# MT1 Physics 1C(2), F16

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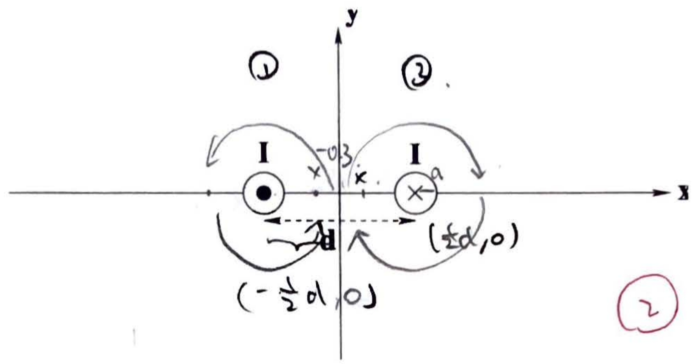
**Seat Number** ____________

| Problem | Grade |
|---|---|
| 1 | 21 /30 |
| 2 | 25 /30 |
| 3 | 23 /30 |
| Total | 69 /90 |

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

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1) A twin-lead conductor is made from two long, straight conducting wires - each of radius $a$ - that lie in the $x, z$-plane, parallel to and symmetric about the $z$-axis. The wires are separated by a center-to-center distance $d$ and carry a uniform electrical current $I$ on their surface into and out of the plane of the page, as shown.

- 1a) (15 points) Find the magnitude and direction of the magnetic field at all points along that $x$-axis that are not physically inside of a wire. Sketch the magnetic field lines produced by the wires on the diagram above. For full credit, the field lines you draw should (accurately) represent (qualitatively) the magnetic field strength at each point.

Ampere's Law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

1). at points on the left side of current ①. $(x < -\frac{1}{2}d)$
for current ①.

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi(x+\frac12 d)}(-\hat{y})$$

$$\vec{B}_2 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi(x+\frac12 d)}(\hat{y})$$

$$\therefore \vec{B}_{left} = \vec{B}_1 + \vec{B}_2 = -\frac{\mu_0 I d}{2\pi(x+\frac12 d)(x-\frac12 d)}\hat{y}$$

$$|\vec{B}_{left}| = \frac{\mu_0 I d}{2\pi(x+\frac12 d)(x-\frac12 d)}$$ direction is down $(-\hat{y})$

2) at points $(-\frac12 d < x < \frac12 d)$.

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi(x+\frac12 d)}(\hat{y})$$

$$\vec{B}_2 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi(\frac12 d - x)}(\hat{y})$$

$$\vec{B}_{mid} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{2\pi}\left(\frac{1}{\frac12 d + x} + \frac{1}{\frac12 d - x}\right)\hat{y}$$

$$|\vec{B}_{mid}| = \frac{\mu_0 I d}{2\pi(\frac12 d + x)(\frac12 d - x)}$$ direction is up $(+\hat{y})$

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3) at points $(x > \frac12 d)$

$$\vec{B}_1 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi(x-\frac12 d)} -\hat{y}$$

$$\vec{B}_2 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi(x+\frac12 d)}\hat{y}$$

$$|\vec{B}_{right}| = \frac{\mu_0 I}{2\pi(x+\frac12 d)(x-\frac12 d)}$$ direction is down $(-\hat{y})$

- 1b) (10 points) Find the $\boxed{\text{flux}}$ of the magnetic field through the rectangular region between the wires described by width $|x| < d/2 - a$ and length $D$.

$$\Phi_B = \int B\, dA$$

$$= \int_{-\frac{1}{2}d-a}^{\frac{1}{2}d-a} \frac{\mu_0 I d}{2\pi(\frac{1}{2}d+x)(\frac{1}{2}d-x)} \cdot D)\, dd.$$

$$= \frac{\mu_0 I D}{2\pi} \int_{-\frac{1}{2}d-a}^{\frac{1}{2}d-a} \left( \frac{1}{\frac{1}{2}d+x} + \frac{1}{\frac{1}{2}d-x} \right) dd$$

$$= \frac{\mu_0 I D}{2\pi} \cdot \left( \ln \tfrac{1}{2}d+x \Big|_{-\frac{1}{2}d-a}^{\frac{1}{2}d-a} + \ln \tfrac{1}{2}d-x \Big|_{-\frac{1}{2}d-a}^{\frac{1}{2}d-a} \right)$$

$$= \frac{\mu_0 I D}{2\pi} \cdot \left( \ln \frac{d-a}{-a} + \ln \frac{-a}{d+a} \right)$$

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$$= \frac{\mu_0 I D}{2\pi} \cdot \ln \frac{d-a}{d+a} \quad \times$$

- 1c) (5 points) Under what approximation(s) will the field in the regions described by $|x| > d/2 + a$ be negligible? Assuming this (these) condition(s) are valid, find the inductance per unit length of this cable.

