Physics 1C Lecture 3, Spring 2014

Midterm Examination 1

October 24, 2014

Exam rules: Please do not forget to write your name and student ID on the front of the exam! No electronic gadgets of any kind, and the exam is closed book and closed notes. Any numerical answers may be given using one or more significant figures. For example, $4\pi = 10$ is acceptable. If a definite integral appears in an answer, but which you do not know how to solve, then continue on with the rest of the question, assigning an arbitrary constant to take the place of the unsolved integral.

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Problem 1.

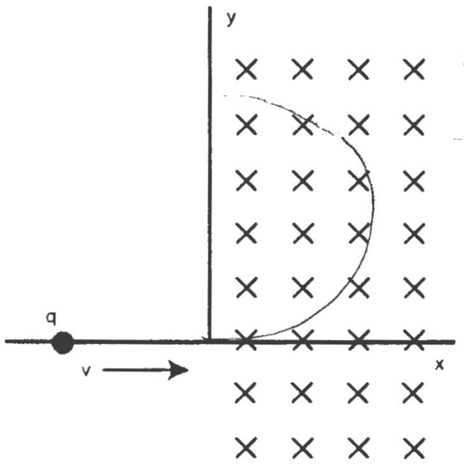
Magnetic fields and magnetic forces (12 points total)

a. Write down the Lorentz Force Law, as applied to a particle of charge q. (2 points)

$$\vec{F}_{Lorentz} = q\,\vec{v} \times \vec{B}$$

velocity magnetic field

b. A particle of charge q, travelling at velocity $\vec{v} = v\hat{x}$ as shown, enters a region of uniform magnetic field $\vec{B} = -B\hat{z}$, with $\hat{x}, \hat{z}$ unit vectors in the x, z direction, respectively. The particle begins to move in a circular trajectory. What is the plane of the circular orbit, and what is the radius of the circle, in terms of q, v, B, and the particle's mass m? **Hint:** Recall that for uniform motion on a circle, the magnitude of the acceleration is v^2/R, with v the speed, and R the radius of the motion. (4 points)



The plane of the circular orbit is the xy plane

The Lorentz Force provides the acceleration necessary for uniform circular motion

$$F = ma_c = m\frac{v^2}{R}$$

$$F_{Lorentz} = q\,\vec{v} \times \vec{B} = qvB\sin\theta = qvB \quad \text{since } \vec{v} \perp \vec{B}$$

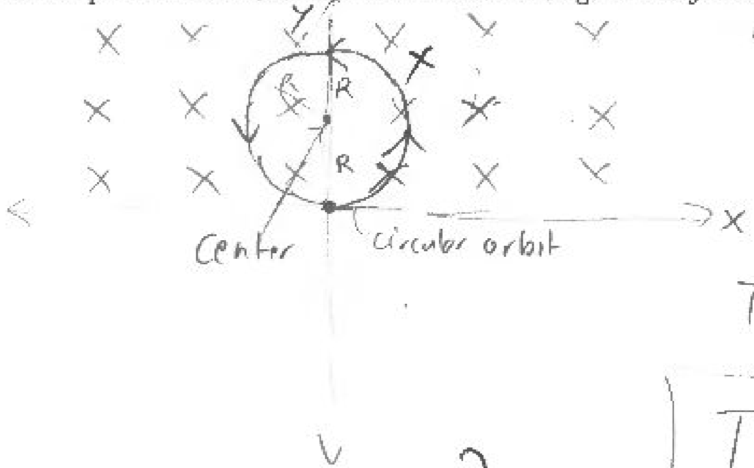
$$F = F_{Lorentz} \Rightarrow m\frac{v^2}{R} = qvB$$

$$R = \frac{mv}{qB}$$

c. Consider that the particle enters the region of nonzero magnetic field at the origin $x = y = z = 0$. Then, $B \neq 0$ for everywhere $y > 0$. What are the coordinates (x,y,z) of the circle center? Make a rough sketch of the trajectory using the figure provided. How much time elapses between the initial and final crossings of the y-axis? (4 points)

$\boxed{\text{Circle center} = (0, R, 0)}$ ✓

since circle lies tangent to origin



$$T = \frac{2\pi R}{v} \quad \text{period} = \frac{\text{dist}}{\text{speed}}$$

$$R = \frac{mv}{qB} \Rightarrow v = \frac{RqB}{m}$$

$$T = \frac{2\pi R}{\left(\frac{RqB}{m}\right)}$$

$$\boxed{T = \frac{2\pi m}{qB}}$$

−2

d. Write down an expression for Gauss' Law for magnetism and explain (no more than ONE sentence!) its meaning. (2 points)

$$\boxed{\oint_S \vec{B} \cdot d\vec{A} = 0}$$

There are no (known) magnetic monopoles, so all magnetic field lines that enter region S must leave. ✓

Problem 2.

