Physics 1C Lecture 1, Fall 2021

Midterm Examination

October 29, 2021

Exam rules: Please do not forget to write your name and student ID on the front of the exam! No electronic gadgets of any kind, and the exam is closed book and closed notes. Any numerical answers may be given using one or more significant figures. For example, $4\pi = 10$ is acceptable. If a definite integral appears in an answer, but which you do not know how to solve, then continue on with the rest of the question, assigning an arbitrary constant to take the place of the unsolved integral.

Some equations

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\Phi_B$$

$$\oint (\vec{E} \cdot \hat{n}) dA = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left[I_{enc} + \varepsilon_0 \frac{d\Phi_E}{dt} \right]$$

$$\oint (\vec{B} \cdot \hat{n}) dA = 0$$

$$\vec{F}_B = q\vec{v} \times \vec{B}; d\vec{F}_B = I d\vec{\ell} \times \vec{B}; \vec{F}_E = q\vec{E}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}; \vec{\mu} = IA\hat{n}$$

$$d\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times (\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3}$$

$$\mathcal{E} = -L\frac{di}{dt}$$

$$U = \frac{1}{2}LI^2$$

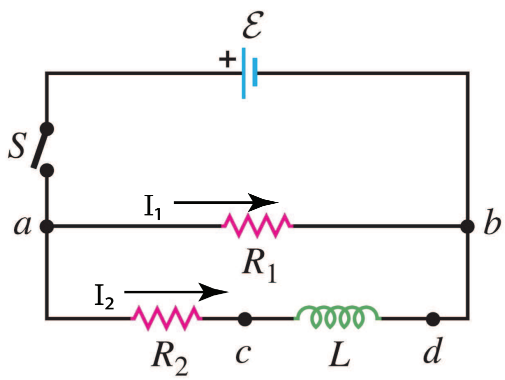
$$X_L = \omega L$$

$$\cos(\omega t + \pi/2) = -\sin(\omega t)$$

$$Z^2 = [R^2 + (X_L - X_C)^2]$$

Problem 1.

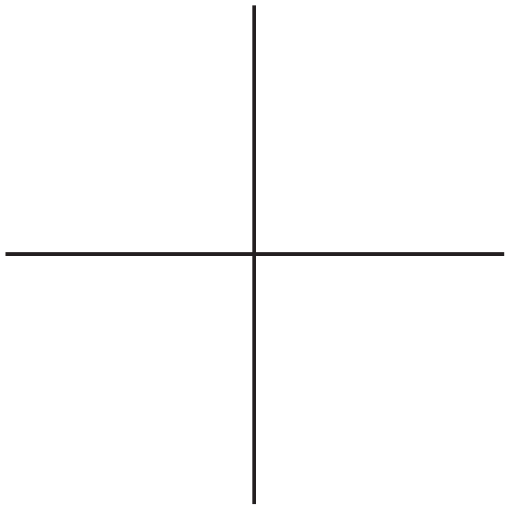
Inductors in circuits. (14 points total)



a. Write down the definition of self inductance, followed by an expression relating the current in an inductor to the potential difference across it. (4 points)

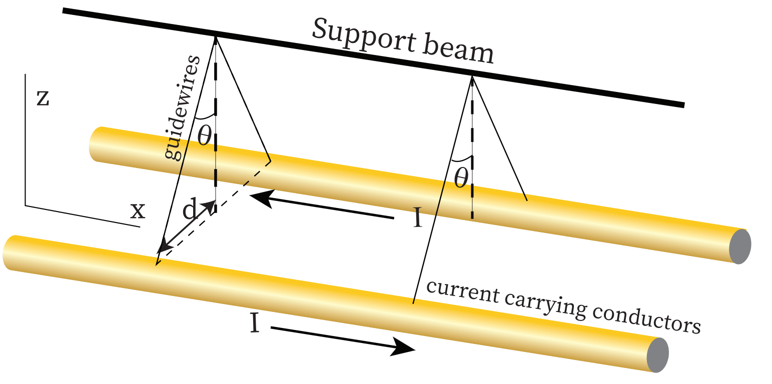
b. An inductor L is connected with two resistors as shown. At $t = 0$, the switch S is closed. *Immediately* after the switch is closed ($t \to 0^+$), what are the currents I_1 and I_2? What are I_1 and I_2 a long time after S is closed? Write down a differential equation for the current $I_2(t)$. Find the solution and sketch it. (4 points)

c. In this part, the battery and switch are replaced by an ac source. Use the axes below to make a phasor diagram for the source potential $v_s(t) = V_s \cos(\omega t + \phi)$, and current $i_2(\cos(\omega t)$. Then use it to solve for the current amplitude I_2, and evaluate the phase angle ϕ. What are the amplitude and phase for $i_1(t)$? (6 points)