$L/D.$

if $d+a >> d-a$

which means. $a >> 0$.

which means the radius of the current is large $\times$

$$L \equiv \left| \frac{\mathcal{E}_i}{\frac{dI}{dt}} \right|.$$

$$\mathcal{E}_i = -\frac{d\Phi_B}{dt}$$

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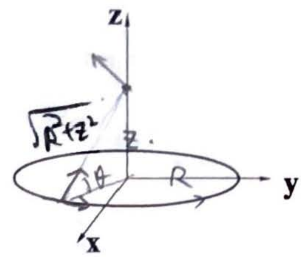
$$L = \frac{\mu_0}{2\pi} \cdot \ln \frac{d-a}{d+a} \quad \times$$




- 2a) (10 points) A circular, conducting loop of radius $R$ lies in the $x$,$y$-plane, centered on the origin. A current $I$ flows through the loop such that at $x = +R$, the current is headed in the $+\hat{y}$ direction, and at $x = -R$ the current is headed in the $-\hat{y}$ direction. Derive the resultant magnetic field (magnitude and direction) at every point along the $z$-axis.

Biot-savart.

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B}_z = \int d\vec{B}_z \cdot \cos\theta$$

$$= \int \frac{\mu_0 I}{4\pi} \frac{|d\vec{l}| \cdot |\hat{r}|}{r^2} \cdot \frac{R}{\sqrt{R^2+z^2}}$$

$$= \frac{\mu_0 I}{4\pi\sqrt{R^2+z^2}} \cdot \frac{R}{(R^2+z^2)} \int d\vec{l}$$

$$= \frac{\mu_0 I R}{4\pi (R^2+z^2)^{\frac{3}{2}}} \cdot 2\pi R$$

$$= \frac{\mu_0 I R^2}{2(R^2+z^2)^{\frac{3}{2}}} \hat{z}$$

✓

- 2b) (10 points) Let's replace the loop with an infinitesimally-thin, uniform ring of electric charge that extends from $r$ to $r + dr$. If the surface charge-density on the ring is given by $\boxed{\sigma}$ and the ring rotates about the $z$-axis with a constant angular velocity $\boxed{\vec{\omega}}$ (recall, the direction of $\hat{\omega}$ is obtained by using the right-hand rule with the physical motion of points on the ring) find the magnitude and direction of the (infinitesimal) magnetic field produced at every point along the $z$-axis.

$$dI = \frac{dq}{T} = \frac{\sigma dA}{T} = \sigma \cdot \frac{\omega}{2\pi} \cdot 2\pi r dr = \sigma \omega r dr.$$

✓

$$\vec{B} = \int d\vec{B}$$

$$= \int \frac{\mu_0 I R^2}{2(R^2+z^2)^{\frac{3}{2}}}$$

$$= \int_r^{r+dr} \frac{\mu_0 \sigma \omega r \, dr \, r^2}{2(r^2+z^2)^{\frac{3}{2}}}$$

$$= \frac{\mu_0 \sigma \omega}{2} \int_r^{r+dr} \frac{r^3 dr}{(r^2+z^2)^{\frac{3}{2}}}$$

✓

- 2c) (10 points) Now lets replace the thin ring of part $b$ with a washer that extends from $r = a$ to $r = b$. Charge is distributed over the washer with a surface charge density

$$\sigma(r) = \frac{2qr^2}{\pi(b^4 - a^4)}$$

and it rotates with a constant angular velocity $\vec{\omega}$. Find the magnitude and direction of the magnetic field produced at every point on the $z$-axis. (The answer is not pretty, but it is pretty doable if you're patient).

$$dI = \frac{dq}{T} = \frac{\sigma dA}{T} = \frac{2qr^2}{\pi(b^4-a^4)} \cdot \frac{\omega}{2\pi} \cdot 2\pi r\, dr.$$

$$= \frac{2qr^3\omega}{\pi(b^4-a^4)} dr$$

$$\int d\vec{B} = \int \frac{\mu_0 I r^2}{2(r^2+z^2)^{\frac{3}{2}}}$$

$$= \int_a^b \frac{\mu_0 r^2}{\cancel{2}(r^2+z^2)^{\frac{3}{2}}} \cdot \frac{\cancel{2}qr^3\omega}{\pi(b^4-a^4)} dr$$

$$= \frac{\mu_0 q\omega}{\pi(b^4-a^4)} \int_a^b \frac{r^5}{(r^2+z^2)^{\frac{3}{2}}} dr.$$