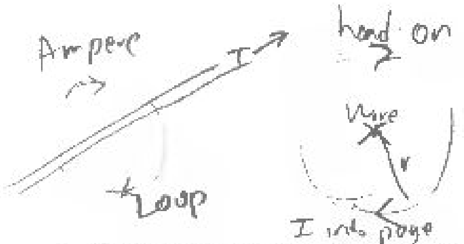
(8)

Sources of magnetic fields. (12 points total)

a. Write down an integral expression for Ampére's Law, and use it to find the magnetic field of a long, straight wire. Make sure to define the direction of the magnetic field. (4 points)

$$\oint_C \vec{B}\, d\vec{l} = \mu_0 I_{encl}$$

+4



$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$ where r is dist. from wire

Direction of B is clockwise for I traveling into page

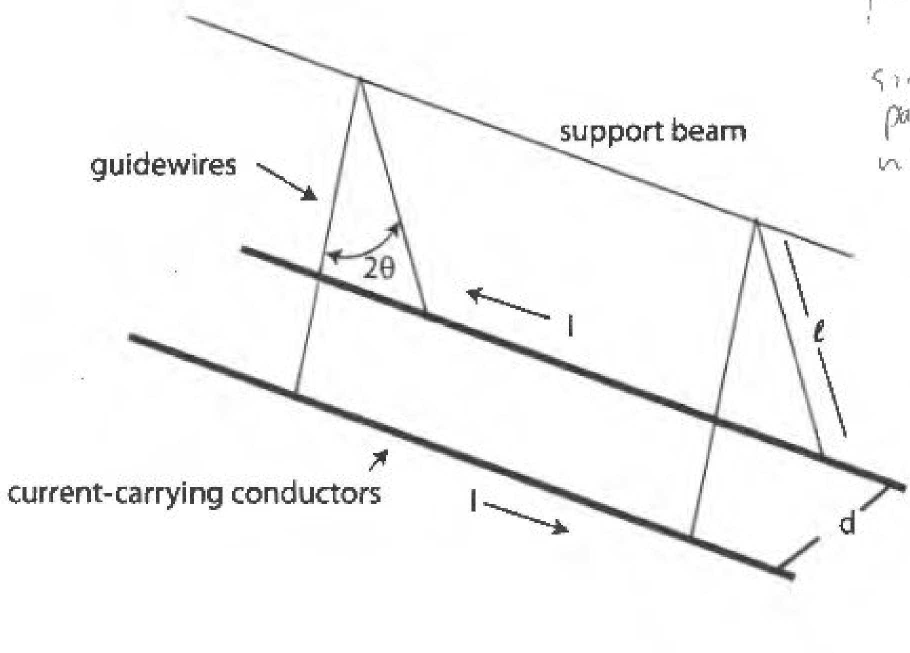
b. Consider the long, current carrying wire segments in the figure. They are carrying current I, but in opposite directions, and they are supported by guidewires, each of length ℓ. The conducting segments are in static equilibrium (stable position), and separated by distance d. What is the magnetic force of interaction *per unit length* f_B between the two segments? You should assume that the limiting case applies, that the length of the segments $L \gg d$. (4 points)

$$f_B = \frac{F}{L} = \frac{\mu_0 I^2}{2\pi d}$$

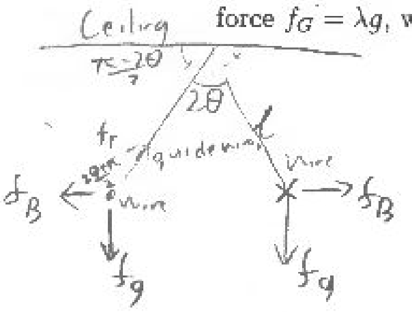
since the force between parallel current-carrying wires is

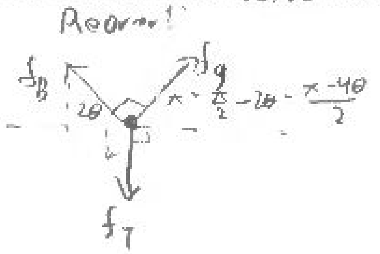
$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

+4



c. Taking the forces (per length) acting on each of the bars to be f_B and the gravitational force $f_G = \lambda g$, write down the ratio f_B/f_G in terms of the angle θ. (2 points)





$$f_{B_x} + f_{g_x} = 0$$

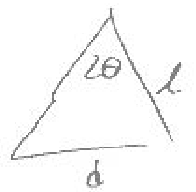
$$f_B\cos(2\theta) = -f_g\cos\left(\frac{\pi-4\theta}{2}\right)$$

$$\boxed{\frac{f_B}{f_g} = -\frac{\cos\left(\frac{\pi-4\theta}{2}\right)}{\cos(2\theta)}}$$