Problem 2. Sources of magnetic fields, magnetic forces

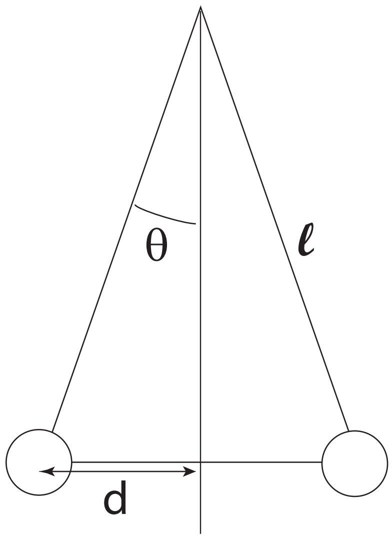
(14 points total)



a. Write down an integral expression for Ampère's Law, and use it to find the magnetic field of a long, straight wire. Make sure to define the direction of the magnetic field. You may choose to make a diagram as part of your answer. (4 points)

b. Consider the long wire segments in the figure above (only a section of the two straight-line segments is shown). Each wire carries current I, but in opposite directions, and they are supported by identical guidewires of length ℓ. The conducting wires are in static equilibrium (stable position), and separated by distance $2d$. What is the magnetic force of interaction *per unit length* f_B between the two segments? You should assume that the length of the segments satisfies the condition $L \gg d$. Your answer should be written in terms of I, and d. (6 points)

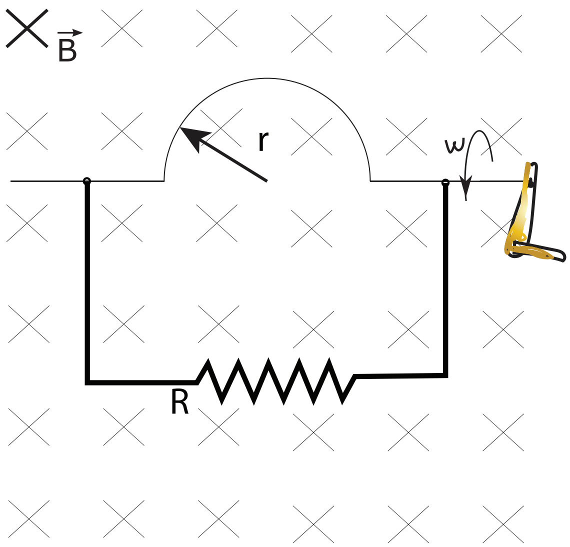
c. In the drawing below, sketch the direction of the forces, f_B, f_g acting on the wire segments. f_g is the gravitational force normalized per unit length. (2 points)



d. Taking the forces (per length) acting on each of the wire segments to be f_B and the gravitational force $f_G = \lambda g$, write down the ratio f_B/f_G in terms of the angle θ. λ is the mass per length of each straight wire. (2 points)

Problem 3. Faraday's Law

Shown is a semi-circular wire loop, free to rotate in a uniform magnetic field. The radius of the semicircular part is r. The direction of the rotations is such that $\vec{\omega}$ is directed horizontal to the right, the magnetic field is directed into the page. Note: the part of the circuit containing the resistor R (heavier lines) is not free to rotate. (14 points total)



a. Write down Faraday's Law, which relates a magnetically induced emf to ____________ (fill in the blank). Define terms introduced below. (2 points)

b. What is the change in magnetic flux through the semi-circular part on rotating it through a quarter turn, from an orientation with the magnetic field directed orthogonal to the plane of the loop (shown in the figure), to where **B** is in the plane of the loop? (4 points)

c. The loop is rotating at angular frequency ω. What is the frequency and amplitude of the induced emf? (4 points)

d. Assuming the mechanical friction between rotating and fixed conductors in the circuit is negligible, how much time-averaged power must be supplied from an external source to rotate the loop at ω? (4 points)