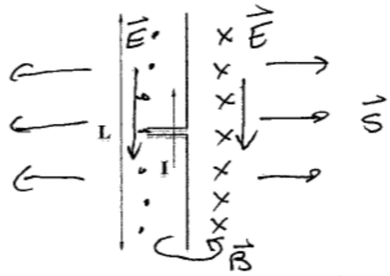


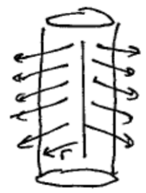
A dipole antenna is constructed from two pieces of wire, fed in the center (you can probably ignore that), that add up to a total length L, as shown. At the moment under consideration, an electric current I flows upward through the dipole and electromagnetic energy radiates radially outward from the dipole.

- 3a) (5 points) In what direction is the electric field pointing? Explain.

Using the right-hand rule with the current gives us the direction of $\vec{B}$. $\vec{S}$ points radially out. $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$, so...

$\vec{E}$ points opposite the direction of I

- 3b) (5 points) How will the intensity of radiated signal vary with distance from the dipole? Explain.



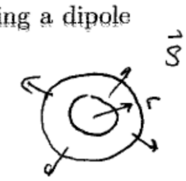
If the antenna radiates energy at a rate P, continuity of energy flow means energy must pass through any closed surface that surrounds the antenna at the same rate - we'll use a cylinder:

$P = \Phi_S = \int \vec{S}\cdot d\vec{A}$

$P = S 2\pi r L$

$S = \frac{P}{2\pi L r} \Rightarrow$ $\boxed{S \propto \frac{1}{r} \quad \text{intensity falls off as } 1/r}$

- 3c) (5 points) An isotropic antenna is a theoretical device that exhibits spherical symmetry (that is, the radiated signal doesn't prefer any direction in space). How would the intensity of a signal radiated from an isotropic antenna vary with distance from the antenna? Briefly discuss the pro's and con's of using a dipole vs. an isotropic antenna.



Same idea, but we'll use a spherical surface...

$P = \Phi_S = \int \vec{S}\cdot d\vec{A}$

$P = S 4\pi r^2$

$S = \frac{P}{4\pi r^2} \Rightarrow$

$S \propto 1/r^2$ intensity falls off as $1/r^2$

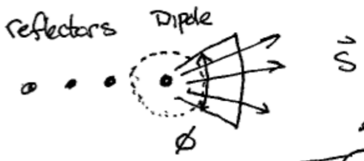
The dipole is more effective at throwing energy over a distance, but it is directional

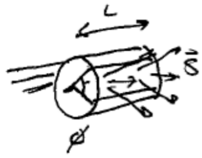
- 3d) (10 points) An observer located a distance r from the dipole measures the average intensity of the received signal to be S_{avg}. What are the **amplitudes** of the electric and the magnetic fields back at the antenna? (For full credit, you will need to derive the solution, not just quote results from a textbook.)

$$\vec{E} = \vec{E}_{max}\sin(kr - \omega t + \phi)$$
$$\vec{B} = \vec{B}_{max}\sin(kr - \omega t + \phi)$$
$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$
$$|\vec{S}| = \frac{1}{\mu_0}E_{max}B_{max}\sin^2(kr - \omega t + \phi) \qquad (\langle \sin^2(\cdot)\rangle = 1/2)$$
$$\langle S\rangle = \frac{1}{2\mu_0}E_{max}B_{max} \qquad (E_{max} = cB_{max})$$
$$\langle S\rangle = \frac{1}{2\mu_0 c}E_{max}^2 = \frac{c}{2\mu_0}B_{max}^2$$

$$\boxed{\begin{aligned} E_{max} &= \sqrt{2\mu_0 c\, S_{avg}} \\ B_{max} &= \sqrt{\frac{2\mu_0}{c}S_{avg}} \end{aligned}}$$

- 3e) (5 points) "Beam" antennas look a bit like those old-fashioned TV antennas you can still find on rooftops. They add passive reflectors to force the dipole to radiate preferentially in a single direction. Suppose the reflective elements behind a dipole restrict the otherwise radial radiation from the antenna to an azimuthal range of ϕ. How does the intensity of the radiated signal fall off with distance from the dipole? Compare your answer to the answer you derived for the simple dipole and explain why a beam antenna is often preferred over a dipole.





$$P = \int \vec{S}\cdot d\vec{A}$$
$$P = SLr\phi$$
$$S = \frac{P}{L\phi r}$$

$$\boxed{S \propto \frac{1}{r} \quad \text{Intensity falls as } 1/r}$$

At first blush, there is no apparent advantage to the beam... However

$\frac{S_{beam}}{S_{dipole}} = \frac{2\pi}{\phi}$ means the intensity of the signal from the beam is multiplied by a large factor if ϕ is small