# MT1 Physics 1C F15

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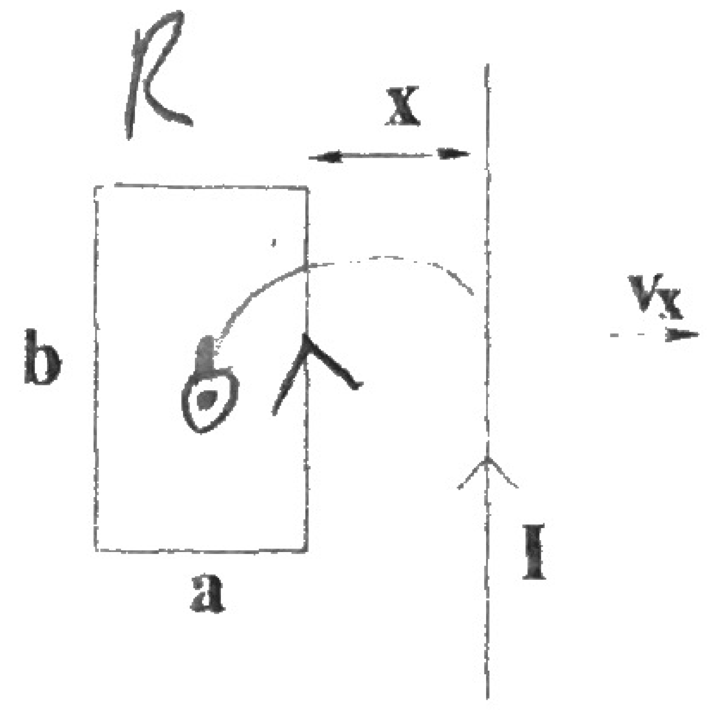
**Seat Number** ______

| Problem | Grade |
|---|---|
| 1 | 11 /30 |
| 2 | 30 /30 |
| 3 | 25 /30 |
| Total | 66 /90 |

Great job! :)

