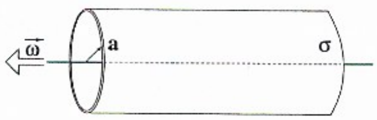


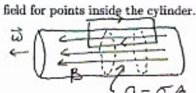

1) Charge is spread uniformly over a long, thin, non-conducting cylinder of radius $a$ so that it has a surface charge density $\sigma$. At the particular moment under consideration, the cylinder is rotating around its symmetry axis with an angular velocity $\vec{\omega}$, which is slowly decreasing.

- 1a) (10 points) Find the magnitude and direction of the magnetic field for points inside the cylinder.

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$

$Bx = \mu_0 \sigma a x \omega$

$\boxed{\vec{B} = \mu_0 \sigma a \vec{\omega}}$




$q = \sigma A$

$q = \sigma 2\pi a x$

$I = \frac{q}{T} = \frac{q\omega}{2\pi}$

$I = \sigma a x \omega$

If $\sigma > 0$, $\vec{B}$ will point in same direction as $\vec{\omega}$

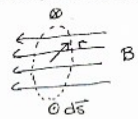
- 1b) (10 points) Find the magnitude and direction of the electric field for points inside the cylinder.

$\oint \vec{E}\cdot d\vec{s} = -\frac{\partial \Phi_B}{\partial t}$

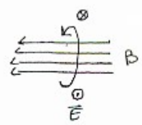
$E 2\pi r = -\frac{\partial}{\partial t}(\mu_0 \sigma a \omega \pi r^2)$

$E = -\frac{1}{2}\mu_0 \sigma a r \frac{d\omega}{dt}$

$\frac{d\omega}{dt} < 0$

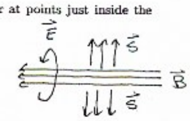



$E = -\frac{1}{2}\mu_0 \sigma a r \frac{d\omega}{dt}$, directed in azimuthal loops around $\vec{B}$ that point into the page at the top and out of the page at the bottom of the diagram.




- 1c) (5 points) Find the magnitude and direction of the Poynting vector at points just inside the cylinder's charged surface.

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ $\quad |\vec{S}| = \frac{1}{\mu_0} EB$




$S(a) = \frac{1}{\mu_0}\left(\frac{1}{2}\mu_0 \sigma a^2 \frac{d\omega}{dt}\right)(\mu_0 \sigma a \omega)$

$S(a) = \frac{1}{2}\mu_0 \sigma^2 a^3 \omega \left|\frac{d\omega}{dt}\right|$ directed radially away from the longitudinal symmetry axis.

- 1d) (5 points) Find the rate at which electromagnetic energy is flowing into/out of the empty volume just inside the cylinder's charged surface, per unit length.

$P = \int \vec{S}\cdot d\vec{A}$

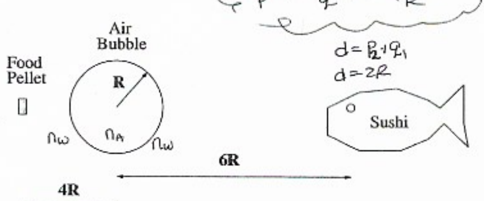
$P = S(a) 2\pi a L$

$\boxed{\frac{P}{L} = \mu_0 \sigma^2 \pi a^4 \omega \left|\frac{d\omega}{dt}\right|}$

$\left(\frac{n_{inc}}{p} + \frac{n_{emg}}{q} = \frac{n_{emg} - n_{inc}}{R}\right)$

$d = p_2 + q_1$

$d = 2R$




$p_1 = 3|R|$

2) My pet fish sits a distance $6R$ from the center of an air-bubble of radius $R$. A pellet of fish-food sits a distance $4R$ on the other side of the bubble (the fish's eye, food pellet and center of the bubble are co-linear). If $n_{air} = 1$ and $n_{water} = \frac{4}{3}$

- 2a) (20 points) Where will the image of the pellet appear to be, relative to the fish's eye?

