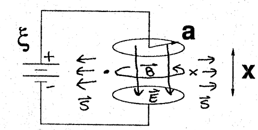


A parallel-plate capacitor made up of circular conducting plates of radius a is connected across a battery of potential difference ξ. At the moment under consideration, the distance between the plates is x and that distance is increasing at a rate $\frac{dx}{dt}$.

- 1a) (10 points) Find the magnitude and direction of the electric and magnetic fields that are present in the region between the capacitor plates. Sketch those fields on the diagram above (use dots and x's to remove any ambiguities for azimuthal fields).

$\Delta V = -\int \vec{E}\cdot d\vec{s}$

$E = \mathcal{E}/x$

(directed from sources to sinks

$\oint \vec{B}\cdot d\vec{s} = \mu_0\epsilon_0 \frac{\partial \Phi_E}{\partial t}$

$B 2\pi r = \mu_0\epsilon_0 \frac{\partial}{\partial t}\left(\frac{\mathcal{E}}{x}\pi r^2\right)$

$B = -\frac{\mu_0\epsilon_0 \mathcal{E} r}{2x^2}\frac{dx}{dt}$

$\frac{dx}{dt} > 0$

(directed opposite the azimuthal direction defined by $\vec{E}$

$$\boxed{\begin{array}{l} E = \mathcal{E}/x \\ B = \frac{\mu_0\epsilon_0 \mathcal{E} r}{2x^2}\frac{dx}{dt} \\ \text{directed as shown in the diagram} \end{array}}$$

- 1b) (5 points) Find the magnitude and direction of the pointing vector present on the boundary of the capacitor's volume and sketch the field on the diagram above).

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ $\quad |\vec{S}| = \frac{1}{\mu_0}EB$ directed outward from side...

$$\boxed{\begin{array}{l} S(a) = \frac{\epsilon_0 \mathcal{E}^2 a}{2x^2}\frac{dx}{dt} \\ \text{directed as shown in the diagram} \end{array}}$$

$\left(\begin{array}{c}\Delta q_{bat} \\ = \\ -\Delta q_{cap}\end{array}\right)$

- 1c) (10 points) Show that the rate at which electromagnetic energy enters or leaves the capacitor is the same as the rate at which energy is entering or leaving the battery (hint: $\Delta U_{bat} = \Delta q_{bat}\xi$).

Capacitor: $P = \int \vec{S}\cdot d\vec{A}$

$P = S(a)\, 2\pi a x$

$$\boxed{P = \frac{\epsilon_0 \pi a^2 \mathcal{E}^2}{x^2}\frac{dx}{dt}}$$

Battery: $\Delta U = -\Delta q_{cap}\mathcal{E}$

$\Delta U = -\Delta C \mathcal{E}^2$

$P = \frac{dU}{dt} = -\frac{dC}{dt}\mathcal{E}^2$

$P = -\frac{d}{dt}\left(\frac{\epsilon_0\pi a^2}{x}\right)\mathcal{E}^2$

$$\boxed{P = \frac{\epsilon_0 \pi a^2}{x^2}\mathcal{E}^2\frac{dx}{dt}}$$

Yup!

- 1d) (5 points) How rapidly is the total energy associated with the capacitor changing? How does your answer compare to the rate at which electromagnetic energy is entering or leaving the capacitor? Discuss.

$U = \frac{1}{2}C\mathcal{E}^2$

$P = \frac{dU}{dt} = \frac{1}{2}\frac{d}{dt}\left(\frac{\epsilon_0\pi a^2}{x}\right)\mathcal{E}^2$

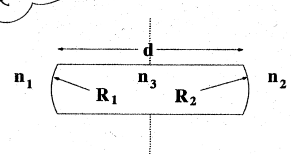
$$\boxed{P = -\frac{1}{2}\frac{\epsilon_0\pi a^2}{x^2}\mathcal{E}^2\frac{dx}{dt}}$$

Interesting! This is half the rate at which electromagnetic energy is leaving the capacitor!

What gives?

