MT1 Physics 1C F18

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Full Name (Signature) [signature]

Student ID Number 004 - 930 - 490

Seat Number ______

Problem	Grade
1	13 /30
2	15 /30
3	30 /30
Total	54 /90

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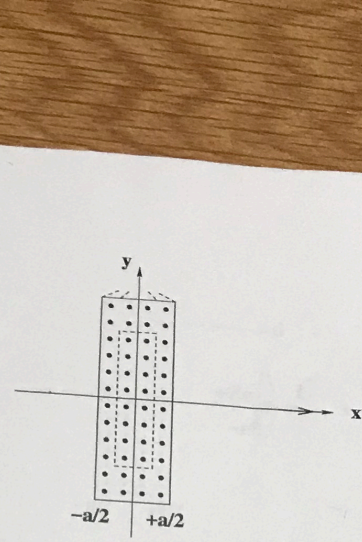
- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



A non-uniform current described by the current density

$$\vec{J} = \vec{J_0}\cos\left(\pi\frac{x}{a}\right)$$

flows out of the plane of the page through a conducting sheet that extends horizontally from $x = -\frac{a}{2}$ to $x = +\frac{a}{2}$ and vertically from $y = -\infty$ to $y = +\infty$.

- 1a) (10 pts) How much current will pass through a rectangular loop that extends horizontally from $-x$ to $+x$ and vertically from $-y$ to $+y$ if *i)* $x < \frac{a}{2}$ and *ii)* $x > \frac{a}{2}$

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i) $I = \oint \vec{J}\cdot d\vec{A} = \int_{-\infty}^{\infty}\int_{-\frac{a}{2}}^{\frac{a}{2}} J\,dx\,dy = \int_{-y}^{y}\int_{-x}^{x} J_0\cos\left(\pi\frac{x}{a}\right)dx\,dy$

$= \int_{-y}^{y} \frac{2J_0 a}{\pi}\sin\left(\pi\frac{x}{a}\right)dy = \frac{J_0 a\sin\left(\pi\frac{x}{a}\right)y}{\pi}\Big|_{-y}$

ii) if $x > \frac{a}{2}$, there is all of the current, so, $J_0 \cdot a(2y) = \boxed{2yJ_0\cos(x)}$ X

- 1b) (10 pts) Find the (vector) magnetic field for *all* points on the x-axis.

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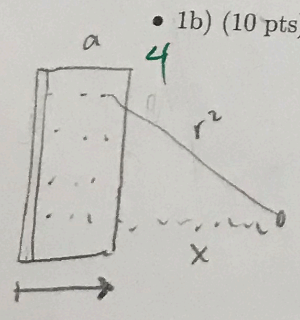


$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$

Infinite line

$\frac{\mu_0 I}{2\pi r}$

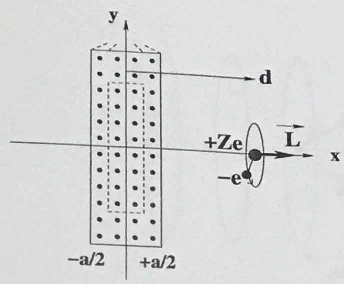
$\vec{B} = \begin{cases} \vec{0} & (x \le \frac{a}{2}) \\ \vec{0} & (x \ge -\frac{a}{2}) \\ = \frac{2\mu_0 y J_0\cos(x)}{x}\hat{j} & (x > \frac{a}{2}) \\ -\frac{2\mu_0 y J_0\cos(x)}{x}\hat{j} & (x < -\frac{a}{2}) \end{cases}$

$\vec{B} = \int_0^a \frac{\mu_0 I}{2\pi\,dr}\,dr$ X

$\int \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 2yJ_0\cos(x) \Rightarrow$

$Bx = \mu_0 2yJ_0\cos(x)$ X

$B = \mu_0$



- 1c) (10 pts) An electron (mass m, charge $-e$) orbits a nucleus (charge $+Ze$) located at the point $< d, 0, 0 >$. If the electron has an orbital angular momentum given by $\vec{L} = L\hat{x}$, find the magnitude and direction of *i*) the magnetic dipole moment associated with the electron's orbital motion and *ii*) the torque on the atom due to the magnetic field created by the current sheet.