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



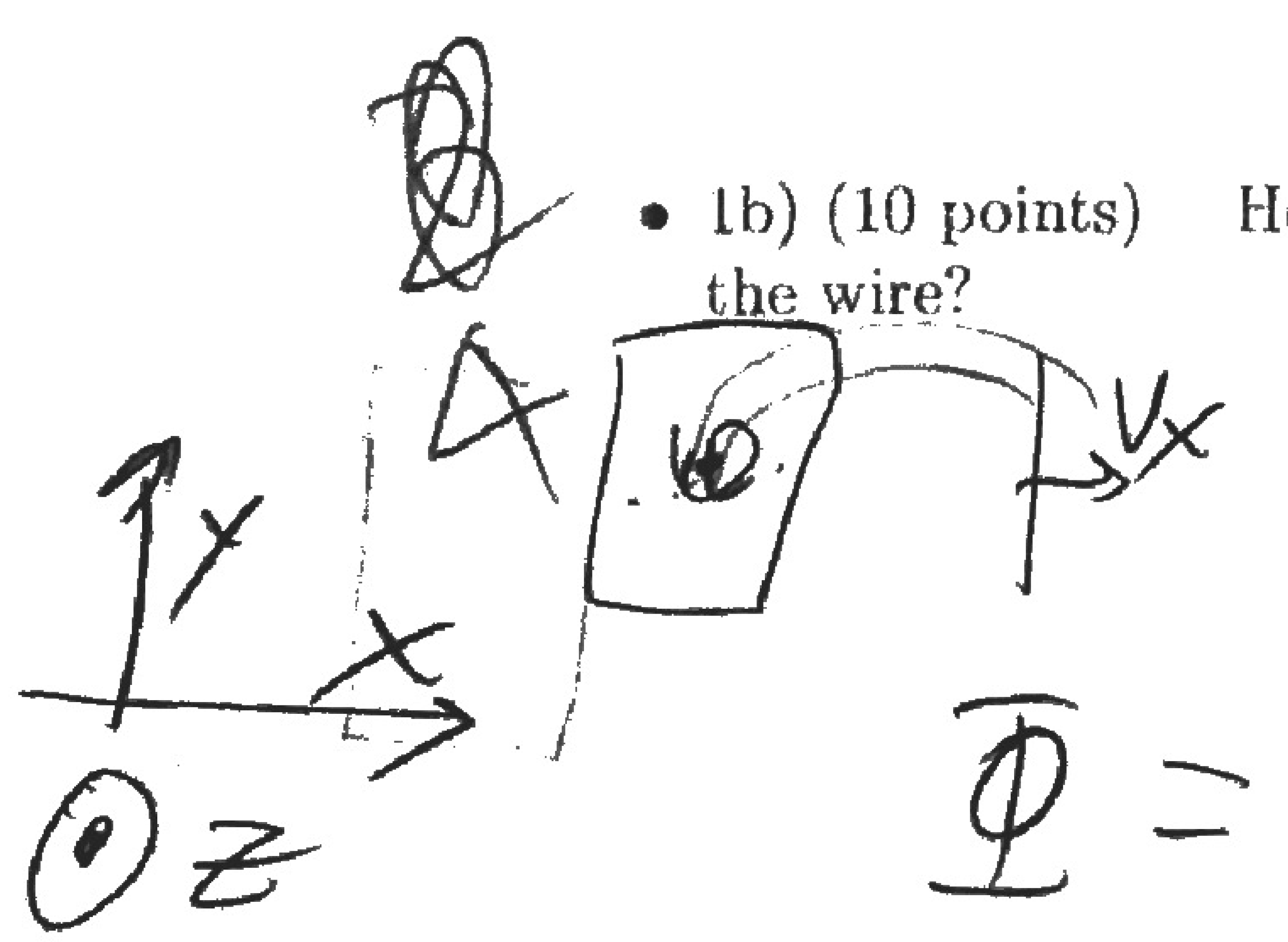

1) In the diagram above, a rectangular conducting loop (dimensions $a$ and $b$, resistance $R$) and a long straight wire that carries an electrical current $I$ are both oriented so that they sit in the plane of the page. They will, for the duration of the problem, remain in the plane of the page with the wire carrying an electrical current $I$ parallel to the right side of the conducting loop at a distance $x$ to that side (as shown).

0

- 1a) (10 points) Assuming the conducting loop remains fixed in space while the wire is pulled away at a speed $v_x$, what is the magnitude of the resulting current induced in the loop? In what direction is that induced current traveling on the side closest to the wire (with or against the current direction in the wire?)

~~the induced current travels with the current, because the induced Emf makes up for the lack of magnetic field out of the page, thus current flows that same way.~~

- 1b) (10 points) How large and in what direction is the force exerted on the loop by the wire?




$$\vec{B}_{loop,I} = \frac{\mu_0 I}{2\pi r} \qquad \hat{A} = \odot$$

$$\Phi = \int \vec{B}\cdot d\vec{A} \qquad A = br \qquad dA = b\,dr$$

$$\Phi = \int_x^{x+a} \frac{\mu_0 I}{2\pi r}\, b\,dr$$

$$\Phi = \frac{\mu_0 I b}{2\pi}\int_x^{x+a} \frac{1}{r}\,dr = \frac{\mu_0 I b}{2\pi}\ln\left(\frac{x+a}{x}\right) = \frac{\mu_0 I b}{2\pi}\ln\left(1+\frac{a}{x}\right)$$

$\vec{F}_{loop} = I\vec{L} \times \vec{B}$

- 1b) *Continued...*

$\mathcal{E}_{i,loop} = -\frac{d\Phi}{dt}$

$= \frac{\mu_0 I b}{2\pi R(1+\frac{a}{x})} V_x \frac{\mu_0 I}{2\pi x}$

$= \frac{\mu_0 I b}{2\pi} \frac{d}{dt}\left(\ln\left(1+\frac{a}{x}\right)\right)$ $\vec{F}_{loop} = \boxed{\frac{\mu_0^2 I^2 b V_x}{4\pi^2 x R(1+\frac{a}{x})} \hat{k}}$

$= \frac{\mu_0 I b}{2\pi} \frac{1}{1+\frac{a}{x}} \cdot \frac{dx}{dt}$ into the page

$\mathcal{E}_{i,loop} = \frac{\mu_0 I b}{2\pi(1+\frac{a}{x})} V_x$ $I_{loop} = \frac{\mathcal{E}_{i,loop}}{R}$

3

- 1c) (5 points) How large and in what direction is the force exerted on the wire by the conducting loop? Is this result consistent with Lenz's Law? Explain.

