MT1 Physics 1C F15

Full Name (Printed) Megan Williams

Full Name (Signature) [illegible]

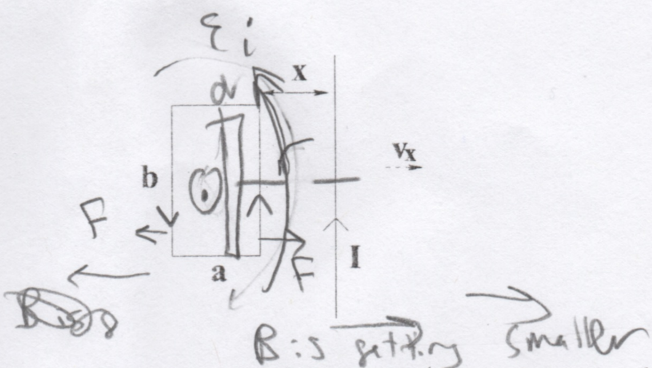
Student ID Number ____________________

Seat Number ____________________

Problem	Grade
1	26 /30
2	28 /30
3	26 /30
Total	80 /90

Wow!

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



1) In the diagram above, a rectangular conducting loop (dimensions a and b, resistance R) and a long straight wire that carries an electrical current I are both oriented so that they sit in the plane of the page. They will, for the duration of the problem, remain in the plane of the page with the wire carrying an electrical current I parallel to the right side of the conducting loop at a distance x to that side (as shown).

1

0

- 1a) (10 points) Assuming the conducting loop remains fixed in space while the wire is pulled away at a speed v_x, what is the magnitude of the resulting current induced in the loop? In what direction is that induced current traveling on the side closest to the wire (with or against the current direction in the wire?)

$\mathcal{E}_i = -\frac{d\Phi}{dt}$ $\quad \Phi = \int B \cdot dA$ $\quad B = \frac{\mu_0 I}{2\pi r}$

$A = b\,dr$ $\quad \Phi = \int_x^{x+a} \frac{\mu_0 I}{2\pi r} \cdot b\,dr$

$\Phi = \frac{\mu_0 I b}{2\pi}\left(\ln(x+a) - \ln x\right)$ $\quad \frac{(x - x+a)}{x(x+a)}$

$\frac{d\Phi}{dt} = \frac{\mu_0 I b}{2\pi}\left(\frac{1}{x+a} - \frac{1}{x}\right)\frac{dx}{dt}$

$\mathcal{E}_i = \frac{\mu_0 I a b V}{2\pi x(x+a)}$

$I = \frac{\mu_0 I a b V_x}{R 2\pi x(x+a)}$

in side closest to wire, current is with the wire (current flows counterclockwise)

1

0

- 1b) (10 points) How large and in what direction is the force exerted on the loop by the wire?

Force on wire 1: $\rightarrow F_1 = IbB$ $\quad F_2 = IbB$

$F = F_1 - F_2$ $\quad = \frac{(\mu_0 I b)^2 a V_x}{2\pi R x(x+a)} \cdot \frac{1}{2\pi}\left(\frac{1}{x} - \frac{1}{x+a}\right)$

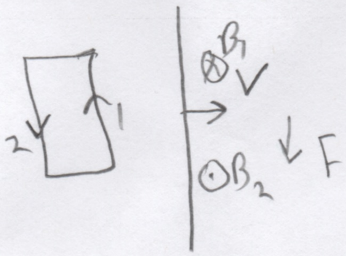
$F = \frac{(\mu_0 I ab)^2 V_x}{4\pi^2 R\, x^2(x+a)^2}$

Direction is to the right (towards wire)

- 1b) *Continued...*

4

- 1c) (5 points) How large and in what direction is the force exerted on the wire by the conducting loop? Is this result consistent with Lenz's Law? Explain.



qvB

$F = \cancel{ILB}$ $\cancel{q}vB$ $= \cancel{IL}\times B$

downward force exerted on the wire

$\cancel{F = ILB}$

$q = Iv$

$$F = \left(\frac{\mu_0 I}{2\pi(x+a)} - \frac{\mu_0 I}{2\pi(x+a)}\right) V \hat{J}$$

Yes it is consistent w/ lenz's law

why?

2

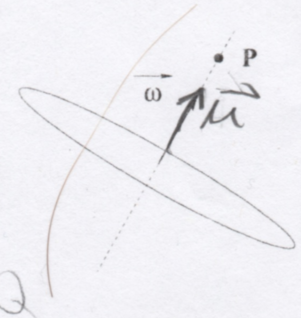
- 1d) (5 points) Find the net torque on the conducting loop. For full credit, the grader must be able to follow the logic of your calculation.