3

i) dipole moment = $\vec{\mu} = NI\vec{A}$

=

ii) $\tau = \vec{\mu} \times \vec{B} = |\vec{\mu}||\vec{B}|\sin\theta = |\vec{\mu}||\vec{B}|$

13

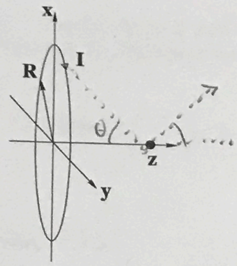


Figure 1

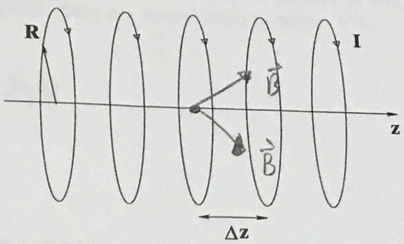


Figure 2

- 2a) (10 points) Figure 1 shows a circular conducting loop of radius R, situated in the x, y plane, centered on the origin. It carries a current I which is directed in the $+y$ direction at the top of the loop. Derive the magnetic field (magnitude and direction) for all points on the z-axis.

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{|d\vec{l}||\hat{r}|}{r^2}\sin\theta_{\hat{z}} = \frac{\mu_0 I}{4\pi}\int \frac{R\ \hat{z}}{(z^2+R^2)^{3/2}}\,dl$$

$$\boxed{\vec{B} = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}\hat{z}}$$

✓

- 2b) (5 points) In Figure 2, we find a collection of conducting loops (Radius R, oriented parallel to the x, y-plane, centered on the z-axis) that extend in steps of Δz from $-N\Delta z$ to $+N\Delta z$. Each carries a current I which is directed in the $+y$ direction at the top of the loop. Derive the magnetic field (magnitude and direction) at the center of the assembly.

$$\vec{B} = \frac{\mu_0 I}{2R} + \frac{\mu_0 I R^2}{(\Delta z^2+R^2)^{3/2}}\cos\theta$$

angle between point and loop

2

$$\int d\vec{B} = \int \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}\,dz$$

θ R

Δz

$$\cos\theta = \frac{\Delta z}{\sqrt{\Delta z^2+R^2}}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2R} + \left(\frac{\mu_0 I R^2}{\Delta z^2+R^2}\right)N\,\hat{z}}$$

- 2c) (5 points) Use $2N\Delta z = L$ and $z = (index) * \Delta z$ to re-write your answer to the previous part in a form suggestive of the Riemann sum. (Hint: there are many ways to write '1'.)

$L = 2N\Delta z, \quad z = i\Delta z$

Hence, the sum is

$$\boxed{\sum_{i=0}^{L/\Delta z} \frac{\mu_0 I R^2}{(z^2+R^2)^{3/2}} \quad \text{\# of steps} = L/\Delta z \quad \text{index from } 0 \text{ to } L/\Delta z}$$

0 ~~4~~ 1

- 2c) (10 points) Convert the Riemann sum to an integral and evaluate $\vec{B}$ at the center of this solenoid of finite length. Evaluate your answer (carefully) in the limit that $L \to \infty$ and show it yields the correct result.

$$\vec{B} = \int d\vec{B} = \int_{-\text{length}/2}^{+\text{length}/2} \frac{\mu_0 I R^2}{(z^2+R^2)^{3/2}}\,dz = 2\int_0^{\text{length}/2} \frac{\mu_0 I R^2}{(z^2+R^2)^{3/2}}\,dz$$

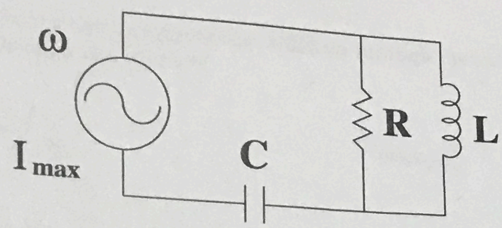
$$= \frac{\mu_0 N I}{\text{length}}\left(\frac{L}{L}\right)$$

2

As $L \to \infty$, we get

$$\vec{B} = \frac{\mu_0 N I}{\text{length}} = \mu_0 n I, \text{ which is the right result.}$$

length ↘ finite length of the solenoid.



3) A sinusoidally-varying current of amplitude I_{max} and angular frequency ω drives the RLC network shown above.

- 3a) (5 pts) Find the impedance of the RL combination.

