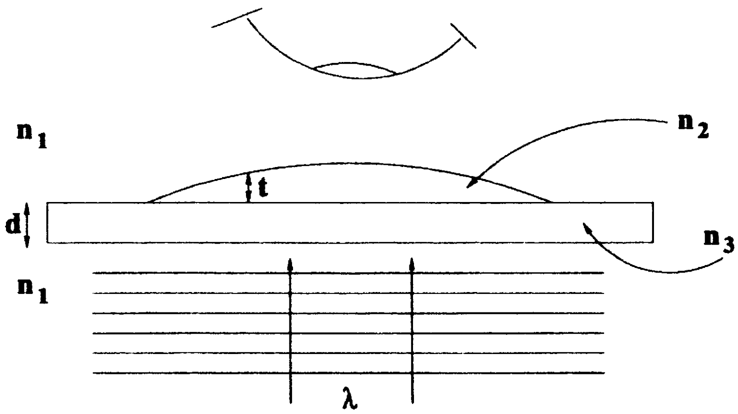
We get a factor of 4 improvement!

- 2e) (5 points) The ability to take large close-up photos comes at a cost. Find (and interpret) the new far-point for the camera when the macro lens is attached.

without the macro, the uncorrected far point is infinity. The new far point occurs when the macro lens drops its virtual image at infinity – in other words, when the (real) object is placed at the focal point of the macro lens:

New far point: $p = \frac{1}{3} f\left(1 + f/\delta x\right)$

→ This is a really shallow "far point"!!



3) Monochromatic light of wavelength λ is directed upward, normal to the surface of a glass slide of refractive index n_3. The light continues upward through an oil drop of refractive index n_2, towards the eyes of an observer. The whole apparatus sits in air (refractive index n_1). For the following, take $n_2 > n_3 > n_1$.

- 3a) (10 points) For what values of thickness (d) will the light be transmitted most efficiently through the glass slide?

$$\Delta\theta_{path} = k_3 \cdot 2d = \frac{2\pi n_3}{\lambda} 2d$$

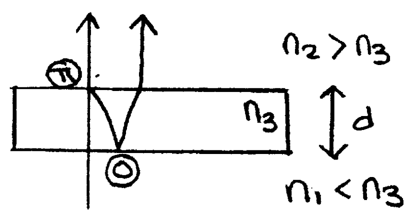
$$\Delta\theta_{IC} = 0$$

$$\Delta\theta_{ref} = \pi - 0$$

$$\Delta\theta_{TOT} = 2N\pi \quad (\text{Max})$$

$$\frac{2\pi n_3}{\lambda} 2d + \pi = 2N\pi$$

$$\boxed{d = \frac{2N-1}{n_3} \frac{\lambda}{4}}$$



The bounce off n_2 picks up π, the bounce off n_1 doesn't

- 3b) (10 points) For what values of thickness (t) will the observer note dark fringes?

$$\Delta\theta_{path} = k_2 \cdot 2t = \frac{2\pi n_2}{\lambda} 2t$$

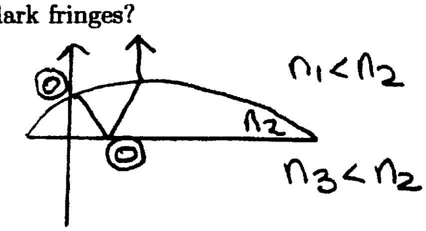
$$\Delta\theta_{IC} = 0$$

$$\Delta\theta_{ref} = 0 - 0$$

$$\Delta\theta_{TOT} = (2N+1)\pi \quad (\text{min})$$

$$\frac{2\pi n_2}{\lambda} 2t = (2N+1)\pi$$

$$\boxed{t = \frac{2N+1}{n_2} \frac{\lambda}{4}}$$



The bounce off n_1 and the bounce off n_2 do not pick up a π

- **3c) (5 points)** What sort of pattern will the observer see, looking down on the oil drop? Note, in particular, what the observer will see along the perimeter of the drop and at the highest point.

The observer will see a topographic map of the drop with rings tracing out the curves where $t = \frac{(2N+1)}{n_2}\frac{\lambda}{4}$

The highest point is not likely to land on a bright line or a dark fringe - it will be something in-between.
The perimeter falls right between the $N=0$ and the (not physical) $N=-1$ order dark fringe.
Indeed, $\Delta\varphi_{TOT} = 0 \Rightarrow$ it will be bright.

- **3d) (5 points)** If the observer counts N dark fringes, how thick is the drop at its highest point? The more accurate your answer, the more points you will get.

The last fringe has order number $N-1$, the next would be order number N

$$\left\{\begin{array}{lcccc} \text{Count}: & 1 & 2 & 3 & \ldots \\ \text{order number}: & 0 & 1 & 2 & \ldots \end{array}\right.$$

$$t(N-1) \leq T \leq t(N)$$

$$\frac{2(N-1)+1}{n_2}\frac{\lambda}{4} \leq T \leq \frac{2N+1}{n_2}\frac{\lambda}{4}$$

$$\boxed{\frac{2N-1}{n_2}\frac{\lambda}{4} \leq T \leq \frac{2N+1}{n_2}\frac{\lambda}{4}}$$

It looks like we can determine the maximum thickness of the drop to within $\frac{\lambda}{2n_2}$!