MT2 Physics 1C S19

Full Name (Printed) David Tung

Full Name (Signature) [signature]

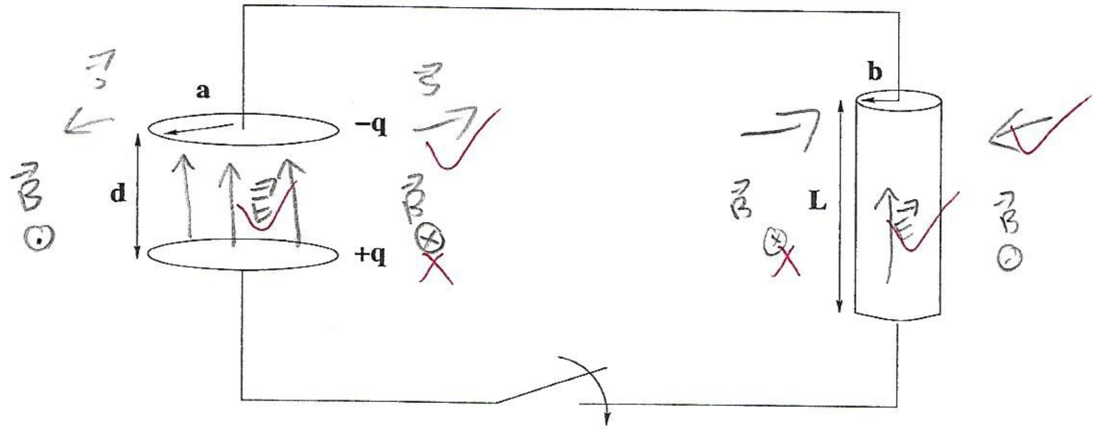
Student ID Number 505113462

Seat Number ______________________

Problem	Grade
1	17 /30
2	28 /30
3	11 /30
Total	56/90

56

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



1) A circular parallel-plate capacitor of plate radius a and plate separation d is connected across a cylindrical resistor of radius b and length L At the instant under consideration, the capacitor has a charge q, with the polarity given in the figure.

You will be asked to draw vectors on the sketch above. When necessary, please use $\odot$ and $\otimes$ to unambiguously resolve the direction of any azimuthal fields.

6

- 1a) (10 pts) Find the magnitude of the magnetic field at points inside the capacitor, located a distance r from the symmetry axis of the capacitor. Clearly sketch and label the direction of the magnetic field, the electric field and the Poynting vector field (in the region around the capacitor) on the figure above. Discuss the consistency between the flow of energy as indicated by the Poynting vector field and the changing state of charge on the capacitor.

$$\oint B \cdot ds = \mu_0 \left(I_{enc} + \varepsilon_0 \frac{d\Phi_E}{dt} \right) \qquad I_{enc} = 0$$

$$q = CV \quad V = \frac{q}{C}$$

$$B \cdot 2\pi a = \mu_0 \varepsilon_0 \frac{d}{dt}\left(E\,(2\pi a d)\right)$$

$$\frac{q}{C} - \frac{dq}{dt} R$$

$$B \cdot \cancel{2\pi a} = \mu_0 \varepsilon_0 \frac{d}{dt}\left(\frac{q\,\cancel{2\pi a}d}{\varepsilon_0 \pi a^2}\right)$$

$$R = \frac{\rho L}{A}$$

$$B = \mu_0 \cancel{\varepsilon_0} \frac{dq}{dt}\left(\frac{d}{\cancel{\varepsilon_0}\pi a^2}\right)$$

$$\frac{q}{C} - \frac{dq}{dt}\frac{\rho L}{\pi b^2} = 0$$

$$\frac{dq}{dt} = \frac{q\pi b^2}{C\rho L}$$

$$B = \mu_0 \frac{q\cancel{\pi} b^2}{C\rho L}\frac{d}{\cancel{\pi} a^2}$$

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$= \boxed{\mu_0 \frac{q b^2 d}{C\rho L a^2} \text{, where } \rho \text{ is resistivity of the resistor}}$$

$$E \pi a^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0 \pi a^2}$$

The Poynting vector points away from the capacitor, which is representative of the decreasing energy from the decreasing charge on the capacitor.