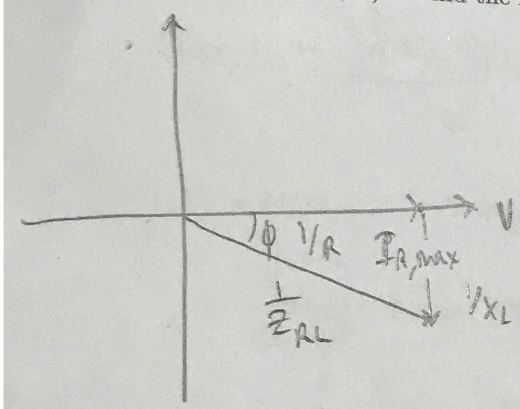
I_{max}

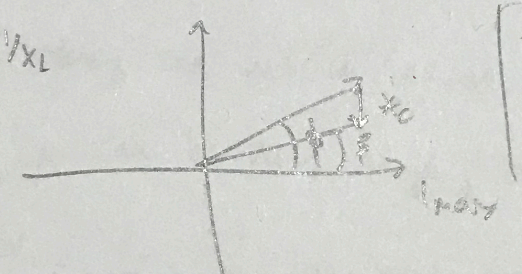
$$Z = \frac{R}{\cos\phi}$$

$$\frac{1}{Z_{RL}} = \sqrt{\left(\frac{1}{X_L}\right)^2 + \left(\frac{1}{R}\right)^2}$$

$$= \sqrt{\frac{1}{\omega^2 L^2}}$$

$$I = I_{max}\cos(\omega t)$$





$$\boxed{Z_{RL} = \frac{1}{\sqrt{\left(\frac{1}{\omega^2 L^2}\right) + \left(\frac{1}{R^2}\right)}}}$$

- 3b) (5 pts) Will the voltage across the RL combination lead or lag the current that passes through it? By how much?

The voltage on the RL will

$$\boxed{\text{lead by } \phi = \tan^{-1}\left(\frac{R}{\omega L}\right)}$$

- 3c) (10 pts) What is the maximum current that will flow through the inductor? What is the maximum current that will flow through the resistor?

$I_{max,L} = V_{max,RL} / X_L$ $\qquad V_{max,RL} = I_{max} Z_{RL}$

$I_{max,R} = V_{max,RL} / R$

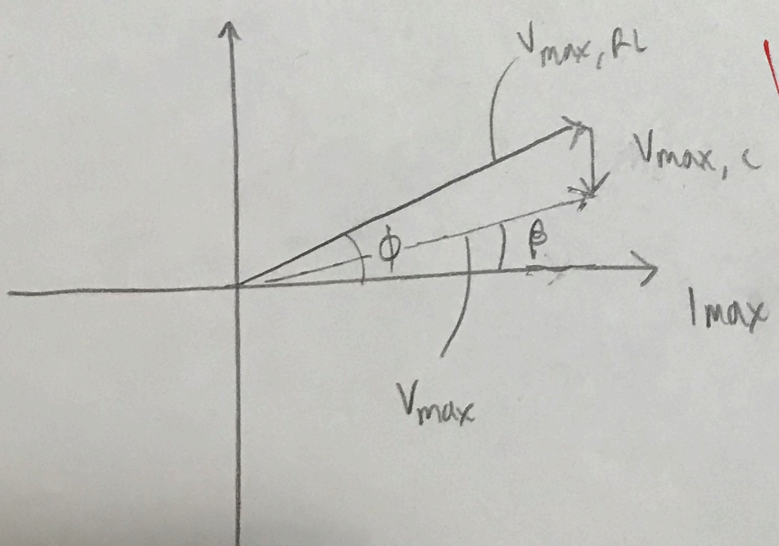
$I_{max,L} = \frac{I_{max} Z_{RL}}{X_L}$ $\boxed{\frac{I_{max}}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}}}$ inductor $\qquad I_{max,R} = \boxed{\frac{I_{max}}{\sqrt{\left(\frac{R}{\omega L}\right)^2+1}}}$ resistor

- 3d) (5 pts) Will $I_{C,max} = I_{L,max} + I_{R,max}$? Explain.

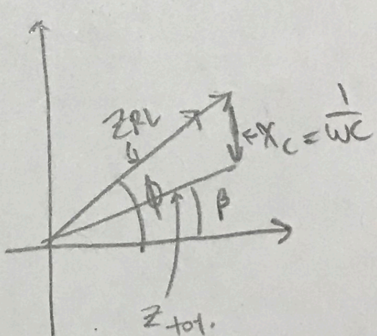
$\boxed{\text{No}}$ since they are out of phase

$V_{max,LR}$ can be very large, leading to large current draw.

- 3e) (5 pts) Draw a phasor diagram (in impedance space) to show how you would combine the capacitor with the RL combination to find the total impedance seen by the source. Label the contributions from the capacitor and the RL combination clearly, and make sure you clearly identify the relevant angles for anything that doesn't sit on an axis.



$\Rightarrow$ divide I_{max}



$\phi = \tan^{-1}\left(\frac{R}{\omega L}\right)$

$$\boxed{\beta = \tan^{-1}\left(\frac{Z_{RL}\cos\phi - \frac{1}{\omega C}}{Z_{RL}\sin\phi}\right)}$$