Problem 1:	13 /15
Problem 2:	6 /10
Problem 3:	12 /13
Problem 4:	12 /12
Total:	43 /50

Problem 1

(1) If the magnetic field of an electromagnetic wave is in the $+x$ direction and the electric field of the wave is in the $+y$ direction, the wave is traveling in the

A) $-x$ direction.
B) $-y$ direction.
C) $-z$ direction.

(2) The energy per unit volume in an electromagnetic wave is

A) equally divided between the electric and magnetic fields.
B) mostly in the electric field.
C) mostly in the magnetic field.

(3) When an electromagnetic wave falls on a surface, it exerts a force F on that surface. If the electric component of the wave is now doubled, what will be the force?

A) $4F$.
B) F.
C) $F/4$.

rad pressure $= \frac{E_0^2}{2\mu_0 c^2} = \frac{E_0 B_0}{2\mu_0 c}$

(4) Standing electromagnetic waves are formed in a cavity. The electric field has a node separation equal to $\lambda/2$. The magnetic field has the same node separation. What is the node separation for the Poynting vector?

$\cos^2(kx - \omega t)$

A) λ.
B) $\lambda/2$.
C) $\lambda/4$.

(5) When light goes from one material into another material having a higher index of refraction,

A) its speed, wavelength, and frequency all decrease.
B) its speed and wavelength decrease, but its frequency stays the same.
C) its speed increases, its wavelength decreases, and its frequency stays the same.

(6) A ray of light strikes a boundary between two materials, from the region with refraction index n_1 to another with refractive index n_2. There is no transmitted ray. What can you conclude?

A) $n_1 > n_2$.
B) $n_1 = n_2$.
C) $n_1 < n_2$.

(7) Which one of the following is true?

A) When light strikes a surface at Brewster's angle, the reflected and transmitted light are both 100% polarized.
B) When light strikes a surface at Brewster's angle, it is completely reflected at the surface.
C) When light strikes a surface at Brewster's angle, only the reflected light is 100% polarized.

(8) An unpolarized light passes through 3 consecutive polarizers with polarization angles θ_1, θ_2 and θ_3. Which of the following will give a lower light intensity outcome?

A) $\theta_1 = 0°, \theta_2 = 10°, \theta_3 = 20°$.
B) $\theta_1 = 0°, \theta_2 = 20°, \theta_3 = 10°$.
C) The two outcomes are the same.

(9) Can light strikes a surface at Brewster's angle and have total internal reflection at the same time?

A) Yes, it can.
B) No, it can't.
C) Only if it enters a region with lower refractive index.

(10) A light travels from water with refractive index n_{water} to air with refractive index $n_{air} = 1$, at some incident angle. Total internal reflection occurs. Now the water is replaced with another medium with a refractive index higher than n_{water}. The incident angle remains unchanged. Which of the following is true?

$\sin\theta \geq \frac{n_{air}}{n_{water}}$ $\quad \frac{n_{air}}{n_{new}} < \frac{n_{air}}{n_{water}}$

$\sin\theta > \frac{n_{air}}{n_{new}}$ ✓

A) Total internal reflection persists.
B) Total internal reflection disappears.
C) Not enough information to answer.

(11) As you walk away from a vertical plane mirror, your image in the mirror

A) decreases in height.
B) always has the same height.
C) is always a real image, no matter how far you are from the mirror.

(12) A convex lens has a focal length f. An object is placed at a distance between f and $2f$ on the optical axis. The image formed is located at what distance from the lens?

A) f.
B) between f and $2f$.
C) farther than $2f$.

$\frac{1}{2f} + \frac{1}{s'} = \frac{1}{f}$

$\frac{1}{s'} = \frac{1}{f} - \frac{1}{2f}$ $\quad \frac{1}{s'} = \frac{1}{f}$

$\frac{1}{f} + \frac{1}{s'} = \frac{1}{f}$

(13) Suppose you place your face in front of a concave mirror.

~~A) No matter where you place yourself, a real image will always be formed.~~
~~B) No matter where you place yourself, your image will always be inverted.~~
C) The two statements above are wrong.