Opposite 1b

$\vec{F}_{wire} = \frac{\mu_0^2 I^2 b V_x}{4\pi^2 x R(1+\frac{a}{x})} \hat{k}$

Yes, consistent with Lenz's law, it is $\boxed{\frac{-d\Phi}{dt}}$ out of the page

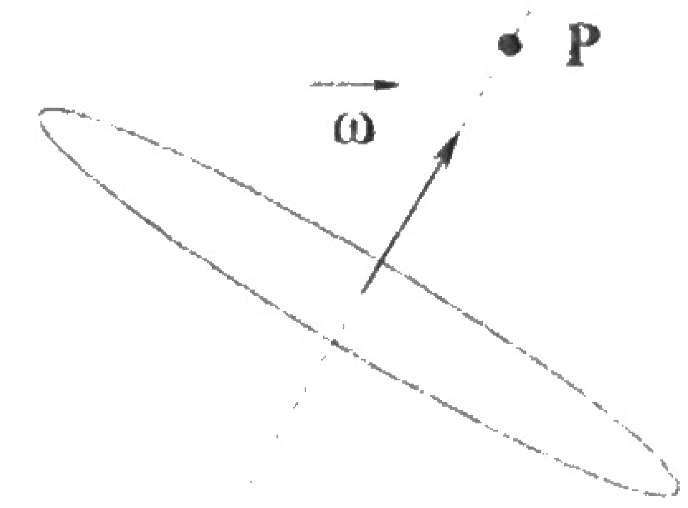
- 1d) (5 points) Find the net torque on the conducting loop. For full credit, the grader must be able to follow the logic of your calculation.

2 $\vec{\mu} = NI\vec{A}$ $\tau = ?$

$= \cancel{\frac{\mu_0 I b}{2\pi R(1+\frac{a}{x})} V_x \cdot ab} \; \hat{k}$

$= \boxed{\frac{\mu_0 I a b^2 V_x}{2\pi R(1+\frac{a}{x})} \hat{k}}$

**Uniform Ring**
**Q, R**




$\mu = NI\vec{A}$

$I = \frac{Q}{T} = \frac{\vec{\omega} Q}{2\pi}$

2) A uniform circular ring of charge $Q$ and radius $R$ rotates around its longitudinal symmetry axis with an angular velocity $\vec{\omega}$, as shown.

$\frac{2\pi}{\omega}$

- 2a) (5 points) Find the (vector) magnetic dipole moment of the ring.

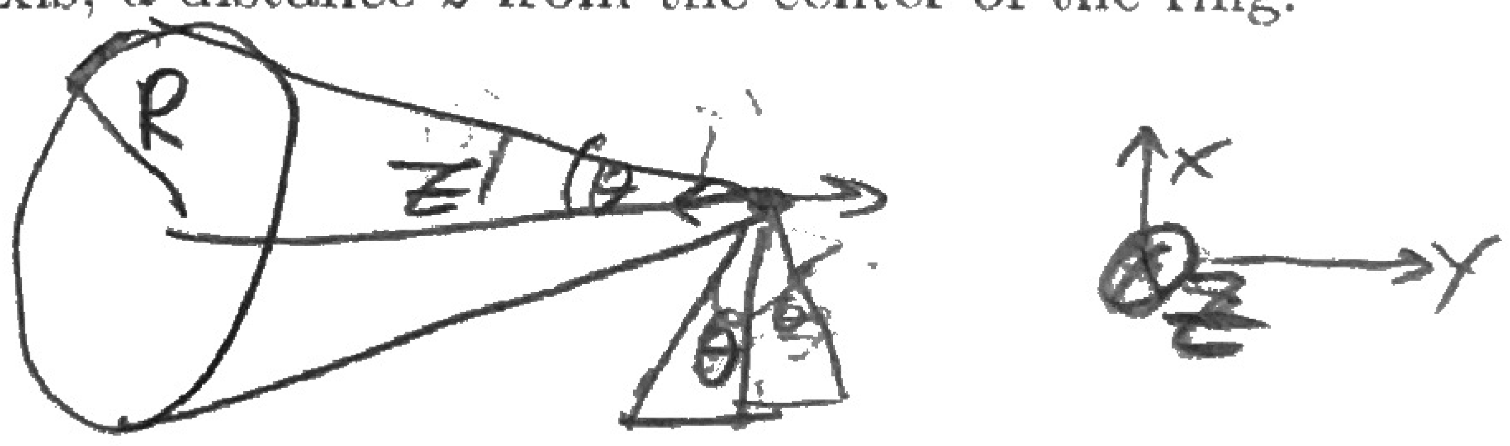
$\vec{\mu} = NI\vec{A}$ $\quad I = \frac{Q}{T}$ $\quad \omega = 2\pi f$

