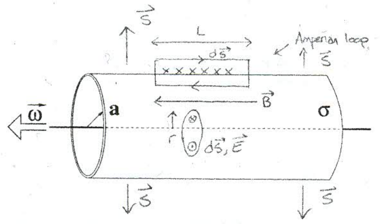


1) Charge is spread uniformly over a long, thin, non-conducting cylinder of radius a so that it has a surface charge density σ. At the particular moment under consideration, the cylinder is rotating around its symmetry axis with an angular velocity $\vec\omega$, which is slowly decreasing.

- 1a) (10 points) Find the magnitude and direction of the magnetic field for points inside the cylinder.

The rate at which charge flows through the Amperian loop: $I = \frac{\sigma 2\pi a L}{2\pi/\omega} = \sigma a L \omega$

Ampere's Law: $\oint \vec B\cdot d\vec l = \mu_0 I_{enc}$

$BL = \mu_0 \sigma a \omega L$

$B = \mu_0 \sigma a \omega$

Use the right-hand-rule with the current → $\vec B$ is directed along $\vec\omega$ if $\sigma > 0$ ...

$$\boxed{\vec B = \mu_0 \sigma a \vec\omega}$$

- 1b) (10 points) Find the magnitude and direction of the electric field for points inside the cylinder.

Faraday's Law: $\oint \vec E\cdot d\vec s = -\frac{d\Phi_B}{dt}$

$E 2\pi r = -\frac{d}{dt}(\mu_0 \sigma a \omega \cdot \pi r^2)$

$E = -\frac{1}{2}\mu_0 \sigma a r \frac{d\omega}{dt}$ ← since $\frac{d\omega}{dt} < 0$, $\vec E$ is directed along $d\vec s$, right-handed about $\vec\omega$

$|\vec E| = -\frac{1}{2}\mu_0 \sigma a r \frac{d\omega}{dt}$
Pointing along an azimuthal circle in the right-handed sense about $\vec\omega$

- 1c) (5 points) Find the magnitude and direction of the Poynting vector at points just inside the cylinder's charged surface.

$\vec S = \frac{1}{\mu_0}\vec E \times \vec B$ ⇒ Right hand rule ~ $\vec S$ is directed radially outward from the longitudinal axis...

$S(a) = \frac{1}{\mu_0}E(a)B(a) = \frac{1}{\mu_0}\left(-\frac{1}{2}\mu_0 \sigma a^2 \frac{d\omega}{dt}\right)(\mu_0 \sigma a \omega)$

$S(a) = -\frac{1}{2}\mu_0 \sigma^2 a^3 \omega \frac{d\omega}{dt}$
directed radially out from the longitudinal axis...

remember: $\frac{d\omega}{dt} < 0$

- 1d) (5 points) Find the rate at which electromagnetic energy is flowing into/out of the empty volume just inside the cylinder's charged surface, per unit length.

$P = \int \vec S \cdot d\vec A$

$= \int S dA (1)$

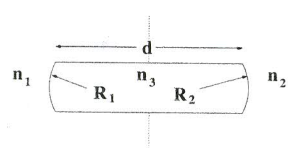
$= -\frac{1}{2}\mu_0 \sigma^2 a^3 \omega \frac{d\omega}{dt} 2\pi a L$

$$\boxed{\frac{P}{L} = -\mu_0 \pi a^4 \sigma^2 \omega \frac{d\omega}{dt}}$$

Check: $U = \frac{1}{2}LI^2$

$\frac{dU}{dt} = LI\frac{dI}{dt}$

$P = \left(\frac{\mu_0 (1)^2 \pi a^2}{L}\right)\left(\sigma^2 a^2 L^2 \omega \frac{d\omega}{dt}\right)$ ✓



2) What happens when a thin lens straddles two different media? To find out, let's examine the case of the thick lens shown above.

- 2a) (10 points) Write a set of equations relating object and image distances to their respective surfaces (that is, p_1 and q_1 to R_1; p_2 and q_2 to R_2). Write an additional equation relating the initial object distance (p_1) to the final image distance (q_2) - you may leave one intermediate quantity (p_2 or q_1) in this equation, but not both.

$\frac{n_1}{p_1} + \frac{n_3}{q_1} = \frac{n_3 - n_1}{R_1}$

$\frac{n_3}{p_2} + \frac{n_2}{q_2} = \frac{n_2 - n_3}{R_2}$

$d = q_1 + p_2$

$\frac{n_1}{p_1} + \frac{n_2}{q_2} + n_3\left(\frac{1}{p_2} + \frac{1}{d - p_2}\right) = \frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}$

- 2b) (10 points) Now, evaluate that last equation in the relevant limit to find the appropriate expression for a thin-lens straddling two media. Find the focal-lengths on both sides of the lens (use the convention that f_i is the focal length when light emerges on the side with index n_i).

