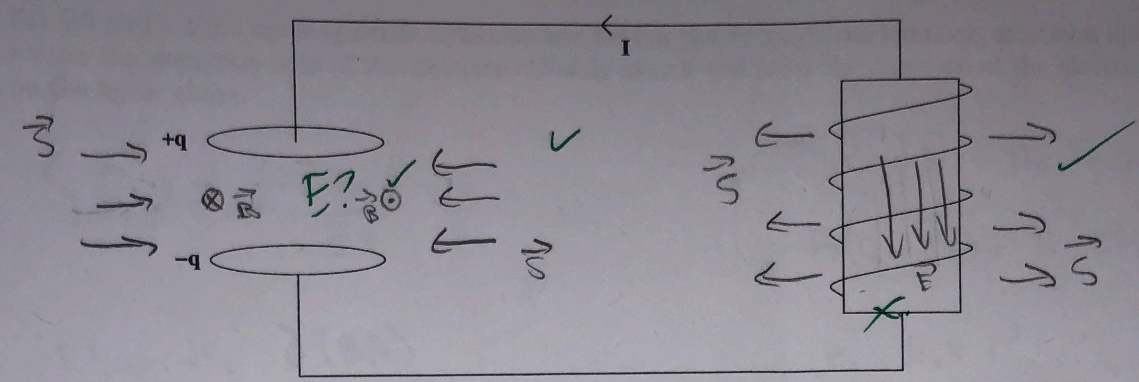


A circular parallel-plate capacitor of plate area A and plate separation d is connected across a solenoidal inductor of radius a, length x and n turns per unit length. At the instant under consideration, the capacitor has a charge q (polarity shown above), and a current I is flowing through the circuit in the direction shown. We do not know the instantaneous rate at which the current is changing, but it may be possible to infer that from the other information we know.

You will be asked to draw vectors on the sketch above. When necessary, please use ⊙ and ⊗ to unambiguously resolve the direction of any azimuthal fields.

6

- 1a) (10 pts) Find the magnitude of the magnetic field at points inside the capacitor, located a distance r from the symmetry axis of the capacitor. Clearly sketch and label the direction of the magnetic field, the electric field and the Poynting vector field (in the region around the capacitor) on the figure above. Discuss the consistency between the flow of energy as indicated by the Poynting vector field and the changing state of charge on the capacitor.

$$\oint \vec{B}\cdot d\vec{s} = \cancel{\mu_0 I_{enc}} + \mu_0\varepsilon_0 \frac{\partial \Phi_E}{\partial t}$$

$$B(2\pi r) = \mu_0\varepsilon_0 \frac{\partial}{\partial t}\left(E(r)\pi r^2\right)$$

$$B = \frac{\mu_0\varepsilon_0 \pi r^2}{2\pi r}\frac{\partial E(r)}{\partial t}$$

$$B = \frac{\mu_0\varepsilon_0 A}{2}\frac{\partial E(r)}{\partial t} \qquad E = \frac{q}{\varepsilon_0 A}$$

$$\boxed{B(r) = \frac{\mu_0\varepsilon_0 A}{2}\frac{\partial E(r)}{\partial t}}$$

where E is the electric field between the plates

The poynting vector points towards the capacitor, which is where the charge is building up. This is consistent with the fact that the poynting vector points towards where energy increases.

- 1b) (10 pts) Find the magnitude of the electric field at points inside the inductor, located a distance r from the symmetry axis of the inductor. Clearly sketch and label the direction of the electric field on the figure above.

6

$$\oint \vec{E}\cdot d\vec{s} = \frac{-\partial \Phi_B}{\partial t}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl}$$

$$n = \frac{N}{x}$$

$$B(2\pi r x) = \mu_0 I\left(\frac{r^2}{a^2}\right)N$$

$$E(2\pi r x) = \frac{-\partial (BA)}{\partial t}$$

$$B = \frac{\mu_0 I r^2 N}{2\pi r x a^2} = \frac{\mu_0 I}{2\pi}\left(\frac{r}{a^2}\right)n$$

$$E(2\pi r \cancel{x}) = \frac{-\partial B}{\partial t}(2\pi r \cancel{x})$$

$$B = \mu_0 n I$$

$$E = -\frac{\partial}{\partial t}\left(\frac{\mu_0 I}{2\pi}\frac{r}{a^2}\right)n = \frac{-\partial I}{\partial t}\frac{\mu_0 I n}{2\pi}\frac{r}{a^2}$$

- 1c) (10 pts) Find the magnitude of the Poynting vector at the boundary of the inductor. Clearly sketch and label the direction of the Poynting vector field on the diagram above. Find the rate at which energy is entering or leaving the inductor, and discuss the result.

4

$$\vec{S}(a) = \frac{1}{\mu_0}\vec{E}\times\vec{B} =$$

$$P = \int \vec{S}$$

The poynting vector field is away from the inductor because the opposing electric field causes energy to leave the inductor

2) Evidence suggests that artists have used lenses and mirrors to assist them from at least as far back as the mid- to early- 1700's. While historians are likely to tell you that lenses, as a technical tool, were still many years off in the future, there are plenty of paintings from the era that portray subjects with spectacles (reading glasses). So the question is, was it possible to use reading glasses to project a (traceable) image of a physical object onto a screen?