$\frac{n_w}{p_1} + \frac{n_A}{q_1} = \frac{n_A - n_w}{|R|}$

$\frac{4}{3\cdot 3|R|} + \frac{1}{q_1} = -\frac{1}{3|R|}$

$\frac{1}{q_1} = -\frac{7}{9|R|}$

$p_2 = 2|R| - q_1 = 2|R| + \frac{9}{7}|R|$

$p_2 = \frac{23}{7}|R|$

$\frac{n_A}{p_2} + \frac{n_w}{q_2} = \frac{n_w - n_A}{-|R|}$

$\frac{7}{23|R|} + \frac{4}{3q_2} = \frac{-1}{3|R|}$

$\frac{4}{3q_2} = \left(-\frac{1}{|R|}\right)\frac{44}{69}$

$\frac{1}{q_2} = -\frac{11}{23|R|}$

$q_2 = -\frac{23}{11}|R|$ ← VI

The image of the pellet is a distance $\frac{23}{11}|R|$ to the left of the right side of the bubble...

The image will appear $\frac{78}{11}|R|$ in front of the fish

- 2b) (5 points) Will that image be upright or inverted?

For an SRS, $M = -\frac{q}{p}\frac{n_{inc}}{n_{emg}}$. In each of the two SRS stages, we had real objects producing virtual images, So... $M_1 > 0$ and $M_2 > 0$

$M = M_1 \cdot M_2 > 0$

So the image has the same orientation as the original object. It is upright

- 2c) (5 points) If the pellet appears to have a height $H$, (as measured by the fish), what is its actual height?

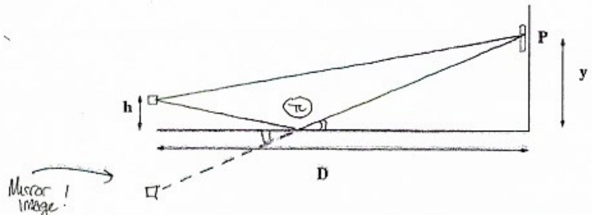
$M_1 = \frac{-q_1}{p_1}\frac{n_w}{n_A}$ $\quad M_2 = \frac{-q_2}{p_2}\frac{n_A}{n_w}$ $\quad M = M_1 M_2$

$M = \frac{q_1 q_2}{p_1 p_2}$ $\quad \left(= \frac{h_i}{h_o}\right)$

$h_o = h_i \frac{p_1 p_2}{q_1 q_2}$

$h_o = H \frac{3|R| \cdot \frac{23}{7}|R|}{\left(-\frac{9}{7}|R|\right)\left(-\frac{23}{11}|R|\right)}$

$\boxed{h_o = \frac{11}{3} H}$




3) A space-heater is placed at a small height $h$ above a reflective floor, a distance $D$ (large, compared to $h$) from an opposing wall. It emits its most intense radiation at a wavelength $\lambda$ in the infrared portion of the electromagnetic spectrum. For the purposes of this problem, assume that that radiation is significantly coherent.

- 3a) (10 points) Write the exact expression for the total phase difference between waves that arrive at point $P$ as a function of $y$, the height of point $P$ on the wall. Hint: you may be able to exploit a symmetry that greatly simplifies the geometry of one path.

$$\Delta\Theta_{path} = \frac{2\pi}{\lambda}\left[\sqrt{D^2+(y+h)^2} - \sqrt{D^2+(y-h)^2}\right]$$

$$\Delta\Theta_{ic} = 0$$

$$\Delta\Theta_{ref} = \pi$$

$$\boxed{\Delta\Theta_{TOT} = \frac{2\pi}{\lambda}\left[\sqrt{D^2+(y+h)^2} - \sqrt{D^2+(y-h)^2}\right] + \pi}$$

- 3b) (10 points) Suppose $h$ and $y$ are both quite small compared to $D$. Use a Taylor expansion to simplify the total phase difference.

$(1+x)^n \simeq 1+nx$ if $|x| \ll 1$

$$\Delta\Theta_{TOT} \simeq \frac{2\pi}{\lambda} D\left[1+\frac{1}{2}\left(\frac{y+h}{D}\right)^2 - 1 - \frac{1}{2}\left(\frac{y-h}{D}\right)^2\right] + \pi$$

$$\boxed{\Delta\Theta_{TOT} \simeq \frac{2\pi}{\lambda}\,\frac{2yh}{D} + \pi}$$

- 3c) (10 points) In that limit where $D$ is large, at what heights will it be safe to hang a candle on the opposing wall?

Looking for minimas

$$\Delta\Theta_{TOT} = (2N+1)\pi$$

$$\frac{2\pi}{\lambda}\,\frac{2yh}{D} + \pi = (2N+1)\pi$$

$$\frac{2yh}{D} = N\lambda$$

$$\boxed{h = N\,\frac{\lambda D}{2y}}$$