$\frac{4}{R} + \frac{1}{s'} = \frac{2}{R}$

$\frac{1}{s'} = -\frac{2}{R}$

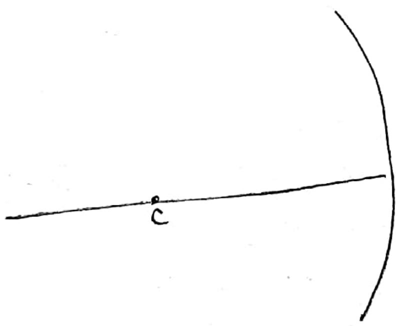
$s' = \frac{-R}{2}$

(14) An object is placed in front of a lens which forms an image of the object.

A) If the lens is convex, the image cannot be virtual.
B) If the image is real, then it is also inverted.
C) If the image is virtual, the lens must be a diverging lens.

(15) A simple refracting telescope provides a large magnification by employing

A) a short focal length objective and a short focal length eyepiece.
B) a long focal length objective and a long focal length eyepiece.
C) a long focal length objective and a short focal length eyepiece.



$\frac{3}{c} + \frac{1}{s'} = \frac{2}{c}$

$\frac{1}{s'} = \frac{2}{c} - \frac{3}{c}$

$\frac{1}{s'} = -\frac{1}{c}$

$s' = -c$ ← virtual

$-\left(\frac{-c}{c/3}\right) \to -\left(-\frac{3\cancel{c}}{\cancel{c}}\right) = 3$

Problem 2

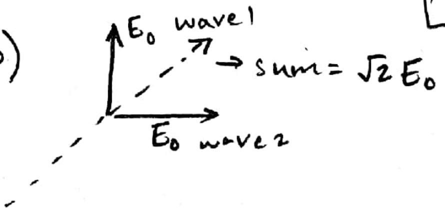
(a) A linearly polarized EM wave travels along the $+x$ direction and has wavenumber k and angular frequency ω. The electric field is polarized along the z-axis with amplitude E_0. (i) Write down the electric field $\vec{E}(x,t)$ and magnetic field $\vec{B}(x,t)$ wave functions. (ii) Find the Poynting vector $\vec{S}(x,t) = \frac{1}{\mu_0}\vec{E} \times \vec{B}$. (iii) What is the average intensity of the wave? Express vector answers in the form of $v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$.

(b) Now, another wave is added and travels along the same direction. The additional wave has the electric field polarized along the y-axis and has the same amplitude E_0. The total wave is now unpolarized. Repeat (i-iii) in part (a) using the total wave.

a) i) $\boxed{\begin{aligned} E(x,t) &= E_0 \cos(kx-\omega t)\,\hat{k} \\ B(x,t) &= -\frac{E_0}{c}\cos(kx-\omega t)\,\hat{j} \end{aligned}}$ 1

ii) $\frac{\vec{E}\times\vec{B}}{\mu_0} = \boxed{\frac{E_0^2 \cos^2(kx-\omega t)\,\hat{i}}{\mu_0 c}}$ ✓ 2

iii) $I = S_{av} = \frac{S}{2} = \boxed{\frac{E_0^2 \cos^2(kx-\omega t)}{2\mu_0 c}\hat{x} + 0\hat{y} + 0\hat{z}}$

b) 

2

i) $\boxed{\begin{aligned} E(x,t) &= E_0\cos(kx-\omega t)\,\hat{j} + E_0\cos(kx-\omega t)\,\hat{k} \\ B(x,t) &= -\frac{E_0}{c}\cos(kx-\omega t)\,\hat{j} + \frac{E_0}{c}\cos(kx-\omega t)\,\hat{k} \end{aligned}}$

ii) $\frac{\vec{E}\times\vec{B}}{\mu_0} = \boxed{\frac{2E_0^2\cos^2(kx-\omega t)\,\hat{i}}{\mu_0 c}}$ X /

iii) $I = S_{av} = \frac{S}{2} = \boxed{\frac{E_0^2\cos^2(kx-\omega t)}{\mu_0 c}\hat{x} + 0\hat{y} + 0\hat{z}}$