$\tau = \mu \times B$ $= \mu B \sin\phi$

$\sin\phi = 1$

$\mu = NIA$ $A = ab$

$$\tau = \frac{\mu_0 I a b V_x}{R\, 2\pi x (x+a)} ab$$



Uniform Ring
Q, R

2) A uniform circular ring of charge Q and radius R rotates around its longitudinal symmetry axis with an angular velocity $\vec{\omega}$, as shown.

- 2a) (5 points) Find the (vector) magnetic dipole moment of the ring.

$$\mu = I\vec{A} \qquad A = \pi R^2$$

$$I = \frac{\Delta Q}{\Delta T} = \frac{Q\omega}{2\pi}$$

$$\mu = \frac{Q\omega}{2\pi}\pi R^2$$

$$\vec{\mu} = \frac{1}{2}Q\omega R^2\hat{\omega}$$

in same direction as $\vec{\omega}$ (see diagram)

5/5

- 2b) (10 points) Find the (vector) magnetic field due to the ring at a point (P) on the longitudinal symmetry axis, a distance z from the center of the ring.

$$dB = \frac{\mu_0 I}{4\pi r}\int \frac{d\vec{L}\times\hat{r}}{r^2}\cos\phi$$

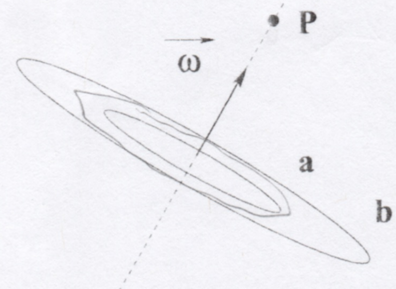
$$\cos\phi = \frac{R}{\sqrt{z^2+R^2}} \qquad r^2 = z^2+R^2 \qquad dL = ds$$

$$I = \frac{Q\omega}{2\pi}$$

$$dB = \frac{\mu_0 I}{4\pi r}\frac{R}{(z^2+R^2)^{3/2}} \cdot \int_0^R 2\pi r\,dr$$

$$= \frac{\mu_0 Q\omega R^2}{4\pi(z^2+R^2)^{3/2}}$$

10/10



- 2c) (15 points) Now, let's replace the ring with a washer of inner-radius a and outer-radius b that carries a surface charge density

$$\sigma(r) = \frac{Q}{2\pi \ln(b/a)} \frac{1}{r^2}$$

where r measures the distance from the center of the washer to a point within the washer. Find the (vector) magnetic field due to the ring at a point (P) on the longitudinal symmetry axis, a distance z from the center of the ring.

$dq = \sigma dr$

$\cancel{dB = \mu_0}$

Let ring have charge density λ

Ring $B(r) = \dfrac{\mu_0 q(r) \omega r^2}{4\pi (z^2 + r^2)^{3/2}}$

$dq = \sigma(r)\, dr\, 2\pi r$

build up disk of ∞ many Rings

$B = \displaystyle\int_a^b \frac{\mu_0 Q \cancel{r^2} \omega\, dr}{2\pi \ln(b/a) \cancel{r^2}\, 4\pi (z^2+r^2)^{3/2}}$

Let $r = z\tan\theta$

$dr = z\sec^2\theta\, d\theta$

$= \dfrac{\mu_0 \omega Q}{8\pi^2 (\ln\frac{b}{a})} \displaystyle\int_a^b \frac{1}{(z^2+r^2)^{3/2}} dr$

$\displaystyle\int \frac{z \sec^2\theta}{z^3 \sec^3\theta} d\theta$

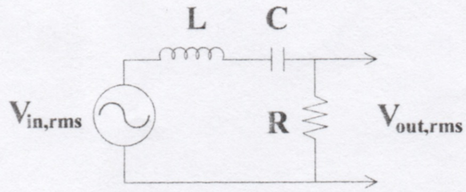
$\dfrac{\mu_0 \omega Q}{8\pi^2 z \ln\frac{b}{a}} \displaystyle\int \cos\theta\, d\theta$

$\sin\theta = \dfrac{r}{\sqrt{z^2+r^2}}$

$\sin\theta$

$$\boxed{= \frac{\mu_0 \omega Q}{8\pi^2 z \ln\frac{b}{a}} \left(\frac{b}{\sqrt{b^2+z^2}} - \frac{a}{\sqrt{a^2+z^2}} \right)}$$

13/15



Filters are classified by the frequency-range of the signals they deliver to their output. High-pass and low-pass filters preferentially pass high- and low-frequency signals, respectively. Band-pass filters preferentially pass signals that have frequencies within some range (or 'band') of frequencies. Notch filters actually remove signals whose frequencies lie within some range of frequencies.

high pass

$L \to \infty$ $X_L \to \infty$

V f

low pass

5

- 3a) (5 pts) Qualitatively discuss how the components in the circuit above will react to input signals over a very broad range of frequencies (say, 0 Hz to ∞ Hz). Explain how their respective behaviors, taken together, will make the circuit shown above behave like a band-pass filter.

