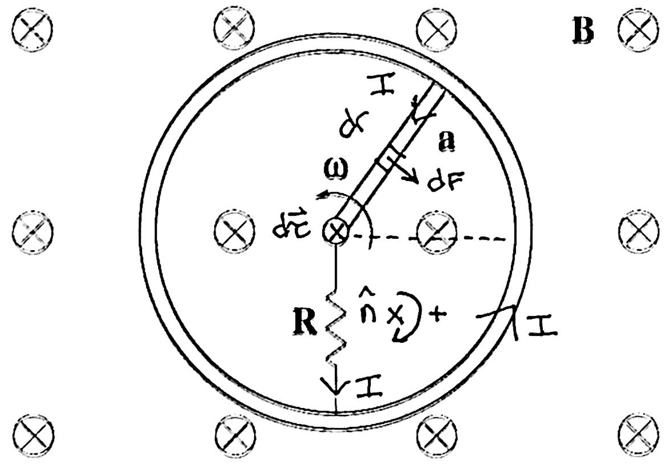

Pizza ¨

$A = \frac{1}{2} B H$

$A = \frac{1}{2} R\theta \cdot R$

$A = \frac{1}{2} R^2 \theta$

A conducting rod of length a extends from a central conducting hub to a conducting ring of radius a. It rotates around the conducting hub with an angular speed ω, in the plane of the page, as shown. The conducting hub is joined to the conducting ring by a resistor of resistance R. The whole apparatus is immersed in a uniform magnetic field $\vec{B}$ which points into the page.

- 2a) (10 points) Find the magnitude and the direction of the induced electric current through the resistor. [Hint: A slice of pizza is almost triangular.]

$$\Phi_B = B \cdot \frac{1}{2} a^2 \theta$$

$$\mathcal{E}_i = -\frac{\partial \Phi_B}{\partial t}$$

$$\mathcal{E}_i = -\frac{1}{2} a^2 B \frac{d\theta}{dt}$$

$$\mathcal{E}_i = -\frac{1}{2} a^2 B \omega$$

$\hat{n}$ points into the page, So the positive azimuthal sense is clockwise

$\Rightarrow \mathcal{E}_i$ is Counter Clockwise

$$\boxed{I_i = \frac{1}{2} \frac{a^2 B}{R} \omega}$$

Counter-Clockwise (down through the resistor)

- 2b) (10 points) Consider a small segment of length dr on the rod, located a distance r from the center of the conducting ring. How large and in what direction will be the magnetic force acting on that segment? How large and in what direction will be the torque (with respect to the center of the hub) acting on that small segment? Are your answers consistent with Lenz's Law? Explain.

$d\vec{F} = I d\vec{L} \times \vec{B}$

$|d\vec{F}| = I dr B \sin 90°$

$\boxed{|d\vec{F}| = \frac{1}{2}\frac{a^2B^2}{R}\omega dr}$

directed perpendicular to the rod in the Clockwise (+azimuthal) direction

$d\vec{\tau} = \vec{r} \times d\vec{F}$

$|d\vec{\tau}| = r dF \sin 90°$

$\boxed{|d\vec{\tau}| = \frac{1}{2}\frac{a^2B^2}{R}\omega r dr}$

directed into the page

Lenz's law could be interpreted to say that $\mathcal{E}_i$ & I_i give rise to effects that run counter to the changes that create them. In this case, $d\vec{F}$ & $d\vec{\tau}$ conspire to oppose the counter-clockwise motion of the rod.

- 2c) (10 points) Show that the rate at which mechanical work would have to be done on the rod to keep it moving with a constant angular velocity is equal to the rate at which electrical energy is dissipated by the resistor. Why would we expect that to be the case?

Mechanical work:

Since all the contributions to the torque are in the same direction,

$\tau = \int d\tau = \frac{1}{2}\frac{a^2B^2}{R}\omega \int_0^a r dr$

$\tau = \frac{1}{4}\frac{a^4B^2\omega}{R}$ ← (net torque caused by $\vec{B}$)

→ an external agent would have to apply this torque in the same direction as the motion of the rod to keep it rotating at a constant speed...

$\vec{\tau}_{ext} = \frac{1}{4}\frac{a^4B^2\vec{\omega}}{R}$

$P = \vec{F}\cdot\vec{v} \rightarrow P = \vec{\tau}_{ext}\cdot\vec{\omega}$

$\boxed{P = \frac{1}{4}\frac{a^4B^2\omega^2}{R}}$

Electrical Work:

$P = I_i \mathcal{E}_i = I_i^2 R$

$\boxed{P = \frac{1}{4}\frac{a^4B^2\omega^2}{R}}$

In short — there is no such thing as a free lunch... The energy the resistor dissipates originates with the external agent keeping everything in motion.