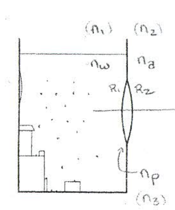
$d \to 0$

$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}$

$f_2 = \frac{1}{n_2}\left[\frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}\right]$

$f_1 = \frac{1}{n_1}\left[\frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}\right]$

Careful! There are no magnitude bars around the R's so sign conventions are in play
In both cases, the sign of R_1 & R_2 is taken relative to light incident from n_1



$n_w = 4/3$
$n_a = 1$
$n_p = 3/2$

$R_1 = +a$
$R_2 = -a$

$p = a/3$

- 2c) (10 points) When I was a kid, I managed to talk my folks into buying me a family of seamonkeys. You may have seen the ads - happy little party animals living in a micro-world on your bookshelf. Well... what you got was a "Seamonkey Habitat" (a small plastic aquarium with a big magnifying glass embedded on one side) and a bag of brine shrimp eggs.

Suppose the radius-of-curvature for the water-side of the bi-convex lens is a and the radius-of-curvature for the air-side of the lens is also a. The water has an index of refraction $n_w = \frac{4}{3}$, the plastic has an index of refraction $n_p = \frac{3}{2}$ and the surrounding air has an index of refraction $n_a = 1$...

Sashimi, a seamonkey, is sitting a distance $\frac{a}{3}$ away from the lens. Where will its image appear as I stare into the tank? Will that image be real or virtual?

$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}$

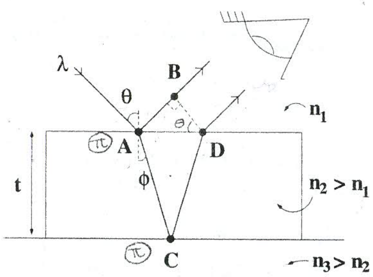
$\frac{4/3}{a/3} + \frac{1}{q} = \frac{3/2 - 4/3}{a} + \frac{3/2 - 1}{a}$

$\frac{4}{a} + \frac{1}{q} = \frac{1}{6a} + \frac{1}{2a}$

$\frac{1}{q} = \left(\frac{2}{3} - 4\right)\frac{1}{a}$

$q = -\frac{3}{10}a$ ⇒

Sashimi will appear to be a distance $3/10\,a$ behind the lens (on the water side).
The image is Virtual



3) Let's analyze the more general case of oblique incidence on a thin film of thickness t.

- 3a) (10 points) Work on the geometry first. Find $\overline{AB}$, $\overline{ACD}$ and $\overline{AD}$ in terms of θ, ϕ and t.

$\overline{AC} = \dfrac{t}{\cos\phi}$

$\overline{AD} = 2t \cdot \tan\phi$

$\overline{AB} = \overline{AD}\sin\theta$

$$\boxed{\begin{aligned} \overline{AB} &= 2t\sin\theta\tan\phi \\ \overline{ACD} &= \frac{2t}{\cos\phi} \\ \overline{AD} &= 2t\tan\phi \end{aligned}}$$

- 3b) (10 points) Now find the phase difference between the rays (as they arrive at the eye) in terms of n_1, n_2, n_3, λ, ϕ and t.

$\Delta\Theta_{1C} = 0$

$\Delta\Theta_{ref} = \pi - \pi = 0$

$\Delta\Theta_{path} = k_2\overline{ACD} - k_1\overline{AB}$

$\Delta\Theta_{TOT} = \dfrac{2\pi}{\lambda}\left[n_2\dfrac{2t}{\cos\phi} - n_1 2t\sin\theta\tan\phi\right]$

$\Delta\Theta_{TOT} = \dfrac{2\pi}{\lambda}\dfrac{2t}{\cos\phi}n_1\left[\dfrac{n_2}{n_1} - \sin\theta\sin\phi\right]$

$\Rightarrow n_1\sin\theta = n_2\sin\phi$

$\dfrac{n_2}{n_1} = \dfrac{\sin\theta}{\sin\phi}$

$\Delta\Theta_{TOT} = \dfrac{2\pi}{\lambda}\dfrac{2t}{\cos\phi}n_1\sin\theta\left[\dfrac{1}{\sin\phi} - \sin\phi\right]$

$\Delta\Theta_{TOT} = \dfrac{2\pi}{\lambda}\dfrac{2t}{\cos\phi}n_1\sin\theta\ \dfrac{\cos^2\phi}{\sin\phi}$

... $n_1\sin\theta = n_2\sin\phi$

$$\boxed{\Delta\Theta_{TOT} = \frac{2\pi}{\lambda}2t\,n_2\cdot\cos\phi}$$

– Check –

in NVI, $\Delta\Theta_{TOT} \simeq \dfrac{2\pi n_2}{\lambda}2t$

✓✓

- 3c) (5 points) Use a modest amount of trigonometry to replace ϕ in your answer to part b with θ.

$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\left(\dfrac{n_1\sin\theta}{n_2}\right)^2}$

$$\boxed{\Delta\Theta_{TOT} = \frac{2\pi n_2}{\lambda}2t\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta}}$$

- 3d) (5 points) You are out walking on a sunny day. You see some water pooled in the gutter next to the sidewalk and notice it is covered by a thin film of motor-oil. You shift your gaze higher and lower as you stare at the thin film, and note that the reflecting light seems to be changing colors. What is happening?

The Colors I see Correspond to wavelengths (in the visible spectrum) that lead to Constructive interference... Moving my head changes the value of θ for the light that makes it to my eyes; Changing λ (and hence the observed Color) of the rays that undergo Constructive interference.