$\vec{\mu} = \frac{\vec{\omega} Q \pi R^2}{2\pi}$ $\quad \vec{\mu} = \gamma \vec{L}$ $\quad \frac{1}{T} = f$

$\vec{\mu} = \frac{QR^2}{2}\vec{\omega}$ $\quad$ 5 $\quad \gamma = \frac{1}{2}\frac{Q}{M}$ $\quad L = MR^2\vec{\omega}$ $\quad T = \frac{2\pi}{\omega}$

- 2b) (10 points) Find the (vector) magnetic field due to the ring at a point ($P$) on the longitudinal symmetry axis, a distance $z$ from the center of the ring.

by symmetry $\vec{B} = B_y \hat{j}$



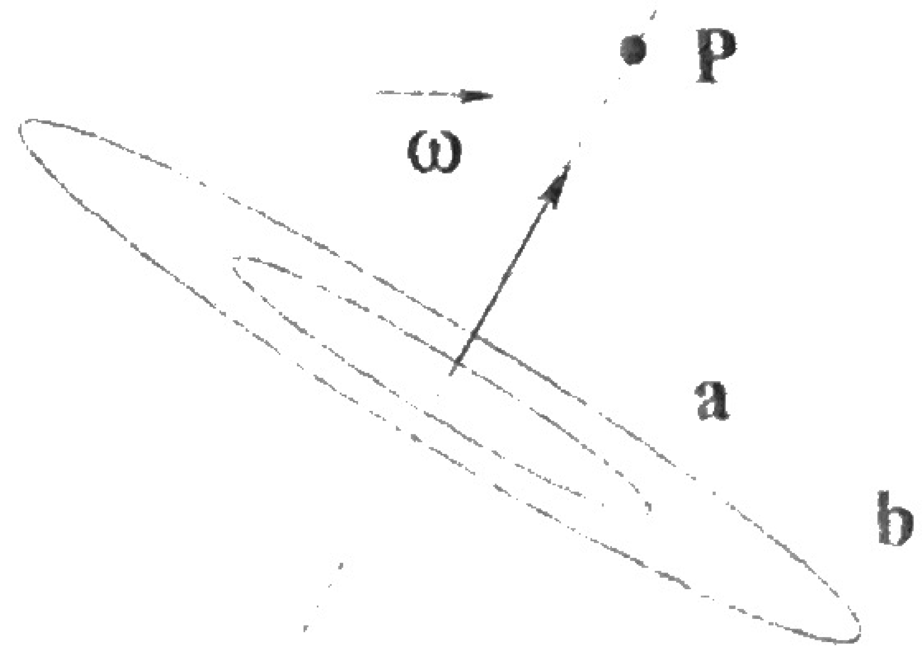

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$ $\quad d\vec{l} \times \hat{r} = |d\vec{l}|\,|\hat{r}| \sin 90°$

$\vec{B} = \frac{\mu_0 \omega Q}{8\pi^2} \int \frac{dl}{r^2} \sin\theta \,\hat{j}$ $\quad \sin\theta = \frac{R}{\sqrt{R^2+z^2}}$ $\quad r = \sqrt{R^2+z^2}$

$= \frac{\mu_0 \omega Q R}{8\pi^2 (R^2+z^2)^{3/2}} \int_0^{2\pi R} dl \,\hat{j}$

$= \boxed{\frac{\mu_0 \omega Q R^2}{4\pi (R^2+z^2)^{3/2}} \hat{j}}$

10




- 2c) (15 points) Now, let's replace the ring with a washer of inner-radius $a$ and outer-radius $b$ that carries a surface charge density

$$\sigma(r) = \frac{Q}{2\pi \ln(b/a)} \frac{1}{r^2}$$

where $r$ measures the distance from the center of the washer to a point within the washer. Find the (vector) magnetic field due to the ring at a point ($P$) on the longitudinal symmetry axis, a distance $z$ from the center of the ring.