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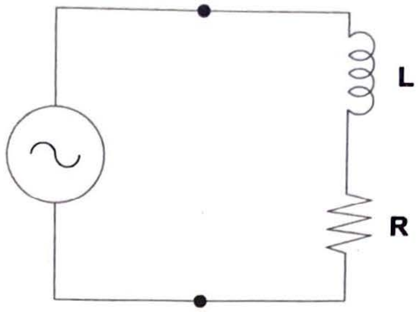


3) An inductor ($L$) and a resistor ($R$) are connected in series across a source of alternating EMF ($\xi(t) = \xi_{max}\cos(\omega t)$). You may do the following calculations in complex space, but keep in mind, the final answers must all be real.

ELI
ICE

- 3a) (10 points) What is the impedance of $LR$ combination? What is the amplitude of the current that passes through it? Does the current through this impedance lead or lag the voltage across it? By how much?

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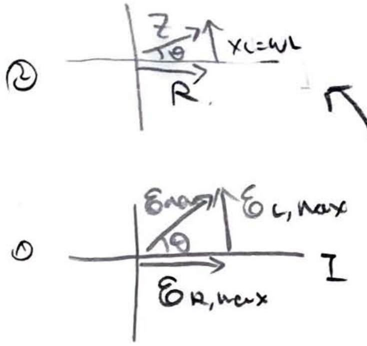



$$Z = \sqrt{R^2 + (\omega L)^2}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$I_{max} = \frac{\mathcal{E}_{max}}{Z} = \frac{\mathcal{E}_{max}}{\sqrt{R^2+(\omega L)^2}}$$

$$I(t) = \frac{\mathcal{E}_{max}}{\sqrt{R^2+(\omega L)^2}} \cdot \cos(\omega t)$$

according to the graph, it leads the voltage across it. ×

$$\cos\theta = \frac{R}{\sqrt{R^2+(\omega L)^2}}$$

- 3b) (10 points) What is the voltage amplitude across the resistor? What is the voltage amplitude across the inductor? Will the sum of these voltage amplitudes equal the voltage amplitude of the source? Explain.

10

$$V_{R,max} = \frac{R}{Z}\mathcal{E}_{max} = \frac{R}{\sqrt{R^2+(\omega L)^2}}\mathcal{E}_{max}$$

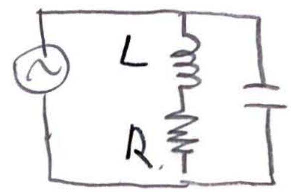
$$V_{L,max} = \frac{\omega L}{Z}\mathcal{E}_{max} = \frac{\omega L}{\sqrt{R^2+(\omega L)^2}}\mathcal{E}_{max}$$

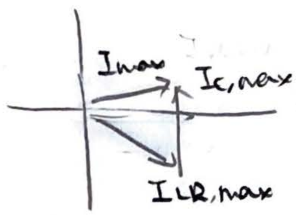
$$V_{R,max} + V_{L,max} = \frac{R+\omega L}{\sqrt{R^2+(\omega L)^2}}\mathcal{E}_{max} \neq \mathcal{E}_{max}$$

So it doesn't.
because for inductors, the voltage leads the current, so they are not in phase.

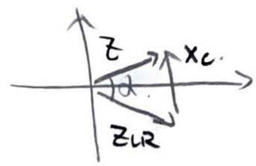
- 3c) (10 points) For a lot of good reasons, it is often desirable to present the source with a purely resistive load. We can tune-out the reactance in the $LR$ network by adding a capacitor in parallel with it (across the two dots in the circuit). What value should the capacitor have? What will be the value of the impedance seen by the source? Will current through this new $LRC$ combination lead or lag the voltage across it? By how much?

4




→ the current will lead the voltage.




$$X_C^2 = Z^2 + Z_{LR}^2 - 2Z\cdot Z_{LR}\sin\alpha.$$

$$\left(\frac{1}{\omega C}\right)^2 = Z^2 + R^2+(\omega L)^2 - 2Z\sqrt{R^2+(\omega L)^2}\cdot\sin\alpha.$$

$$\sin\alpha = \frac{\left(\frac{1}{\omega C} - Z^2 - R^2+(\omega L)^2\right)}{2Z\sqrt{R^2+(\omega L)^2}}$$