The remaining energy is associated with the mechanical work being done on the system :'

$\frac{n_{inc}}{p} + \frac{n_{emg}}{q} = \frac{n_{emg} - n_{inc}}{R}$



2) What happens when a thin lens straddles two different media? To find out, let's examine the case of the thick lens shown above.

- 2a) (10 points) Write a set of equations relating object and image distances to their respective surfaces (that is, p_1 and q_1 to R_1; p_2 and q_2 to R_2). Write an additional equation relating the initial object distance (p_1) to the final image distance (q_2) - you may leave one intermediate quantity (p_2 or q_1) in this equation, but not both.

$$\boxed{\begin{array}{l} \frac{n_1}{p_1} + \frac{n_3}{q_1} = \frac{n_3 - n_1}{R_1} \\ \frac{n_3}{p_2} + \frac{n_2}{q_2} = \frac{n_2 - n_3}{R_2} \end{array}}$$

$(d = q_1 + p_2)$

$$\boxed{\frac{n_1}{p_1} + \frac{n_2}{q_2} + n_3\left(\frac{1}{q_1} + \frac{1}{d - q_1}\right) = \frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}}$$

- 2b) (10 points) Now, evaluate that last equation in the relevant limit to find the appropriate expression for a thin-lens straddling two media. Find the focal-lengths on both sides of the lens (use the convention that f_i is the focal length when light emerges on the side with index n_i).

$d \to 0$

$$\boxed{\begin{array}{l} \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2} \\ \frac{1}{f_2} = \frac{1}{n_2}\left(\frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}\right) \\ \frac{1}{f_1} = \frac{1}{n_1}\left(\frac{n_3 - n_1}{R_1} - \frac{n_3 - n_2}{R_2}\right) \end{array}}$$

$f_i \propto n_i$ ✓

$n_1 = n_w = 4/3$

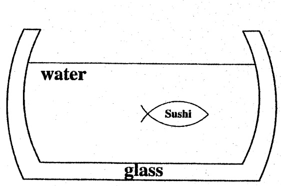
$n_2 = n_a = 1$

$n_3 = n_g = 3/2$

$p = \frac{|a|}{3}$

$R_1 = R_2 = -|a|$

air

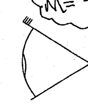


$M = M_1 \cdot M_2$

$M = \left(\frac{-q_1}{n_g}\frac{n_w}{p_1}\right)\cdot\left(\frac{-q_2}{n_a}\frac{n_g}{p_2}\right)$

$d = q_1 + p_2 \to 0 \quad q_1 = -p_2$

$M = -\frac{q_2}{n_a}\frac{n_w}{p_1}$



Could have guessed that! :)

- 2c) (10 points) I keep my pet fish *Sushi* in a (thin) glass bowl for which the inner-radius and the outer-radius of curvature are both given (in magnitude) as a.

  The water has an index of refraction $n_w = \frac{4}{3}$, the glass has an index of refraction $n_g = \frac{3}{2}$ and the surrounding air has an index of refraction $n_a = 1$...

  If *Sushi* is located a distance $\frac{a}{3}$ away from the tank wall, where will its image appear as I stare through that same tank wall eye-to-eye with the fish? How will the size of the image I see compare to *Sushi*'s actual size?

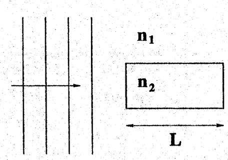
$\frac{n_w}{p} + \frac{n_a}{q} = \frac{n_g - n_w}{-|a|} - \frac{n_g - n_a}{-|a|}$

$\frac{4}{3}\frac{3}{|a|} + \frac{1}{q} = -\frac{1}{6|a|} + \frac{1}{2|a|} \rightarrow q = -\frac{3}{11}|a|$ (VI) ✓✓