$$A = 2\pi r\,dr$$

$$d\vec{B} = \frac{\mu_0 \omega\, dQ\, r^2}{4\pi (r^2+z^2)^{3/2}} \hat{j}$$

$$dQ = \sigma\, dA = \frac{Q}{\cancel{2\pi} \ln(b/a)} \frac{1}{r^{\cancel{2}}} \cancel{2\pi} \cancel{r}\, dr$$

$$= \frac{Q\, dr}{\ln(b/a)\, r}$$

$$d\vec{B} = \frac{\mu_0 \omega Q}{4\pi \ln(\frac{b}{a})} \frac{r}{(r^2+z^2)^{3/2}}\, dr\, \hat{j}$$

$$\vec{B} = \frac{\mu_0 \omega Q}{4\pi \ln(\frac{b}{a})} \hat{j} \int_a^b \frac{r}{(r^2+z^2)^{3/2}}\, dr$$

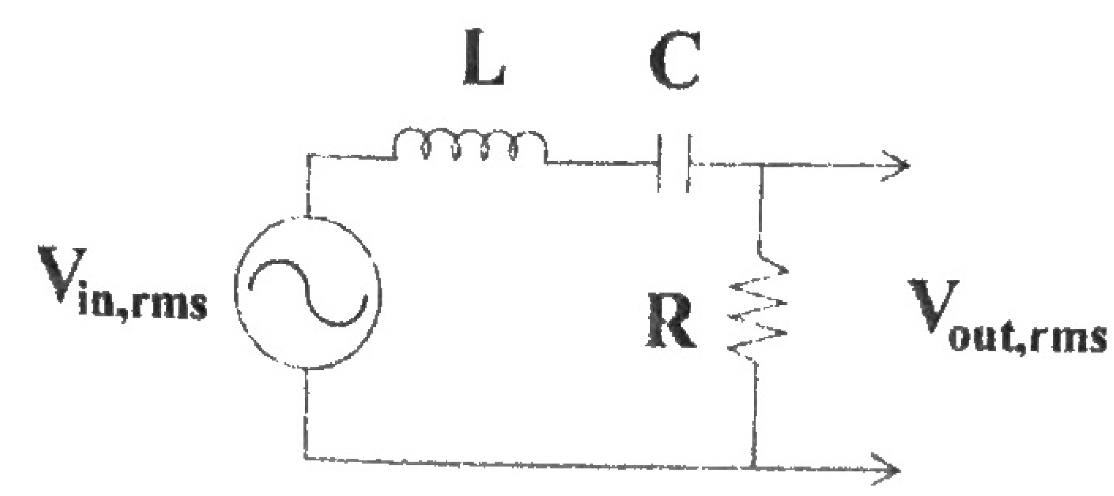
$$u = r^2 + z^2$$
$$du = 2r\,dr$$
$$\frac{du}{2} = r\,dr$$

$$\int \frac{1}{u^{3/2}} \frac{du}{2}$$
$$= -\frac{\cancel{2}}{\sqrt{u}} \frac{\cancel{du}}{\cancel{2}} + C$$

$$= \frac{\mu_0 \omega Q \hat{j}}{4\pi \ln(\frac{b}{a})} \left( -\frac{1}{\sqrt{r^2+z^2}} \right) \Bigg|_a^b$$

$$\boxed{\vec{B} = \frac{\mu_0 \omega Q}{4\pi \ln(\frac{b}{a})} \left( \frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}} \right) \hat{j}}$$

15




Filters are classified by the frequency-range of the signals they deliver to their output. High-pass and low-pass filters preferentially pass high- and low-frequency signals, respectively. Band-pass filters preferentially pass signals that have frequencies within some range (or 'band') of frequencies. Notch filters actually remove signals whose frequencies lie within some range of frequencies.

3 • 3a) (5 pts) Qualitatively discuss how the components in the circuit above will react to input signals over a very broad range of frequencies (say, 0 Hz to $\infty$ Hz). Explain how their respective behaviors, taken together, will make the circuit shown above behave like a band-pass filter.