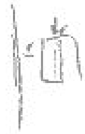
d. Show that

$$\frac{d}{2\ell} = \frac{f_B}{[f_B^2 + f_G^2]^{1/2}},$$

with λ the mass per length of each rod. **Hint:** Following the previous part, use what you know about the geometry of the problem, and how it relates to the forces acting on the conductors. (2 points)



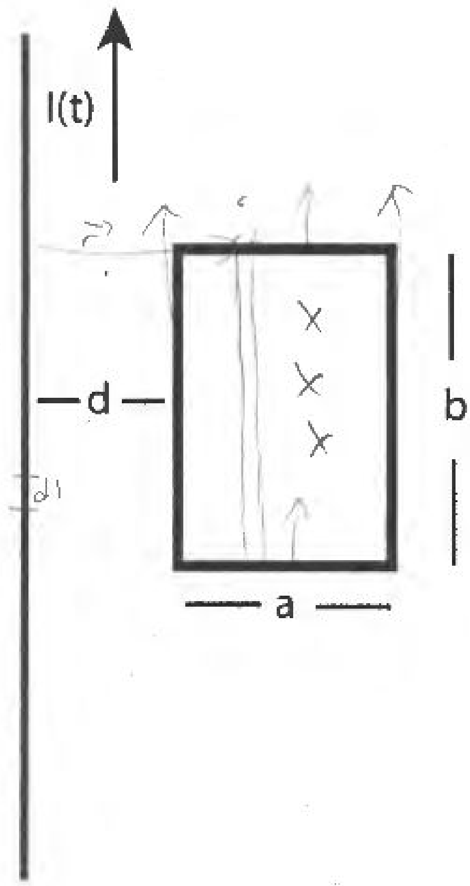
$I > 0 \quad \frac{dI}{dt} > 0$



13 **Problem 3.**

Faraday's Law (16 points total)

3 a. Write down Faraday's Law, and define each of the terms appearing in your expression. (3 points)



$$\boxed{\mathcal{E} = -\underbrace{\frac{d\Phi_B}{dt}}_{\text{change in magnetic flux over time}}}$$

Φ_B ← Magnetic Flux

t ← Time

$\mathcal{E}$ ↑ induced electromotive force.

Hw?

$dI =$

4 b. In the figure above is a long straight wire, carrying current that varies with time, $I(t)$. Adjacent to it is a conducting loop of cross-section $A = ab$ and total resistance around the loop R. Find the magnetic flux passing through the loop as a function of the distance d. Verify that the flux tends to zero in the limit of large d(4 points).

$dI = r\,dr$

$d\Phi_B = \vec{B}\cdot d\vec{A} = BA \qquad d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$

$B_{\perp A}$

$\Phi_B = \frac{\mu_0 I}{4\pi}\int_d^{d+a}\frac{dr}{r}\cdot ab \qquad dB = \frac{\mu_0 I}{4\pi}\frac{r\,dr\sin(90^\circ)}{r^2}$

$= \frac{\mu_0 I}{4\pi}\ln(r)\Big|_d^{d+a}\cdot ab \qquad = \frac{\mu_0 I}{4\pi}\frac{dr}{r}$

$$\boxed{\Phi_B = \frac{\mu_0 I ab}{4\pi}\ln\left(\frac{d+a}{d}\right)}$$

$\lim_{d\to\infty}\Phi_B = \lim_{d\to\infty}\ln\left(\frac{d+a}{d}\right) = \lim_{d\to\infty}\ln\left(\frac{d}{d}\right) = 0$ ✓

2 c. Use Faraday's Law to find the induced emf in the loop. In what direction is the resulting current flowing and evaluate it in terms of given parameters. ~~(Note that the sign of the derivative, $I' = dI/dt$, is not specified in what's written above.)~~ (3 points) $\frac{dI}{dt} > 0$

$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 ab}{4\pi}\ln\left(\frac{d+a}{a}\right)\cdot\frac{dI}{dt}$

The current flows counterclockwise to oppose the increase in flux

4 d. Write down an expression for the induced dipole moment of the loop in terms of the induced current and dimensions of the loop. Is there a torque on the induced dipole moment? Is there a force on it? Explain your answer (ONE sentence!), and if there is a torque or force, give the direction. (6 points)

$\mu = I\cdot A \qquad \tau = \vec{\mu}\times\vec{B}$

No torque, because $\vec{\mu}$ and $\vec{B}$ are antiparallel and thus in (unstable) equilibrium.

$\mu =$