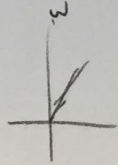
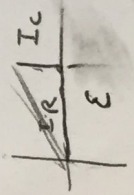
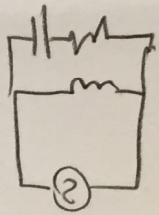


- 3c) (10 points) For a lot of good reasons, it is often desirable to present the source with a purely resistive load. We can tune-out the reactance in the  $RC$  network by adding an inductor in parallel with it (across the two dots in the circuit). What value should the inductor have? What will be the value of the new impedance seen by the source? Will current through this new  $LRC$  combination lead or lag the voltage across it? By how much?

$\Sigma I = I < \epsilon$



3

Reactance part  $(\omega L)^2 - (\frac{1}{\omega C})^2$

$\omega L = \frac{1}{\omega C}$

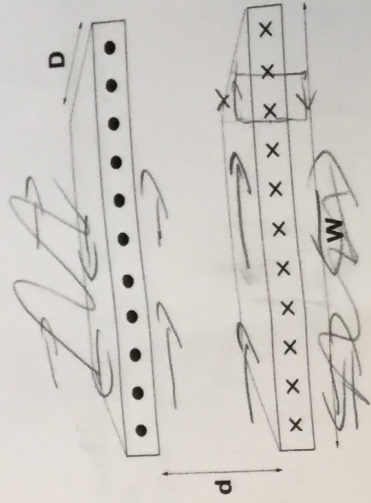
$L = \frac{1}{\omega^2 C}$

this fits in series always

$Z = R$

Current in phase w  $\epsilon$

2



- 1) Two thin, parallel, conducting sheets of dimension  $D \times W$  are separated by a distance  $d$  as shown ( $D \gg W \gg d$ ). The sheets each carry a linear current density  $K$ , one into the plane of the page, one out, as shown.

- 14 • 1a) (15 points)

Use Faraday's law to calculate the self-inductance of the arrangement.

$$L = \left| \frac{\Phi_i}{dI/dt} \right|$$

$$\mathcal{E}_i = - \frac{d\Phi_{\text{ext}}}{dt}$$

$$\mathcal{E}_i = - \frac{d(BA \cos \theta)}{dt} \quad \theta = 90^\circ?$$

$$2 \times B = \mu_0 K \times$$

$$B = \frac{\mu_0 K}{2}$$

for plate 2

$$B = \mu_0 K \times 2$$

in between

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}}$$

Area  $\perp$  to field

$$\frac{dK}{dt} = \frac{dI}{dt}$$

if const. area

$$\mathcal{E}_i = dD \mu_0 \frac{dK}{dt}$$

$$L = \frac{dD \mu_0 \frac{dI}{dt}}{\frac{dI}{dt}}$$

$$L = dD \mu_0 \quad + 4$$

- 1b) (10 pts)

Verify your answer to the first part using energy considerations.

$$U = \frac{1}{2} L I^2 + 1$$

$$P = I \mathcal{E}$$

$$\frac{dI}{dt} = \frac{\mathcal{E}_i}{L}$$

$$I = K W$$

$$\int P dt = \int I L \frac{dI}{dt} dt$$

$$U = \frac{1}{2} I^2 dD \mu_0$$

$$U = \frac{1}{2} I^2 L$$

$$C = \frac{q}{\mathcal{E}_i}$$

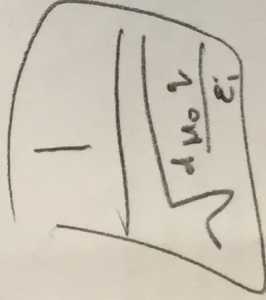
$$q = V W$$

$$L = dD \mu_0$$

- 1c) (5 pts) Define  $\hat{L}$  as the inductance per unit length (measured along the current) and  $\hat{C}$  as the capacitance per unit length. Calculate  $\frac{1}{\sqrt{\hat{L}\hat{C}}}$ . This quantity plays an important role in the practical evaluation of transmission lines. Care to guess what it is?

$$\hat{L} = \mu_0$$

$$\hat{C} = \frac{\epsilon_1}{d}$$



Resistance?

