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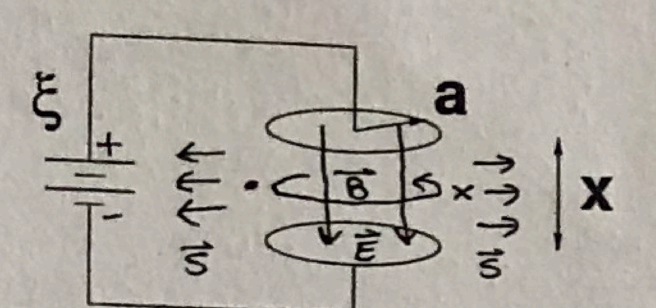


A parallel-plate capacitor made up of circular conducting plates of radius a is connected across a battery of potential difference ξ. At the moment under consideration, the distance between the plates is x and that distance is increasing at a rate $\frac{dx}{dt}$.

- 1a) (10 points) Find the magnitude and direction of the electric and magnetic fields that are present in the region between the capacitor plates. Sketch those fields on the diagram above (use dots and x's to remove any ambiguities for azimuthal fields).

$\Delta V = -\int \vec{E}\cdot d\vec{s}$ $\qquad \oint \vec{B}\cdot d\vec{s} = \mu_0\epsilon_0 \frac{\partial \Phi_E}{\partial t}$ $\qquad \frac{dx}{dt} > 0$

$E = \mathcal{E}/x$ $\qquad B 2\pi r = \mu_0\epsilon_0 \frac{\partial}{\partial t}\left(\frac{\mathcal{E}}{x}\pi r^2\right)$

(directed from sources to sinks) $\qquad B = \frac{-\mu_0\epsilon_0 \mathcal{E} r}{2x^2}\frac{dx}{dt}$ $\qquad$ (directed opposite the azimuthal direction defined by $\vec{E}$

$$\boxed{\begin{array}{l} E = \mathcal{E}/x \\ B = \frac{\mu_0\epsilon_0\mathcal{E} r}{2x^2}\frac{dx}{dt} \\ \text{directed as shown in the diagram} \end{array}}$$

- 1b) (5 points) Find the magnitude and direction of the pointing vector present on the boundary of the capacitor's volume and sketch the field on the diagram above).

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ $\qquad |\vec{S}| = \frac{1}{\mu_0}EB$ $\qquad$ directed outward from side...

$$\boxed{\begin{array}{l} S(a) = \frac{\epsilon_0 \mathcal{E}^2 a}{2x^2}\frac{dx}{dt} \\ \text{directed as shown in the diagram} \end{array}}$$

- 1c) (10 points) Show that the rate at which electromagnetic energy enters or leaves the capacitor is the same as the rate at which energy is entering or leaving the battery (hint: $\Delta U_{bat} = \Delta q_{bat}\,\xi$).

$\left(\begin{array}{c}\Delta q_{bat} \\ = \\ -\Delta q_{cap}\end{array}\right)$

Capacitor: $P = \int \vec{S}\cdot d\vec{A}$ $\qquad$ Battery: $\Delta U = -\Delta q_{cap}\mathcal{E}$

$P = S(a)2\pi a x$ $\qquad \Delta U = -\Delta C\mathcal{E}^2$

$\boxed{P = \frac{\epsilon_0\pi a^2\mathcal{E}^2}{x^2}\frac{dx}{dt}}$ $\qquad P = \frac{dU}{dt} = -\frac{dC}{dt}\mathcal{E}^2$

$P = -\frac{d}{dt}\left(\frac{\epsilon_0\pi a^2}{x}\right)\mathcal{E}^2$

yup!

$\boxed{P = \frac{\epsilon_0\pi a^2}{x^2}\mathcal{E}^2\frac{dx}{dt}}$

- 1d) (5 points) How rapidly is the total energy associated with the capacitor changing? How does your answer compare to the rate at which electromagnetic energy is entering or leaving the capacitor? Discuss.

$U = \frac{1}{2}C\mathcal{E}^2$

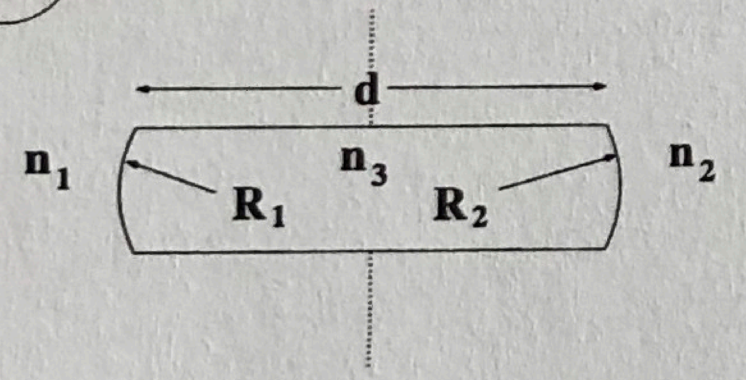
$P = \frac{dU}{dt} = \frac{1}{2}\frac{d}{dt}\left(\frac{\epsilon_0\pi a^2}{x}\right)\mathcal{E}^2$

$\boxed{P = -\frac{1}{2}\frac{\epsilon_0\pi a^2}{x^2}\mathcal{E}^2\frac{dx}{dt}}$

Interesting! This is half the rate at which electromagnetic energy is leaving the capacitor! what gives?

The remaining energy is associated with the mechanical work being done on the system :)

$$\frac{n_{inc}}{p} + \frac{n_{emg}}{q} = \frac{n_{emg}-n_{inc}}{R}$$



2) What happens when a thin lens straddles two different media? To find out, let's examine the case of the thick lens shown above.

