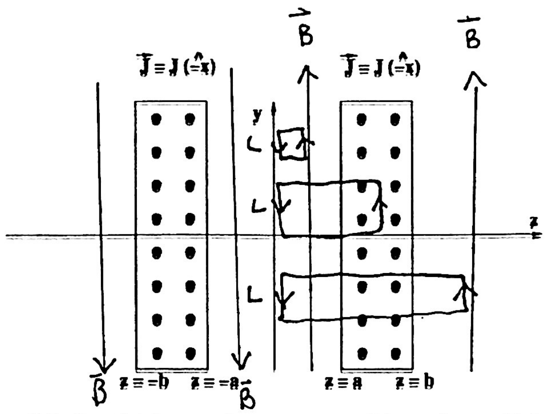


A pair of large, wide current sheets of finite thickness $b - a$ are positioned parallel to the x, y-plane, symmetric about the origin, as shown. Each carries a current described by the density $\vec{J} = J(-\hat{x})$ out of the page.

- 1a) (10 points) How would you expect the magnetic field to behave (that is, how do you expect it to depend on the various rectangular coordinates, and in what direction(s) will it point)? Back up your statements with arguments based on symmetry.

• Because the sheets are long & tall we can't distinguish one location in x from another, nor can we distinguish one location in y from another. The magnetic field can only depend on z $\Rightarrow B = B(z)$

• Magnetic field lines must form closed loops around lines of current. If a field line were to have a $+\hat{x}$ or $+\hat{z}$ component somewhere, it would have to have a $-\hat{x}$ or $-\hat{z}$ component somewhere else to bring it back... That would imply dependence on x or y that we don't have $\Rightarrow \vec{B} = \hat{B}(z)y$

• If we rotate the entire arrangement 180° in the y-z plane nothing changes - this symmetry requires $\vec{B}(z) = -\vec{B}(-z)$

$$\boxed{\begin{gathered}\vec{B} = B(z)\hat{y} \\ \vec{B}(z) = -\vec{B}(-z)\end{gathered}}$$

- 1b) (10 points) Find the (vector) magnetic field at all points on the $+z$-axis.

$\vec{B}(z) = -\vec{B}(-z) \Rightarrow \vec{B}(0) = 0 \rightarrow$ Put one side of amperian loop on y-axis!

$(0<z<a)$ $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

$BL = \mu_0 (0)$

$B = 0$

$(a<z<b)$ $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

$BL = \mu_0 J \cdot L(z-a)$

$B = \mu_0 J(z-a)$

$(b<z)$ $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

$BL = \mu_0 J(b-a)L$

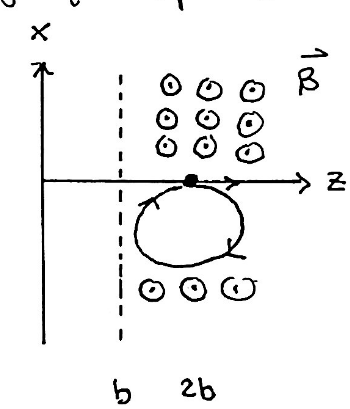
$B = \mu_0 J(b-a)$

$$\boxed{\vec{B} = \begin{cases} 0 & (0<z<a) \\ \mu_0 J(z-a)\hat{y} & (a<z<b) \\ \mu_0 J(b-a)\hat{y} & (b<z) \end{cases}}$$

• 1b) (*continued*)

• 1c) (10 points) A particle of mass m and electric charge $+q$ is placed at $\vec{r} = <0, 0, 2b>$ and given an initial velocity $\vec{v}_0 = <0, 0, v_0>$ directed away from the current sheets. Describe, in as much detail as you can, the subsequent motion of the particle (a sketch of the trajectory will definitely help!). Find the range of initial speeds that will end with the particle colliding with a current sheet.

Since there was no motion along the field lines, the helix you might have expected devolves into a circle in the x-z plane. $\vec{F} = q\vec{v}\times\vec{B}$ means the particle veers off to the right if we're looking down the $+y$ axis.



Boundary Conditions (V_0 along the z axis) mean we start out at the top of that circle, so we'll collide with the plane if $r > 2b - b = b$

$$r = \frac{mv}{qB} > b$$

$$\boxed{V > \frac{qBb}{m}}$$