$V_L = IX_L$

$V = \frac{I}{\sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}}$

$V = IZ_C$

The reactance of an inductor is $X_L = \omega L$, The inductor has a strong opposition to high frequencies. It acts as a low pass filter. When $\omega \to 0$, $X_L \to 0$. when $\omega \to \infty$, $X_L \to \infty$
The reactance of the capacitor is $X_C = \omega C$.
The reactance of the capacitor has a strong opposition to low frequencies
It has a strong opposition to low frequencies so it is a high pass filter. When $\omega \to 0$, $X_C \to \infty$ when $\omega \to \infty$, $X_C \to 0$

first the signal encounters the inductor. If the signal is low it will continue to the capacitor, where it is filtered out by the high pass filter. If the signal is high it is filtered by the inductor. This only leaves a range of frequencies neither too high nor too low

10

- 3b) (10 pts) If the signal on the input has an rms voltage $V_{in,rms}$ and a frequency ω, how large is the rms voltage at the output? At what frequency will this output voltage be greatest?

$$V = V_{in,rms} \sin(\omega t)$$

$$V_{out,rms,max} = \frac{V_{inrms,max} R}{2}$$

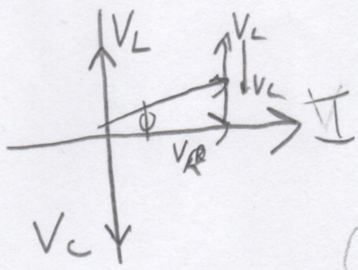
$$V_{out,max} = \frac{V_{in,rms} R}{\sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}}$$

this will be greatest when $\omega L = \frac{1}{\omega C}$

$$\omega = \frac{1}{\sqrt{LC}}$$

4

- 3c) (5 pts) By what phase angle will the output voltage lead or lag the input voltage? Under what conditions will it lead? ... lag?



$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$\omega L < \frac{1}{\omega C}$

the voltage will lead current when $\omega L - \frac{1}{\omega C} > 0$
when: $\omega^2 > \frac{1}{LC}$

the voltage will ~~lead~~ lag current when $\omega L - \frac{1}{LC} < 0$
$\omega^2 < \frac{1}{LC}$

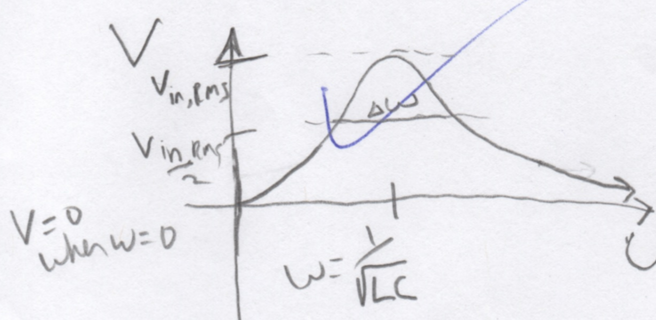
Backwards -1

7

- 3d) (10 pts) Do a quick, qualitative sketch of the output voltage vs. the frequency of the input signal. The width of that peak, $\Delta\omega$, is usually taken to be the "Full Width at Half-Maximum" (FWHM) - the distance (in frequency-space) between the two points at which the output voltage amplitude is half the input amplitude. Find the bandwidth of this filter, and discuss how one might achieve a sufficiently narrow peak without sacrificing output amplitude.

-3

$$V = \frac{V_{max} R}{\sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}}$$



$\omega C,\ \omega L - \frac{1}{\omega C}$

$\frac{\omega^2 CL - 1}{\omega C}$

$$\frac{V_{in,max}}{2} = \frac{V_{max} R}{\sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}}$$

$$\sqrt{(\omega L - \tfrac{1}{\omega C})^2 + R^2} = \cancel{4} R^2$$

$$\sqrt{(\omega L - \tfrac{1}{\omega C})^2} = \sqrt{3}R^2$$

$$\omega L - \frac{1}{\omega C} = R\sqrt{3}$$

$$\frac{1}{C} - \omega R\sqrt{3} = 0$$

$$\omega = \frac{R\sqrt{3} \pm \sqrt{3R^2 + 4L/C}}{2L}$$

Bandwith:

$$\Delta\omega = \frac{R\sqrt{3} + \sqrt{3R^2 + 4L/C}}{2L} - \frac{R\sqrt{3} - \sqrt{3R^2 + 4L/C}}{2L}$$

Almost...