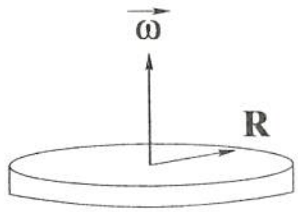
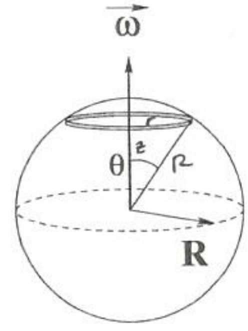




- 1a) (10 points) A uniform **disk** of charge Q and radius R spins about its perpendicular symmetry axis with an angular velocity $\vec{\omega}$ (as shown in the left diagram above). Derive the (vector) magnetic dipole moment of the distribution.

$\vec{\mu} = \gamma \vec{L} = N I \vec{A}$ $\quad \gamma = \frac{1}{2}\frac{Q}{M}$?

$\int_0^R I\, dA$

$d\vec{A} = 2\pi r\, dr \quad I(r) = \frac{2\pi r\, dr}{\pi R^2} Q \cdot f = \frac{2\pi r\, dr}{\pi R^2} Q \cdot \frac{\omega}{2\pi}$

$\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi}$ +3

$\vec{L} = I_{disk}\vec{\omega} \quad I_{disk} = \int r^2\, dm$

$dm = 2\pi r\, dr\, \sigma$

$I_{disk} = 2\pi\sigma \int_0^R r^3\, dr$ +3

$= \frac{1}{2}\pi\sigma R^4$

$\sigma = \frac{M}{\pi R^2} \quad \downarrow \quad \frac{1}{2} M R^2$

$\vec{L} = \frac{1}{2} M R^2 \vec{\omega}$

$\vec{\mu} = \gamma \vec{L} = \frac{1}{2}\frac{Q}{M} \cdot \frac{1}{2} M R^2 \vec{\omega}$ +4

$= \boxed{\frac{1}{4} Q R^2 \vec{\omega} = \vec{\mu}}$

- 1b) (10 points) A uniform **sphere** of charge Q and radius R spins about an axis that passes through its center with an angular velocity $\vec{\omega}$ (as shown in the right diagram above). How large a contribution to the total magnetic dipole moment of the sphere will the infinitesimally-thin disk shown in the right diagram make?

$d\vec{\mu} \sim IA$? $\quad \cancel{dI = dq \cdot \frac{\omega}{2\pi}} \quad \left(\frac{\omega}{2\pi} = f\right)$

$dq = \pi r^2\, dz\, \rho$ +3

$\boxed{d\vec{\mu} = \frac{1}{4}\, dq\, r^2 \vec{\omega}}$

$r = R\sin\theta$ +2

$z = R\cos\theta$

$dz = -R\sin\theta$ +1

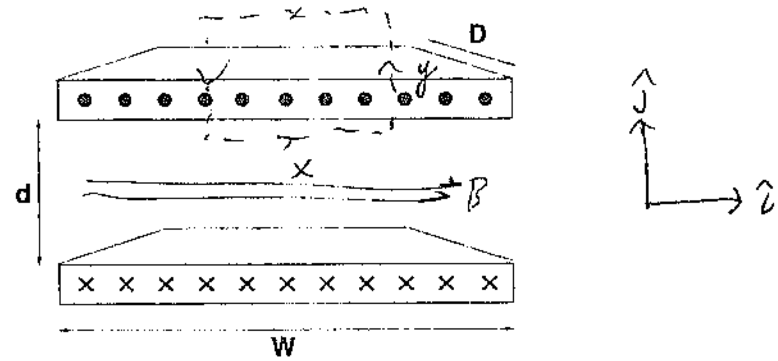
$\rho = \left(\frac{4}{3}\pi R^3\right)^{-1} Q$

- 1c) (5 points) Derive the (vector) magnetic dipole moment of the spinning spherical distribution.

+0

- 1d) (5 points) When placed in a uniform magnetic field B, the sphere precesses with an angular frequency Ω. Assuming its mass is also uniformly distributed, find the charge-to-mass ratio for the sphere.

+0



2) Two thin, parallel, conducting sheets of dimension $D \times W$ are separated by a distance d as shown ($D \gg W \gg d$). The sheets each carry a *linear* current density K, one into the plane of the page, one out, as shown.

Amps/meter

- 2a) (10 points) Use your knowledge of the magnetic field in and around long, wide current sheets to obtain a qualitative description of the magnetic field in the region between the sheets and in the regions outside the sheets. Follow that up by actually finding the magnetic field in each of those regions.

By right-hand rule, magnetic field points to ~~the~~ the right between the sheets. Since field strength doesn't depend on distance, magnetic field uniform between sheets and zero outside sheets.

top sheet: $\vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$Bx = \mu_0 K x$

$B = \mu_0 K$

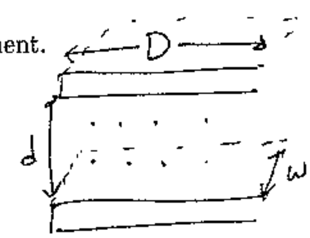
one sheet $2Bx = \mu_0 K x$

$\Rightarrow B_{tot} = \mu_0 K \hat{x}$

both sheets: $B = 2\mu_0 K$

$\boxed{\vec{B} = 2\mu_0 K \hat{i}}$ (between) $\quad \boxed{\vec{B} = 0}$ (outside)

- 2b) (10 points) Use Faraday's law to calculate the self-inductance of the arrangement.



$\mathcal{E} = -\frac{d\Phi_B}{dt}$

$\Phi_B = BA = 2\mu_0 K\, Dd$

$\cancel{\frac{d\Phi_B}{dt} = 2\mu_0 Dd \frac{dK}{dt}}$

$K = \eta I$?

