MT2 Physics 1C(2), F16

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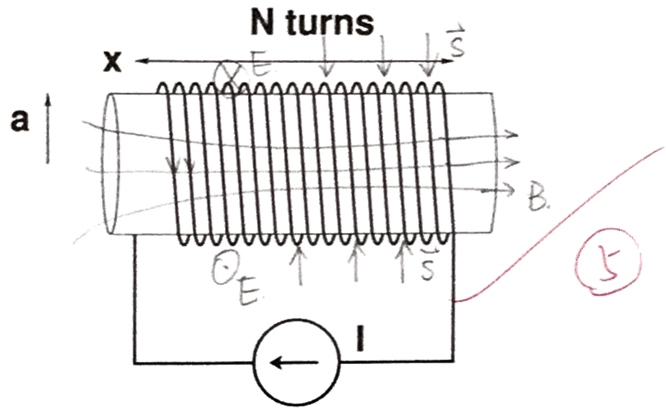
Seat Number ______________________

Problem	Grade
1	21 /30
2	19 /30
3	14 /30
Total	54 /90

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



An inductor is wound over a cylindrical form of radius a and has N turns evenly spaced over a length x as shown. It is connected across a source that supplies a constant current I in the direction shown.

- 1a) (5 points) Find the magnitude and direction of the magnetic field produced by the inductor. Sketch it in on the diagram.

$B = \mu_0 n I = \mu_0 \frac{N}{x} I$ To the right

- 1b) (10 points) Suppose we (slowly) decrease the length of the solenoid at a rate $\frac{dx}{dt}$. Find the magnitude and direction of the electric field that results. Sketch the electric field on the diagram using $\odot$ and $\otimes$, if necessary, to resolve any ambiguity and explain how that direction is consistent with what you know from Maxwell's equations.

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$E \cdot 2\pi a = -\frac{d\left(\mu_0 \frac{N}{x} I \cdot \pi a^2\right)}{dt}$$

$$E = -\frac{1}{2}\mu_0 N I a \frac{d\frac{1}{x}}{dt} = \cdots$$

From Maxwell's equations.

- **1c) (10 points)** Find the magnitude and direction of the Poynting vector at the boundary of the inductor. Sketch it on the diagram. (Again, use ⊙ and ⊗, if necessary.)

$$|\vec{S}| = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$= \frac{1}{\mu_0} |\vec{E}| \cdot |\vec{B}|$$

$$= \frac{1}{\mu_0} \frac{1}{2} \mu_0 N I a \frac{dI}{dt} \cdot \mu_0 \frac{N}{x} I$$

$$= \frac{N^2 I^2 a}{2x} \frac{dI}{dt}$$

Pointing inward as sketched

$= \cdots$

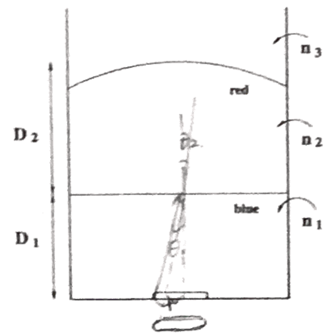
(5)

- **1d) (5 points)** Show that electromagnetic waves are delivering (or removing) energy at the same rate that the energy associated with the inductor is changing.

$$\text{Energy} = |\vec{S}| \cdot A = \frac{N^2 I^2 a}{2x} \cdot \frac{dI}{dt} \cdot 2\pi a x$$

(1)

For an inductor,



A beaker contains two layers of fluid. The blue fluid that sits on the bottom has an index of refraction n_1 and fills the beaker to a height D_1. The red fluid on top has an index of refraction n_2 and spans a depth D_2. The interface between the fluids is flat. The interface between the top fluid and the air above it (of index n_3) is curved upward with a radius of curvature of magnitude $|R|$ (Large compared to either value of D, but small enough to be noticeable).

- 2a) (15 pts) A small object sits on the bottom of the beaker. What will be the apparent depth of the object, as seen by an observer looking straight down from the top?

From $n_1 \to n_2$,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \tan\theta_1 = n_2 \tan\theta_2$$

$$n_1 \frac{x}{D_1} = n_2 \frac{x}{D_1'}$$

$$\Rightarrow D_1' = -\frac{n_2 D_1}{n_1}$$

From $n_2 \to n_3$. $p = D_2 + D_1' = D_2 + \frac{n_2 D_1}{n_1}$ ✓

Since n_3 is air, n_2 is fluid

$n_3 < n_2$

concave $R < 0$

10

$$\frac{n_2}{p} + \frac{n_3}{q} = \frac{n_3 - n_2}{-|R|}$$

$$\frac{n_3}{q} = \frac{n_3}{-|R|} - \frac{n_2 n_1}{n_1 D_2 + n_2 D_1}$$

$$\frac{n_3}{q} = \frac{n_1 n_3 D_2 + n_2 n_3 D_1 + n_2 n_1 |R|}{-(n_1 D_2 + n_2 D_1)|R|} \Rightarrow q = \frac{-(n_1 n_3 D_2 + n_2 n_3 D_1)|R|}{n_1 n_3 D_2 + n_2 n_3 D_1 + n_2 n_1 |R|}$$

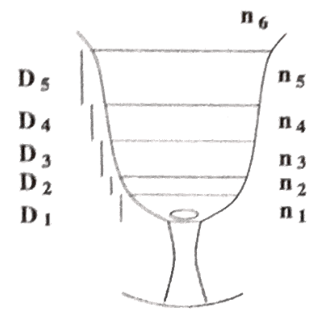
- 2b) (5 pts) By what factor (overall) will the object appear to be magnified?

