MT1 Physics 1C S19

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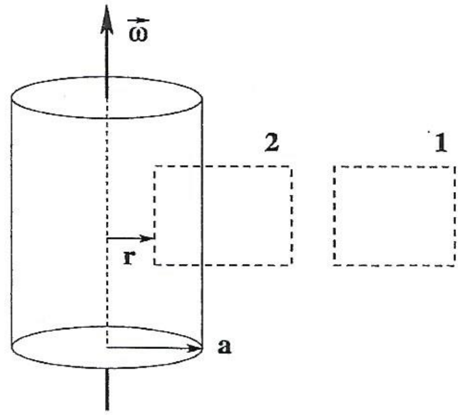
Seat Number ______

Problem	Grade
1	14 /30
2	13 /30
3	21 /30
Total	48 /90

48

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



$\mu = NIA$

1) A *very long*, solid, uniform cylinder of radius a and uniform electric charge density ρ rotates around its longitudinal symmetry axis with an angular velocity $\vec{\omega}$

2

- 1a) (5 points) Using symmetry arguments and first principles, predict the **direction** of the magnetic field the cylinder produces at points inside and outside the cylinder.

The magnetic field direction is determined by the right hand rule.

Inside The magnetic field will point in the "asmuthal" direction (into the page on the right side and out of the page on the left side).

Outside The magnetic field produced is 0 because there is no charge enclosed

1

- 1b) (5 points) Using the loop labeled '1', discuss the rate at which the magnetic field outside the cylinder changes with respect to changes in radial distance from the symmetry axis of the cylinder. Make an argument for the value of the magnetic field at large distances from the symmetry axis and deduce the value of the magnetic field at points outside the cylinder.

The magnetic field drops off at a rate proportional to $\frac{1}{r}$.

$$B = \frac{\mu_0 I}{2\pi r}, \text{ where } I = \frac{q}{T} = \frac{\pi r^2 \rho w}{2\pi} = \frac{r^2 \rho w}{2}$$

$$B = \frac{\mu_0 r^2 \rho w}{4\pi r}$$

$$= \boxed{\frac{\mu_0 r \rho w}{4\pi}}$$

2

- 1c) (5 points) Now, if you stop to think about it, there's a significant chance that your answer to part b is in conflict with your answer to part a. Explain. Do your best to resolve this discrepancy.

$\oint B\,ds = \mu_0 (I_{enc} + \varepsilon_0 \frac{d\Phi_E}{dt})$.

There is no "charge enclosed" but there is a changing electric flux that is happening.

By the mathematical interpretation of ampere's law, there is a discrepancy.

7

- 1d) (10 points) Find the rate at which **charge** passes through the loop marked '2' as a function of the distance from the inner boundary of the loop to the symmetry axis (r).

The charge that passes through is $\rho \pi (a^2 - r^2)$.

$T = \frac{2\pi}{\omega}$

$$i = \frac{q}{T} = \frac{\rho \pi (a^2 - r^2)\,\omega}{2\pi}$$

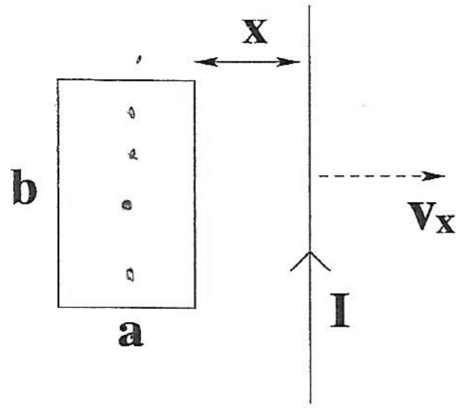
$$= \boxed{\frac{\rho (a^2 - r^2)\,\omega}{2}}$$

2

- 1e) (5 points) Find the magnetic field at all values of the radial distance from the symmetry axis.

$\oint B\,ds = \mu_0 I_{enc}$

$$B = \frac{\mu_0 i}{2\pi r} = \boxed{\frac{\mu_0 \rho (a^2 - r^2)\,\omega}{4\pi r}}$$



2) In the diagram above, a rectangular conducting loop (dimensions a and b, resistance R) and a long straight wire that carries an electrical current I are both oriented so that they sit in the plane of the page. They will, for the duration of the problem, remain in the plane of the page with the wire carrying an electrical current I parallel to the right side of the conducting loop at a distance x to that side (as shown).

4

- 2a) (10 points) Assuming the conducting loop remains fixed in space while the wire is pulled away at a speed v_x, what is the magnitude of the resulting current induced in the loop? In what direction is that induced current traveling on the side closest to the wire (with or against the current direction in the wire?)

When the wire is pulled away, this creates a decreasing magnetic flux.
the resulting current has magnitude $i = \frac{B b V_x}{X}$

$B = \frac{\mu_0 I}{2\pi x}$

The wire causes a B-field in the direction out of the page on the left side. When it is pulled away, there is a decreasing magnetic flux. Therefore, the induced emf (by Lenz's law) will work to restore this, so it will run in the counterclockwise direction (with the current direction in the wire).