- 2a) (10 points) Write a set of equations relating object and image distances to their respective surfaces (that is, p_1 and q_1 to R_1; p_2 and q_2 to R_2). Write an additional equation relating the initial object distance (p_1) to the final image distance (q_2) - you may leave one intermediate quantity (p_2 or q_1) in this equation, but not both.

$$\boxed{\begin{array}{l} \frac{n_1}{p_1} + \frac{n_3}{q_1} = \frac{n_3-n_1}{R_1} \\ \frac{n_3}{p_2} + \frac{n_2}{q_2} = \frac{n_2-n_3}{R_2} \end{array}}$$

$(d = q_1 + p_2)$

$$\boxed{\frac{n_1}{p_1} + \frac{n_2}{q_2} + n_3\left(\frac{1}{q_1} + \frac{1}{d-q_1}\right) = \frac{n_3-n_1}{R_1} - \frac{n_3-n_2}{R_2}}$$

- 2b) (10 points) Now, evaluate that last equation in the relevant limit to find the appropriate expression for a thin-lens straddling two media. Find the focal-lengths on both sides of the lens (use the convention that f_i is the focal length when light emerges on the side with index n_i).

$d \to 0$

$$\boxed{\begin{array}{l} \frac{n_1}{p} + \frac{n_2}{q} = \frac{n_3-n_1}{R_1} - \frac{n_3-n_2}{R_2} \\ \frac{1}{f_2} = \frac{1}{n_2}\left(\frac{n_3-n_1}{R_1} - \frac{n_3-n_2}{R_2}\right) \\ \frac{1}{f_1} = \frac{1}{n_1}\left(\frac{n_3-n_1}{R_1} - \frac{n_3-n_2}{R_2}\right) \end{array}}$$

$f_i \propto n_i$ ✓

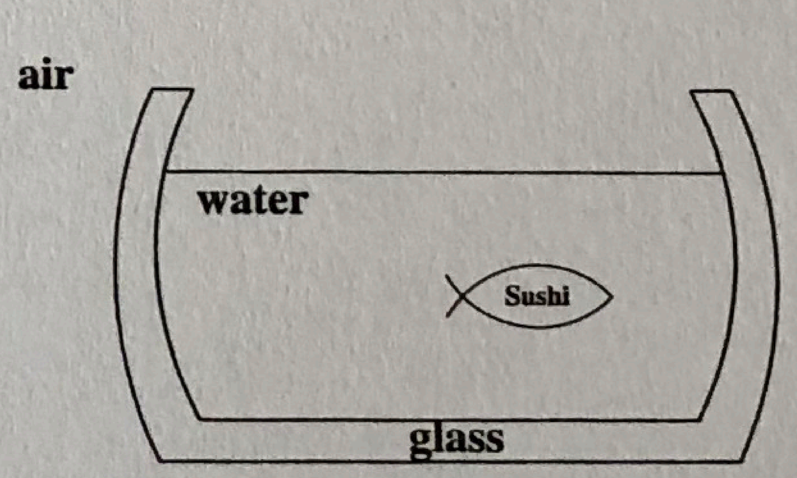
$n_1 = n_w = 4/3$

$n_2 = n_a = 1$

$n_3 = n_g = 3/2$

$p = \frac{|a|}{3}$

$R_1 = R_2 = -|a|$

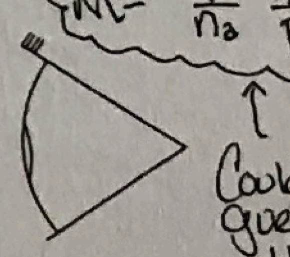


$M = M_1 \cdot M_2$

$M = \left(\frac{-q_1}{n_g}\frac{n_w}{p_1}\right)\cdot\left(\frac{-q_2}{n_a}\frac{n_g}{p_2}\right)$

$d = q_1 + p_2 \to 0 \qquad q_1 = -p_2$

$M = \frac{-q_2}{n_a}\frac{n_w}{p_1}$



Could have guessed that! :)

- 2c) (10 points) I keep my pet fish *Sushi* in a (thin) glass bowl for which the inner-radius and the outer-radius of curvature are both given (in magnitude) as a.

The water has an index of refraction $n_w = \frac{4}{3}$, the glass has an index of refraction $n_g = \frac{3}{2}$ and the surrounding air has an index of refraction $n_a = 1$...

If *Sushi* is located a distance $\frac{a}{3}$ away from the tank wall, where will its image appear as I stare through that same tank wall eye-to-eye with the fish? How will the size of the image I see compare to *Sushi*'s actual size?

$\frac{n_w}{p} + \frac{n_a}{q} = \frac{n_g - n_w}{-|a|} - \frac{n_g - n_a}{-|a|}$

$\frac{4}{3}\frac{3}{|a|} + \frac{1}{q} = -\frac{1}{6|a|} + \frac{1}{2|a|} \quad \rightarrow \quad q = -\frac{3}{11}|a| \qquad$ (VI) ✓✓

$M = \frac{-q}{n_a}\frac{n_w}{p}$

$M = \frac{+3}{11}|a|\,\frac{4\cdot 3}{3|a|} \quad \longrightarrow \quad M = \frac{12}{11} \qquad$ (same orientation roughly same size) ✓

The image appears at a distance $\frac{3}{11}|a|$ behind the tank wall (on the water side) and appears to be $\frac{12}{11}$ * the actual size of the fish...