From $n_1 \to n_2$ There is ~~no~~ ✓ magnification.

4

From $n_2 \to n_3$ $M = -\frac{q}{p} = \frac{\dfrac{n_1(n_1 n_3 D_2 + n_2 n_3 D_1)|R|}{n_1 n_3 D_2 + \cdots + \cdots}}{n_1 D_2 + n_2 D_1}$

$$= \frac{n_1 n_3 |R|}{n_1 n_3 D_2 + n_2 n_3 D_1 + n_2 n_1 |R|}$$



- 2c) (5 pts) *(Refering to the previous page)* Under what condition(s) will the object appear to our observer to be immersed in the red fluid? Explain.

When $q_R > -D_2$

$$\frac{(n_1 n_3 D_2 + n_2 n_3 D_1)|R|}{n_1 n_3 D_2 + n_2 n_3 D_1 + n_2 n_1 |R|} < D_2$$

$$\therefore \frac{1}{D_2} < \frac{1}{|R|} + \frac{n_2 n_1}{n_1 n_3 D_2 + n_2 n_3 D_1}$$

0

- 2d) (5 pts) The good folks at the cantina on Mos Eisley make a mean multi-layered cocktail that looks something like the diagram above (depths and indices are given). Assuming all the interfaces are flat, find the apparent depth of the space-olive on the bottom of the glass. (This does not have to involve a lot of work if you look for a pattern in your answer to part *a* in some appropriate limit).

From $n_1 \to n_2$, $D_1' = \frac{n_2 D_1}{n_1}$

From $n_2 \to n_3$. $D_2' = \frac{n_3}{n_2}(D_1' + D_2)$

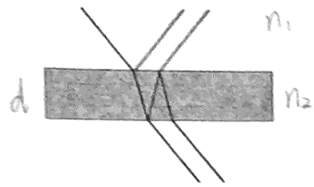
$$= \frac{n_3}{n_2}\left(\frac{n_2 D_1}{n_1} + D_2\right)$$

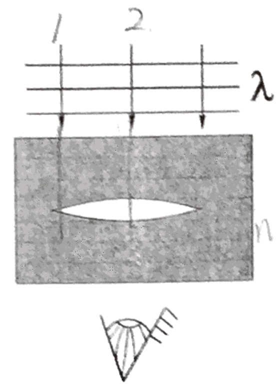
$$= \frac{n_3}{n_1} D_1 + \frac{n_3}{n_2} D_2$$

5

$\cdots\cdots n_6$

$$D_6' = \frac{n_6}{n_1} D_1 + \frac{n_6}{n_2} D_2 + \cdots \frac{n_6}{n_5} D_5$$





- 3a) (10 points) Monochromatic light strikes a thin film in near-vertical incidence. Using interference principles, show that when the light reflected off the top of the film undergoes constructive interference, the light transmitted through the film undergoes destructive interference, and vis-versa.

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We assume $n_2 > n_1$

1° Constructive for reflect $\Delta\theta_{TOT} = \Delta\theta_{path} + \Delta\theta_{ref}$
$= k_2 \Delta L + \pi$
$= \frac{2\pi}{\lambda_2} \cdot 2d + \pi = 2N\pi$

Destructive for transmit $\Delta\theta_{TOT} = \Delta\theta_{path} + \Delta\theta_{ref}$
$= k_2 \Delta L + 0$
$= \frac{2\pi}{\lambda_2} \cdot 2d + 0 = (2N-1)\pi$

$\Rightarrow \frac{4d}{\lambda_2} + 1 = 2N$ $\xLeftrightarrow{\text{Same}}$ $\Rightarrow \frac{4d}{\lambda_2} = 2N-1$

2° Destructive for reflect $\Rightarrow \frac{4d}{\lambda_2} + 1 = 2N+1$ $\xLeftrightarrow{\text{Same}}$ Constructive for transmit $\Rightarrow \frac{4d}{\lambda_2} = 2N$

- 3b) (10 points) Suppose we illuminate a piece of translucent material with monochromatic light of wavelength λ. If the light encounters a thin crack (running parallel to the flat faces of the material) describe the pattern of fringes seen by an observer on the opposite side. Assume near-vertical incidence, and be sure to mention whether the edges of the crack are bright or dark.

4

Compare light 1 & 2. $\Delta\theta_{TOT} = \Delta\theta_{path} = (k_n - k)\Delta L = (\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda})\Delta L$

Constructive $(\frac{n}{\lambda} - \frac{1}{\lambda})\Delta L = N$
$\Delta L = \frac{\lambda N}{n-1}$

Destructive $(\frac{n}{\lambda} - \frac{1}{\lambda})\Delta L = N + \frac{1}{2}$
$\Delta L = \frac{\lambda(N+\frac{1}{2})}{n-1}$

ΔL is the difference distance between the crack.

The pattern is many circles of bright and dark fringes with edge bright if the thick of crack is $\frac{\lambda N}{n-1}$ or dark if crack is $\frac{\lambda(N+\frac{1}{2})}{n-1}$ thick

- 3c) (10 points) Suppose (looking at the side opposite the source) we count M dark fringes in all. Estimate both the maximum thickness of the crack and the potential error in that estimate.