−6

$B = \frac{\mu_0 I}{2\pi R}$ $B^2 = \frac{\mu_0^2 I^2}{4\pi^2 R^2}$ $F_B =$

6

- 2b) (10 points) How large and in what direction is the force exerted on the loop by the wire? How large and in what direction is the force exerted on the wire by the conducting loop? Is this result consistent with Lenz's Law? Explain.

The force exerted on the wire is in the same direction of v_x, with magnitude $F_B = iL \times B = \frac{Bbv_x}{x} bB F \frac{B^2 b^2 v_x}{x}$;

-4

By Newton's second law, the force exerted on the wire is of the same magnitude ($F = \frac{B^2 b^2 v_x}{x}$) in the $-v_x$ direction

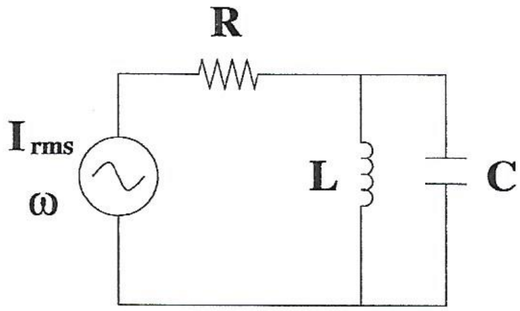
This is consistent with Lenz's law because there is a force working to oppose this changing magnetic flux (wires will "attract").

3

- 2c) (10 points) Find the net torque on the conducting loop. For full credit, the grader must be able to follow the logic of your calculation.

$$\vec{\tau} = I(\vec{A} \times \vec{B})$$
$$= \frac{Bbv_x}{x} \left(\vec{ab} \times \frac{\mu_0 I}{2\pi x} \right)$$
$$= \frac{Bbv_x}{x} \left(ab \frac{\mu_0 I}{2\pi x} \right) \sin\theta$$

-7



3) An RLC network is driven by a sinusoidally-varying electric current of root-mean-square value I_{rms} and angular frequency ω.

3

- 3a) (5 points) Find the root-mean-square values of the voltage that appears across the resistor and the voltage that appears across the LC-network.

$V = iR$

$V = iR = iZ$

$V_{rms\ resistor} = \boxed{i_{rms} R}$ ✓

$V_{rms\ LC} = i_{rms}\left(\frac{1}{X_L} + \frac{1}{X_C}\right) = \boxed{i_{rms}\left(\frac{1}{\omega L} + \omega C\right)}$ ✗

4

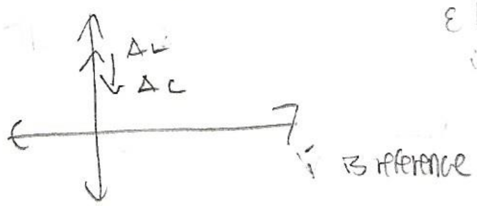
- 3b) (5 points) Will the sum of the rms-voltages across the resistor and the LC-network add up to the rms-voltage across the current source? Why or why not? Explain.

No, because there is an imagnary component and only the current is the same.

5

- 3c) (5 points) Under what conditions will the voltage across the LC network lead the driving current? Under what conditions will it lag? By how much will it lead or lag in each case?

LC



ε leads I in inductor

$\frac{1}{X_C} = \frac{1}{\omega L}$

$\uparrow$

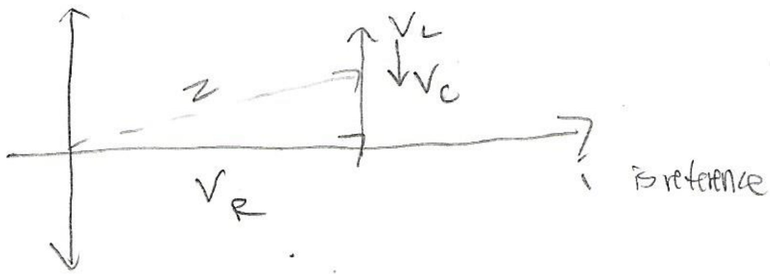
$\downarrow \frac{1}{X_L} = \omega C$

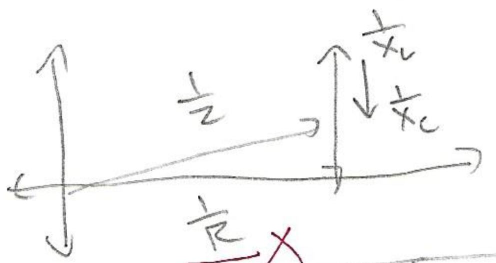
$X_C = \frac{1}{\omega C}$ $X_L = \omega L$

voltage will lead when $\frac{1}{\omega L} > \omega C$ and lag when $\frac{1}{\omega L} < \omega C$, by 90° in each case. ✓

6

- 3d) (10 points) What is the impedance seen by the source?





$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

$$\boxed{Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}}}$$

3

- 3e) (5 points) Find the root-mean-square value of of the voltage across the current source. How does your answer compare to your response in part *b*?

$V = iZ$

$$\boxed{V_{rms} = \frac{I_{rms}}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}}}$$

$V_{rms\ resistor} = i_{rms} R$ $V_{rms\ LC} = i_{rms}\left(\frac{1}{\omega L} + \omega C\right)$

The sum of the rms voltages across resistor and LC network do not add up to the rms across the voltage source, which is consistent with part b.