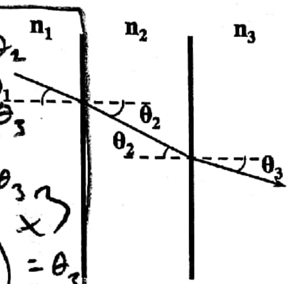
X

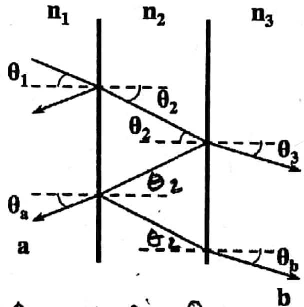
+12

Problem 3

(a) A light passes through three regions with refractive index n_1, n_2 and n_3, respectively, as shown in the left figure. θ_1 and θ_3 are the incident and transmitted angles, respectively (angles not to scale). (i) Write down two Snell's equations for the two interfaces. Express θ_3 in terms of θ_1. (ii) Suppose total internal reflection happens at the n_1-to-n_2 interface. Find the critical incident angle $\theta_{1,c}$. What is the condition for n_1 and n_2? (iii) Suppose total internal reflection happens at the n_2-to-n_3 interface. Find the critical incident angle $\theta'_{1,c}$. What is the condition for n_1 and n_3? Can it occur when $n_1 < n_2$?

(b) Consider the light after multiple reflections shown in the right figure. (i) Find θ_a and θ_b in terms of θ_1, θ_2, θ_3. (ii) If $\tan\theta_2 = n_3/n_2$, will the light ray a and b be polarized or unpolarized?





a) i)

$$\boxed{\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ n_2 \sin\theta_2 &= n_3 \sin\theta_3 \\ n_1 \sin\theta_1 &= n_3 \sin\theta_3 \\ \sin^{-1}\left(\frac{n_1}{n_3}\sin\theta_1\right) &= \theta_3 \end{aligned}}$$

x3

ii) $n_1 \sin\theta_{1c} = n_2 \sin\theta_2$

$\theta_2 = 90°$

$n_1 \sin\theta_{1c} = n_2$

$\sin\theta_{1c} = \frac{n_2}{n_1}$

$\theta_{1c} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ x3

given that $n_2 < n_1$

iii) $n_1 \sin\theta_{1c} = n_3 \sin\theta_3$

$\theta_3 = 90°$

$\sin\theta_{1c} = \frac{n_3}{n_1}$

$\theta_{1c} = \sin^{-1}\left(\frac{n_3}{n_1}\right)$ x3

given that $n_3 < n_1$ and $n_2 > n_3$

can occur when $n_1 < n_2$

x1

b) $n_2 \sin\theta_2 = n_1 \sin\theta_a = n_1 \sin\theta_1$

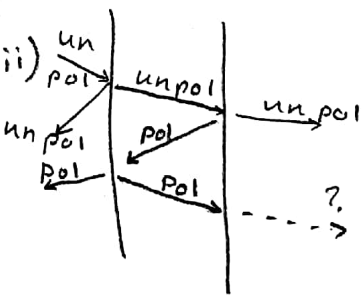
$\theta_a = \theta_1$

$n_2 \sin\theta_2 = n_3 \sin\theta_b = n_3 \sin\theta_3$

$\theta_b = \theta_3$

i) $\boxed{\theta_a = \theta_1 \text{ x1} \quad \theta_b = \theta_3 \text{ x1}}$

ii)



ray a is polarized

~~ray b is unpolarized~~

+12

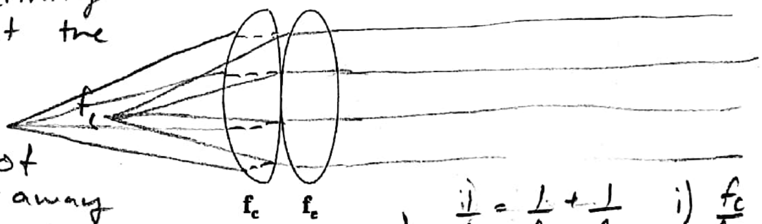
Problem 4

Bob's pupil has a focal length f_e and he is wearing a contact lens with an unknown focal length f_c. We can model them as two thin lenses in contact. (a) Prove that the total focal length f of the combination obeys:

$$\frac{1}{f} = \frac{1}{f_e} + \frac{1}{f_c}.$$

(b) What are the values of f/f_e when $f_c/f_e =$ (i) 1, (ii) 0, (iii) ∞, (iv) -1, respectively? (c) Plot f/f_e as a function of $f_c/f_e \in (-\infty, \infty)$.

a) An object at infinity will have an image at the focal point.



Thus the image of an infinitely far away object will be at a point f_c in front of the two touching lenses, since $\frac{1}{\infty} + \frac{1}{s'} = \frac{1}{f_c}$; $s' = f_c$. Now the image point through the second lens is $\frac{-1}{f_c} + \frac{1}{s'} = \frac{1}{f_e}$. The focal point of lens C is negated because for convex lenses, the focal point will be on the side that the light exits, and its positive focal length must be negative, and for concave lenses, the focal point is on the side the light enters, so its negative focal length must be made positive. Solving for s' yields $\frac{1}{s'} = \frac{1}{f_e} + \frac{1}{f_c}$, and since the original object is at infinity, s' is the focal length of the combined lenses. Thus $\frac{1}{f} = \frac{1}{f_e} + \frac{1}{f_c}$

b) $\frac{1}{f} = \frac{1}{f_e} + \frac{1}{f_c}$

$\frac{f_c}{f} = \frac{f_c}{f_e} + 1$

i) $\frac{f_c}{f} = 2$

$f_c = 2f$

$\frac{f}{f_e} = \frac{1}{2}\frac{f_c}{f_e} = \boxed{\frac{1}{2}}$ +1

ii) $\frac{f_c}{f} = 1$ $f_c = f$

$\frac{f}{f_e} = \frac{f_c}{f_e} = \boxed{0}$ +1

iii) $\frac{f_c}{f} = \infty$ $f_c = \infty$

$\frac{1}{f} = \frac{1}{f_e} + \frac{1}{\infty}$

$\frac{1}{f} = \frac{1}{f_e}$

$\boxed{\frac{f}{f_e} = 1}$ +1

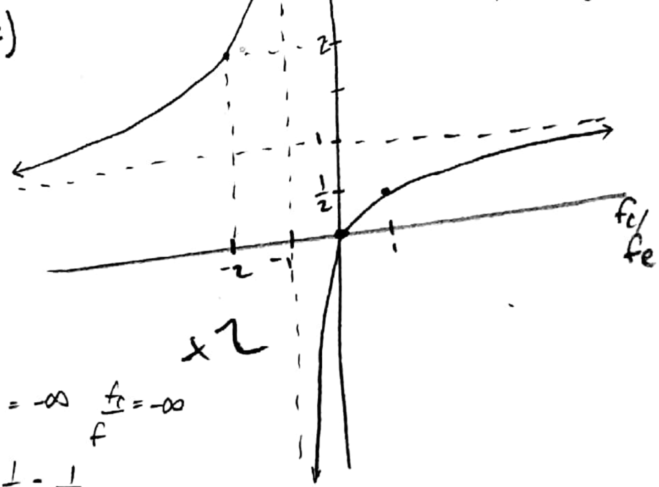
iv) $\frac{f_c}{f} = 0$ $f = \infty$

$\boxed{\frac{f}{f_e} = \infty}$ +1

$\frac{f_c}{f_e} = -2$ $\frac{f_c}{f_e} = -1$

$\frac{f}{f_e} = \frac{-f_c}{f_e} = +2$

c)



$\frac{f_c}{f_e} = -\infty$ $\frac{f_c}{f} = -\infty$

$\frac{1}{f} = \frac{1}{f_e} - \frac{1}{\infty}$

$f_e = f$ $\frac{f}{f_e} = 1$