$Z = \sqrt{(X_L - X_C)^2 + R^2}$

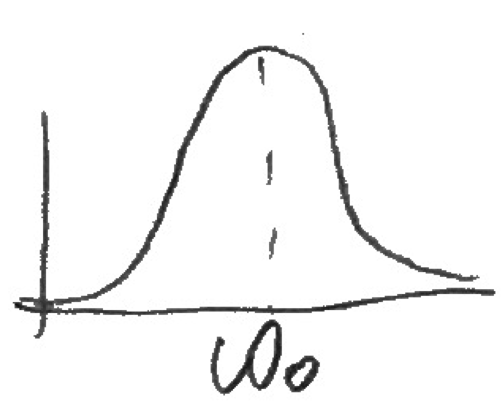
P

$\Delta V_R = \frac{X_R}{Z}$

$\Delta V_C = \frac{X_C}{Z}$

$\Delta V = \frac{X_L}{Z}$

Because of the impedence of this RLC circuit, the "band-pass" occurs when ressistivity is at a minimum, since $V_{out,rms} = V_{in}\frac{R}{Z}$, causing a hump at minimum X




Inductor, capacitor!

10 • 3b) (10 pts) If the signal on the input has an rms voltage $V_{in,rms}$ and a frequency $\omega$, how large is the rms voltage at the output? At what frequency will this output voltage be greatest?

$V_{out,rms} = V_{in,rms}\frac{X_R}{Z}$ ✓ $= V_{in,rms}\frac{R}{\sqrt{(X_L - X_C)^2 + R^2}}$

$\boxed{V_{out,rms} = V_{in,rms}\frac{R}{\sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}}}$

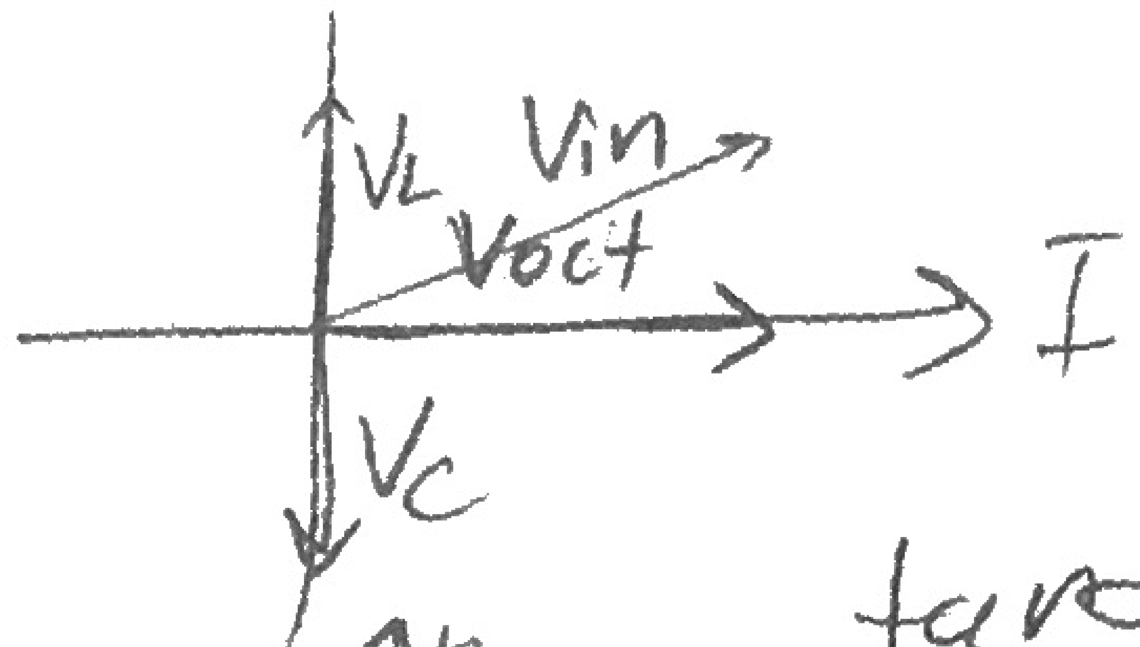
$V_{out}$ is max when $\omega L = \frac{1}{\omega C}$ $\quad \omega_{res} = \sqrt{\frac{1}{LC}}$

$f = \frac{\omega}{2\pi}$

$\boxed{f_{res} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}}$

EII ICE

4 • 3c) (5 pts) By what phase angle will the output voltage lead or lag the input voltage? Under what conditions will it lead? ... lag?