Recall that reading glasses are used to correct for the migration of the "near point" away from the eye with age. For the following, we'll assume that a typical pair of reading glasses is designed for an uncorrected near-point that is a distance D from the lenses of the reading glasses, and a desired near-point that is a distance d from the lenses.

$(NP)_{act} = D$

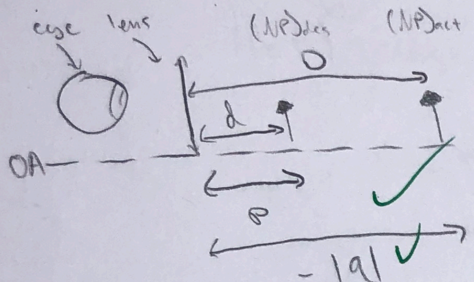
$(NP)_{des} = d$

- **2a) (10 points)** Think very carefully about this before you answer (a sketch might definitely help). If a person uses a typical pair of reading glasses to examine the fine detail on some object, where are they going to hold that object (in relation to the lens)? Where will the corresponding image appear? Will the image be real or virtual? Clearly identify the object and image distances (p and q, respectively) for this case - use absolute value bars and explicit signs.

10

$$\frac{n_{inc}}{p} + \frac{n_{ans}}{q} = \frac{n_{ans} - n_{inc}}{R}$$

radius of curvature



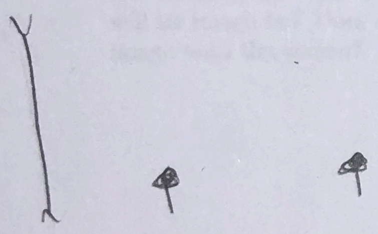
1) They will hold the object at d.

2) The image will appear at D.

3) The image will be virtual

4) $p = |p| \rightarrow |p| = d$

$q = -|q| \rightarrow |q| = D$



- **2b) (5 points)** Find the focal length of the lens. Is it converging or diverging? Explain.

3

Lens-makers' equation: $\frac{1}{f} = \frac{n_L - n_m}{n_m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$f = ?$

The lens is converging, otherwise you wouldn't be able to see a clear picture.

- 2c) (10 points) Let's see if we can project an image of a physical object on a screen using this lens. If the object is located a distance $L >> D$ from the lens, will a projectable image be formed (explain), if so, where, and what will be its magnification?

4

A projectable image means that the focal point can be displayed on the screen, which is possible because the focal length is shorter than L.

The image will be inverted.

It's magnification is $\frac{D}{d}$.

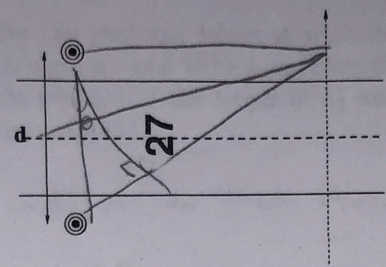
- 2d) (5 points) Ok, I apologize, but we need some numbers to gauge the feasibility of the technique. Typical values for the relevant quantities are $L = 4$ m, $D = 0.5$ m and $d = 0.1$ m. How much distance will there be between the lens and the screen? If the physical object located at L is 2 m tall, how large will its image be? Does it seem likely an artist could use this technique to trace a nearly photographic image onto the screen?

1

H = 20 m tall

Yes an artist could have used this ✓.

(a very talented one)



A simple navigation system consists of two radio-frequency antennas fed in-phase with a signal of wavelength λ. The antennas lie along a line that runs perpendicular to the runway, they are equidistant from the center of the runway, and they are separated by a distance d. Incoming aircraft are supposed to locate the central maximum generated by the resultant signal and follow it to a safe landing on the airstrip below.

- **3a) (5 points)** Find the full angular width of the central maximum (assume small angle approximations are relevant).

$$\Delta\Theta_{TOT} = \Delta\Theta_{PATH} + \Delta\Theta_{IC}$$

- **3b) (5 points)** Under what (approximate) conditions will there only be a central maximum?

There will only be a central maximum when

$\Delta\Theta_{TOT} \approx \pi/2$

2

- **3c) (5 points)** It might seem like a good idea to set things up so that there are no higher-order maxima to confuse the pilot, but it really isn't. Why? What compromise needs to be made?

Because if you eliminate higher order maxima, the central maxima becomes wider and then it becomes unclear where the center of the strip is. There needs to be a compromise between the number of higher order maxima and the width of the central maxima.

- 3d) (5 points) We can resolve the problem inherent with higher-order maximas by feeding each antenna with *two* in-phase radio signals - one with a wavelength λ_1 and the other with a wavelength λ_2. Explain how this will fix the problem if the larger of λ_1 and λ_2 is not an integer multiple of the smaller (why is this condition important?).

the condintion is important so that there is not regular interference between the two waves

This will fix the problem because we essentially get two interference patterns: one for λ_1 and one for λ_2. Each of which gives a central max at the same spot, which will be large due to constructive interference between the two. None of the higher order maxima will have this constructive interference (b/c of condition) and so the central max will be much bigger.

- 3e) (10 points) A pilot decides to calibrate her groundspeed indicator. She flies along a path parallel to the line joining the antennas, a distance D from that line and notes the signal strength at wavelength λ_1 is fluttering at a rate F_1 beats per second. How fast is she flying?

fast enough to stay afloat but slower than speed of light