6 • 1b) (10 pts) Find the magnitude of the electric field and the magnitude of the magnetic field at points inside the resistor, located a distance r from the symmetry axis of the resistor. Clearly sketch and label the direction of the electric field and the magnetic field on the figure above.

$$\oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

$$E\, 2\pi b L = \frac{q_{enc}}{\varepsilon_0}$$

$$\boxed{E = \frac{q_{enc}}{2\pi\varepsilon_0 b L}}$$

$$\oint B \cdot ds = \mu_0 \left(I_{enc} + \frac{d\Phi_E}{dt}\right)$$

$$B \pi r^2 = \mu_0 I \frac{\pi r^2}{\pi b^2}$$

$$B \cancel{\pi} \cancel{r^2} = \mu_0 \frac{q \cancel{\pi} \cancel{b^2}}{c\rho L} \left(\frac{\cancel{r^2}}{\cancel{b^2}}\right)$$

$$\boxed{B = \frac{\mu_0 q}{c\rho L}, \text{ where } \rho \text{ is resistivity of resistor}}$$

5 • 1c) (10 pts) Find the magnitude of the Poynting vector at the boundary of the resistor. Clearly sketch and label the direction of the Poynting vector field on the diagram above. Find the rate at which energy is entering or leaving the resistor, and discuss the result.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

From part b,

$$\vec{S} = \frac{1}{\mu_0} \frac{q}{2\pi\varepsilon_0 b L} \frac{\mu_0 q}{c\rho L}$$

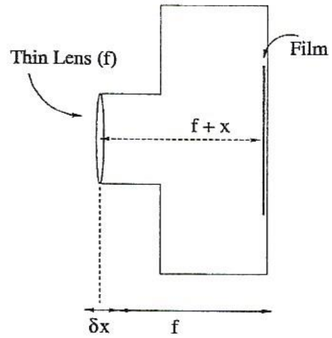
$$= \boxed{\frac{q^2}{2\pi\varepsilon_0 b c\rho L^2} \quad \text{where } \rho \text{ is resistivity of resistor}}$$

Poynting vector points towards the resistor, energy is entering

$$P = \int \vec{S}\, dA = \frac{q^2}{2\pi\varepsilon_0 b c\rho L^2}\, 2\pi b L$$

$$= \boxed{\frac{q^2}{\varepsilon_0 c\rho L}}$$

Energy is leaving the capacitor and entering the resistor



2) The important parts of the camera shown above are the film and the converging lens of focal length f. The distance between the lens and the film is given by $f + x$, where x can be adjusted to any value between 0 and $\delta x = f/10$ to bring the subject to be photographed into focus.

5

- 2a) (5 points) Find the near-point (the closest point at which you can take a focused picture of an object) and the far-point (the farthest point at which you can take a focused picture) for this camera.

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$q = f+x$

$\frac{1}{p} + \frac{1}{f+x} = \frac{1}{f}$

$\frac{1}{p} = \frac{f+x}{f(f+x)} - \frac{f}{f(f+x)}$

$\frac{1}{p} = \frac{x}{f(f+x)}$

$p = f\left(\frac{f+x}{x}\right)$

$= f\left(1+\frac{f}{x}\right)$

x ranges from 0 to δx

Near point when $x = \delta x$ =

$\boxed{NP: f\left(1+\frac{f}{\delta x}\right)}$

Far point when $x = 0$

$\Rightarrow f\left(1+\frac{f}{0}\right)$

$\boxed{FP = \infty}$

5

- 2b) (5 points) Suppose we're focused on an object and the lens is located at a distance $f + x$ from the film. What is the magnification of the object? Where would we have to place the object to get the greatest magnification and what would that magnification be?

$M = \frac{-q}{p}$ $\quad q = f+x$

From part a, $p = f\left(\frac{f+x}{x}\right)$

$M = -\frac{f+x}{\frac{f(f+x)}{x}} = -\frac{1}{\frac{f}{x}} = \frac{-x}{f}$

$M(x) = \boxed{\frac{-x}{f}}$

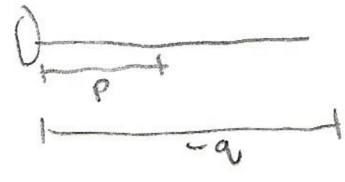
Greatest magnification at the near point, where $x = \delta x$

So $M = -\frac{\delta x}{f}$

10

- 2c) (10 points) In order to take close-up shots of an object, one may place a "*macro*" lens adjacent to the lens of the camera. In practice, the macro lens produces an image of the close object at the near-point of the camera and the camera takes a picture of that image. **What type of image must the macro lens produce?** If the desired distance between the object and the two lenses is $\frac{1}{5}$ of the camera's original near-point, **what focal length should the macro lens have? Is the macro lens converging or diverging?** [Hint: Sketch the macro lens, the object at the new near-point and the image at the old near-point. Mind your p's and q's].

Macro lens must produce a virtual image



$p = -\frac{1}{5}q$, where $q < 0$

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$-\frac{5}{q} + \frac{1}{q} = \frac{1}{f}$

$-\frac{4}{q} = \frac{1}{f}$

$f = \frac{-q}{4}$, which is >0

$$\boxed{f_m = \frac{f}{4}\left(1 + \frac{f}{8x}\right)}$$

macro lens is converging

5

- 2d) (5 points) With the macro lens in place and the object located at the new effective near-point ($\frac{1}{5}$ of the camera's original near-point distance), what is the total magnification of the object?