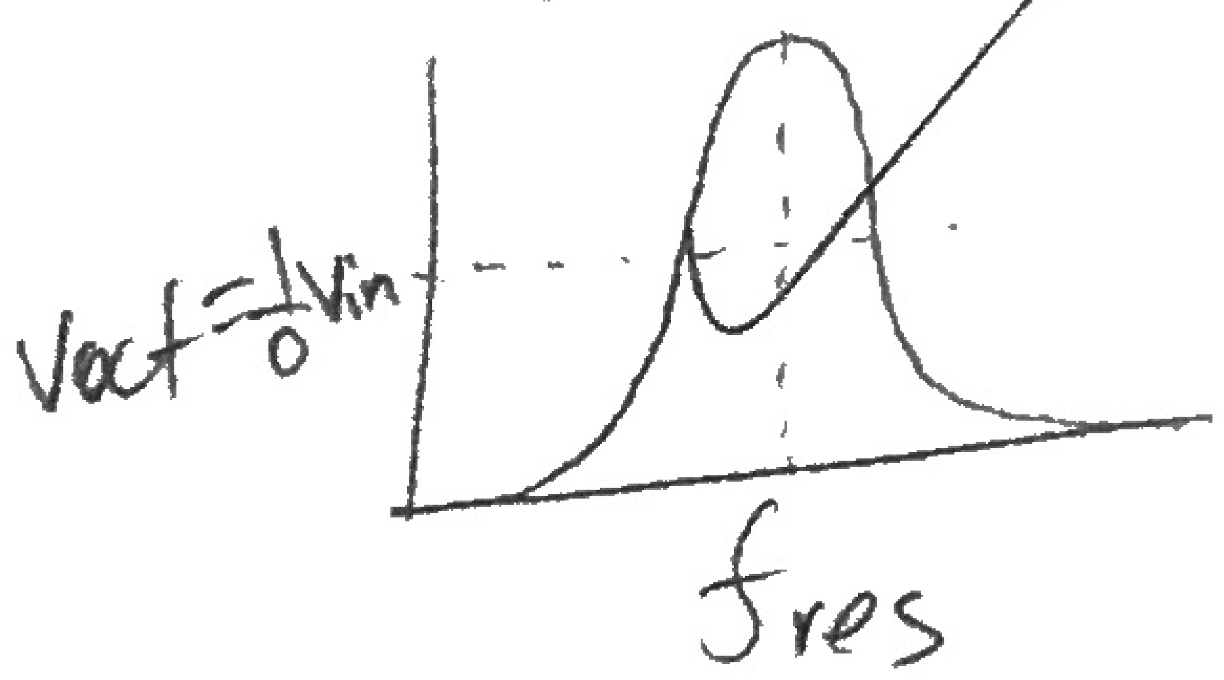

$$\tan\alpha = \frac{X_L - X_c}{R}$$


$$\boxed{\alpha \pm \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)}$$

when $\omega L > \frac{1}{\omega C}$, the output voltage lags the input voltage, when $\omega L < \frac{1}{\omega C}$, the output voltage leads the input voltage

opposite -1

8 • 3d) (10 pts) Do a quick, qualitative sketch of the output voltage vs. the frequency of the input signal. The width of that peak, $\Delta\omega$, is usually taken to be the "Full Width at Half-Maximum" (FWHM) - the distance (in frequency-space) between the two points at which the output voltage amplitude is half the input amplitude. Find the bandwidth of this filter, and discuss how one might achieve a sufficiently narrow peak without sacrificing output amplitude.




At half:

$$\frac{R}{\sqrt{(X_L - X_c)^2 + R^2}} = \frac{1}{2}$$

$$4R^2 = (X_L - X_c)^2 + R^2$$

$$\sqrt{3R^2 = \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$

$$\sqrt{3}R = 2\pi f L - \frac{1}{2\pi f C}$$

$$\frac{1}{2\pi f C} = 2\pi f L - \sqrt{3}R$$

$$0 = (4\pi^2 LC) f^2 - (\sqrt{3}R2\pi C) f$$

$$\Delta\omega = |f_1 - f_2|$$

$$= \left| \frac{-b + \sqrt{b^2 - 4ac}}{2a} - \frac{-b - \sqrt{b^2-4ac}}{2a} \right|$$

$$= \frac{\sqrt{b^2 - 4ac}}{a} = \frac{\sqrt{12\pi^2 R^2 C^2 - 16\pi^4 LC}}{4\pi^2 LC}$$

$$\boxed{\Delta\omega = \frac{\sqrt{3R^2C^2 - 4}}{\pi LC}}$$

close...

-2