$K \sim I$

let $K = I/W$

~~[illegible]~~

$\Phi_B = 2\mu_0 \frac{I}{W} Dd$

$\frac{d\Phi_B}{dt} = \frac{2\mu_0 Dd}{W} \frac{dI}{dt}$

$$\boxed{L = 2\mu_0 \frac{Dd}{W}}$$

- 2c) (10 points) Verify your answer for the self-inductance of the arrangement by using energy considerations.

$$U = \frac{1}{2} L I^2 = \cancel{\frac{1}{2}}\left(\cancel{2}\mu_0 \frac{Dd}{w}\right)(\cancel{2}Kw)^2$$

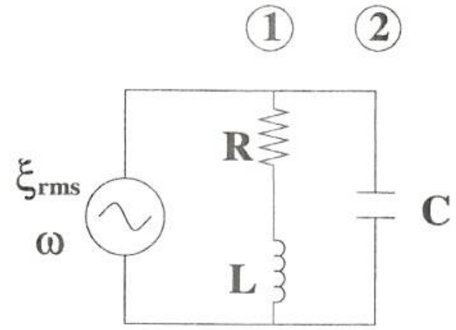
$$U = \mu_0 D d w K^2 \quad (\text{per sheet})$$

$$U = 2\mu_0 D d w K^2$$

also, $\frac{U}{V} = \frac{B^2}{2\mu_0}$ $\quad V = WDd$

$$U = WDd \cdot (2\mu_0 K)^2 \cdot \frac{1}{2\mu_0}$$

$$U = 2\mu_0 K^2 W D d$$



3) The following questions all refer to the circuit shown above. For convenience, we'll refer to the branch that contains the resistor and inductor as branch 1 and the branch that contains the capacitor as branch 2.

- 3a) (5 points) Find the RMS current through branch 1 and through branch 2.

$\mathcal{E} = IZ \qquad \mathcal{E} = I_1 Z_1 \qquad Z_1 = \sqrt{(\omega L)^2 + R^2}$

$$\boxed{I_{1\,rms} = \frac{\mathcal{E}_{rms}}{\sqrt{(\omega L)^2 + R^2}}}$$

+5

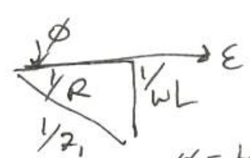
$\mathcal{E} = I_2 Z_2 \qquad Z_2 = \frac{1}{\omega C}$

$$\boxed{I_{2\,rms} = \frac{\mathcal{E}_{rms}}{1/(\omega C)}}$$

- 3b) (5 points) Will the current through branch 1 lead or lag the voltage across it? By how much?

$I_1 = \frac{\mathcal{E}}{Z_1}$ Voltage leads current

$\mathcal{E}\ L\ I$



tan $\phi = \tan^{-1}\left(\frac{1/\omega L}{1/R}\right)$

$\boxed{\text{current lags voltage by } \tan^{-1}\left(\cancel{\frac{R}{\omega L}}\right)}$ $\left(\frac{\omega L}{R}\right)$

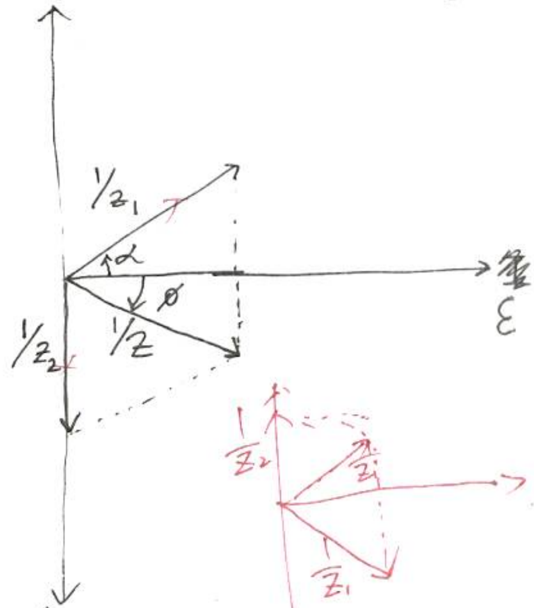
+2

- 3c) (5 points) Will the current through ~~branch 2 lead~~ or lag the voltage across it? By how much?

$\boxed{\text{current leads voltage by } 90^\circ}$

+5

- 3d) (5 points) Sketch the phasor diagram that will describe how the impedance in branch 2 combines with the impedance in branch 1 to give us the effective impedance as seen by the voltage source. Clearly label the diagram with the relevant branch impedances (or inverse-impedances) and phase angles.



Z_1 : impedance branch 1

Z_2 : impedance branch 2

Z : total impedance

α : phase angle impedance 1

ϕ : phase angle of total impedance

* (ϕ is not necessarily negative, as depicted in this drawing)

+2

- 3e) (5 points) At what frequency will the effective impedance seen by the source look purely resistive?

natural frequency $\boxed{\omega_0 = \sqrt{\frac{1}{LC}}}$

+1

- 3f) (5 points) How large is the effective impedance when the driving frequency is tuned so that it is purely resistive?

$Z_1 = R \qquad Z_2 = \frac{1}{\omega C}$

$Z_1 : 0°$

$Z_2 : 90°$

$\cancel{Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}}$

$\cancel{Z = \frac{1}{\sqrt{(1/R)^2 + (1/\omega C)^2}}}$

$$Z = \frac{1}{\sqrt{(1/Z_1)^2 + (1/Z_2)^2}} = \boxed{Z = \frac{1}{\sqrt{(1/R)^2 + (\omega C)^2}}}$$