$M = \frac{-q}{n_a}\frac{n_w}{p}$

$M = \frac{+3}{11}|a|\,\frac{4\cdot 3}{3|a|} \longrightarrow M = \frac{12}{11}$ (same orientation, roughly same size) ✓✓

$$\boxed{\begin{array}{l} \text{The image appears at a distance } \frac{3}{11}|a| \\ \text{behind the tank wall (on the water side)} \\ \text{and appears to be } \frac{12}{11} * \text{ the actual size} \\ \text{of the fish...} \end{array}}$$



$k_i = \frac{2\pi}{\lambda_i}$

$k_i = \frac{2\pi n_i}{\lambda}$

3) Coherent light, of wavelength λ (in vacuum), is incident in a medium of index n_1 on a translucent block of index n_2. The block is a regular rectangular prism of length L, and the light arrives normal to its upstream face, as shown.

- **3a) (5 points)** Looking back towards the block from the far-right, what phase-difference will the rays that traveled through the block pick up relative to the rays that missed the block? Check your answer in the limits *i)* $n_2 = n_1$ and *ii)* $L = 0$.

$$\Delta\theta_{path} = \Delta(KL)$$
$$= \Delta k \cdot L$$
$$= \frac{2\pi}{\lambda}(n_2 - n_1) L$$

$$\boxed{\Delta\theta_{path} = \frac{2\pi}{\lambda}(n_2 - n_1) L}$$

$\Delta\theta_{path} \to 0$ if $n_2 = n_1$ ✓
if $L \to 0$ ✓

- **3b) (5 points)** At a distant observation point (defined here by the difficulty encountered in resolving the block from its surroundings) the rays that emerge from the block will interfere with the rays that miss the block. For what values of L will the block be easiest to see? Explain.

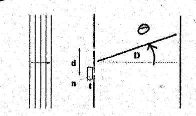
For maximum contrast, the block should be dark - Look for destructive interference

$$\Delta\theta_{TOT} = (2N+1)\pi$$
$$\frac{2\pi}{\lambda}(n_2 - n_1) L = (2N+1)\pi$$

$$\boxed{L_N = (2N+1)\frac{\lambda}{2(n_2 - n_1)}}$$

Treat the additional phase provided by the block as an initial condition present at the slits...

$\Delta\theta_{IC} = \Delta\theta_{block}$



$n_1 \to 1$
$n_2 \to n$
$L \to t$

$\Delta\theta_{block} = \frac{2\pi}{\lambda}(n-1)t$

The apparatus shown above consists of a slit-screen (a barrier with two straight slits, separated by a distance d) and a projection screen (oriented parallel to the slit-screen, at a distance D from the slit screen). The whole thing sits on a bench-top in a room filled with air and it is lit up, in the usual manner, with light of wavelength λ (in vacuum).

- **3c) (10 points)** A small translucent block of index n and thickness t is placed over the lower slit. By what distance will the central maximum be displaced (relative to where it would appear if the block weren't there) and in what direction?

$\Delta\theta_{TOT} = 2N\pi \to$ Central max $\Rightarrow N = 0$

$$\Delta\theta_{path} + \Delta\theta_{IC} + \cancel{\Delta\theta_{ref}} = 0$$
$$Kd\sin\theta + \frac{2\pi}{\lambda}(n-1)t = 0$$
$$d\sin\theta = -(n-1)t$$
$$\sin\theta = -(n-1)\frac{t}{d}$$

The central max is shifted from $\theta = 0$ to θ that satisfies: $\sin\theta = -(n-1)t/d$. Since $\theta < 0$, that shift is to the right (looking from the slits to the projection screen)

- **3d) (5 points)** Could you use an apparatus like this to measure the thickness of the block? How would you do it (or why won't it work)?

Maybe... if you were certain that the shift distance was small overall... the potential problem is that, if the shift were actually large, you would probably have a hard time identifying which maximum was the "central max" (eg- if the first order max slides to negative θ as well...)

- **3e) (5 points)** Because propagation differs so much from daylight to night, most AM radio stations have to shift the direction in which they broadcast when night falls. Explain how a broadcaster might configure a two-antenna system to do just that.

The pattern of maximas and minimas can be shifted by adding extra phase to one antenna. This is usually accomplished by adding extra feedline for a temporal delay...