$M_{macro} = \frac{-q}{p} = \frac{-q}{-\frac{1}{5}q} = 5$ $\quad M_{lens} = \frac{-x}{f}$

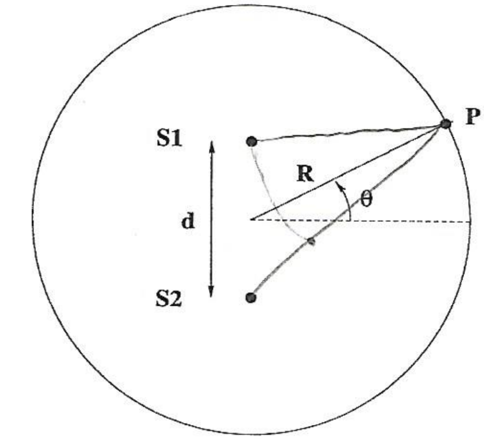
$M = M_{macro} M_{lens} = \boxed{-\frac{5x}{f}}$

five times initial magnification

3

- 2e) (5 points) The ability to take large close-up photos comes at a cost. Find (and interpret) the new far-point for the camera when the macro lens is attached.

The new far point for the camera is now at the focal length $f_m = \frac{f}{4}(1 + \frac{f}{8x})$, which makes it hard to take photos of further away objects.



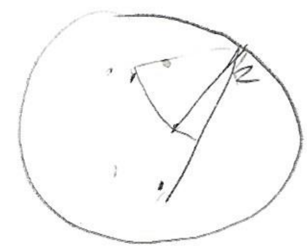
3) It is possible to use interference to create antenna arrays that preferentially radiate radio-frequency energy along preferred directions ("lobes") in the azimuth.

Let's analyze the simplest case - take a pair of vertically-oriented antennas separated by a horizontal distance d, driven in phase by a radio signal that has a wavelength λ in vacuum. The illustration shows a top-down view of the arrangement.

3

- 3a) (10 points) Find the exact phase difference between the waves that originate at sources S_1 and S_2 when they arrive at point P, located a distance R from the midpoint between the sources, in the azimuthal direction θ.

$$\Delta\theta_{path} = k\Delta L = \frac{2\pi}{\lambda}\,\frac{R\tan\theta\, d}{2} = \boxed{\frac{\pi R\tan\theta d}{\lambda}}$$



$R\tan\theta$

$$\Delta\theta_{TOT} = \underbrace{\Delta\theta_{path}}_{k\Delta L} + \Delta\theta_{IC} + \Delta\theta_{REF}$$

2

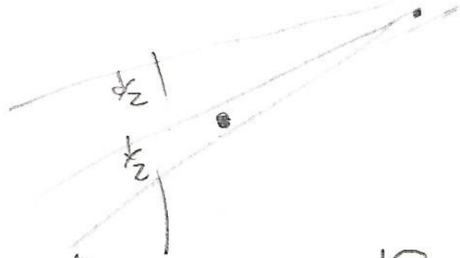
- 3b) (5 points) Show that, in the limit that $R \gg d$, that total phase difference is no longer a function of R and θ, but now, simply a function of θ only (remember, the values of d and λ are fixed). For the rest of the problem, we will assume that we are in this so-called *far-field* limit.

When $R>>d$, θ becomes a small angle, R can be approximated as d

$$\Delta\theta = \boxed{\frac{\pi}{\lambda}\,\frac{d^2\sin\theta}{}}$$

2

- 3c) (5 points) In what azimuthal direction(s) will the lobes point if $d = 2\lambda$? How many azimuthal lobes are there? (*Careful!* A quick qualitative sketch might help.)



$\frac{\pi}{\lambda} d^2 \sin\theta$

$4\lambda \pi \sin\theta$

The lobes will point in the positive azimuthal direction

There are 4 azimuthal lobes ✗

- 3d) (5 points) In what azimuthal direction(s) will the lobes point if $d < \lambda$? How many azimuthal lobes are there? (Again: *Careful!*)

$\frac{\pi}{\lambda} d^2 \sin\theta$

The lobes will point in the negative azimuthal direction

There are no azimuthal lobes ✗

48

- 3e) (5 points) Large trucks often have antennas strapped to the side mirrors on either side of the truck - $d \approx 2$ m, $f \approx 30$ MHz. Where is most of the energy going? How might this be advantageous to the truck driver? Much like biological evolution, technological solutions that work well tend to persist. Police cars usually have a single radiator that emits uniformly in all azimuthal directions. How did they evolve differently?

In the truck, most of the energy is emitted along the lobes in the azimuth, which could help transmit a longer radio signal.

Police cars emit energy in a uniformly direction across all azimuthal directions, which is better for that certain task, so it can be heard by everyone.

They evolved differently because of their different specific uses.