7/10

- 2a) (10 points) A circular, conducting loop of radius  $R$  lies in the  $x, y$ -plane, centered on the origin. A current  $I$  flows through the loop such that at  $x = +R$ , the current is headed in the  $+y$  direction, and at  $x = -R$  the current is headed in the  $-y$  direction. Derive the resultant magnetic field (magnitude and direction) at every point along the  $z$ -axis.

along  $z$ -axis, magnetic field always oriented on positive  $z$ -axis ( $R$  for symmetry)

$$B_z = B \cos \theta \rightarrow B \cos 90^\circ - \theta \cdot \text{wrong angle}$$

$$r \times dB = \frac{\mu_0 I}{4\pi} \frac{dL \times \hat{r}}{r^2}$$

$$\cos \theta = \frac{z}{\sqrt{R^2 + z^2}}$$

$$B = \int \frac{\mu_0 I}{4\pi} \frac{dL}{r^2} \cdot \frac{1}{r^2} \sin \theta \cos \theta$$

different  $\theta$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{2\pi r z}{\sqrt{R^2 + z^2} (R^2 + z^2)}$$

$$\int dL = 2\pi R$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{1}{r^2} \cos \theta dL$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{(z^2}{(R^2 + z^2)^{3/2}} dL$$

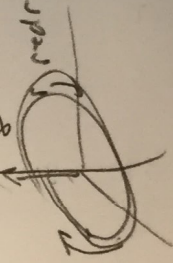
$$B = \frac{\mu_0 I z^2}{2(R^2 + z^2)^{3/2}} \hat{z}$$

- 2b) (10 points) Let's replace the loop with an infinitesimally-thin, uniform ring-of-charge that extends from  $r$  to  $r + dr$ . If the surface charge-density on the ring is given by  $\sigma$  and the ring rotates about the  $z$ -axis with a constant angular velocity  $\omega$  (recall, the direction of  $\omega$  is obtained by using the right-hand rule with the physical motion of points on the ring) find the magnitude and direction of the (infinitesimal) magnetic field produced at every point along the  $z$ -axis.

5/10

cancel  $x, y$  components  
only  $z$

$$I = \frac{dq}{dt} = \sigma dr \omega$$



$$B = \int \frac{\mu_0}{4\pi} \frac{I dL \times \hat{r}}{r^2}$$

$$B = \int \frac{\mu_0}{4\pi} \frac{\sigma \omega dr}{r^2} dL \times \hat{r}$$

everything else same

$$B = \frac{\mu_0 \sigma \omega dr z^2}{2(R^2 + z^2)^{3/2}} \hat{z}$$

$$\int dA = \int \frac{dq}{T} = \frac{\sigma dA = \sigma 2\pi r dr}{T}$$

depends on direction of rotation

but leave as  $dB = \dots dr$

integrate in part (c).

4/10

- 2c) (10 points) Now lets replace the thin ring of part b with a washer that extends from  $r = a$  to  $r = b$ . Charge is distributed over the washer with a surface charge density

$$\sigma(r) = \frac{qab}{2\pi(b-a)} \frac{1}{r^3}$$

and it rotates with a constant angular velocity  $\omega$ . Find the magnitude and direction of the magnetic field produced at every point on the z-axis.

+

+

$$\int_a^b \frac{\mu_0}{4\pi} \sigma (b-a) \omega dr \frac{dl \times \vec{r}}{r^2}$$

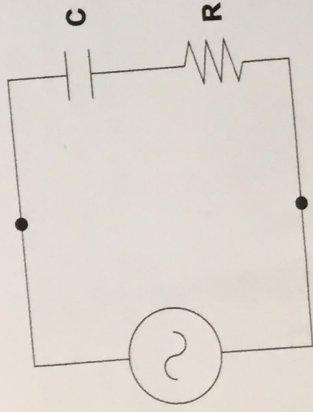
$$I = \frac{dq}{dt} = \sigma (b-a) \omega dr$$

$$\int_a^b \frac{\mu_0}{4\pi} \sigma (b-a) \omega dr \frac{dl \times \vec{r}}{r^2}$$

$$B = \int_a^b \frac{\mu_0}{4\pi} \frac{qab\omega}{8\pi^2 r^3} dr \frac{2\pi r \cos\theta}{r^2}$$

$$B = \frac{\mu_0 q ab \omega}{4\pi r^4} dr \cos\theta \hat{\omega}$$

+

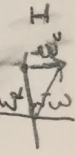


3) An capacitor (C) and a resistor (R) are connected in series across a source of alternating EMF ( $\xi(t) = \xi_{max} \cos(\omega t)$ ). You may do the following calculations in complex space, but keep in mind, the final answers must all be real.

- 3a) (10 points) What is the impedance of RC combination? What is the amplitude of the current that passes through it? Does the current through this impedance lead or lag the voltage across it? By how much?

$$\xi = \xi_{max} \cos(\omega t)$$

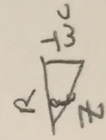
$$I = I_{max} \cos(\omega t + \phi)$$



$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\tan \phi = \frac{-1}{\omega C R}$$

$$\phi = \tan^{-1} \left( \frac{-1}{\omega C R} \right)$$



$$\xi = IR$$

$$I_{max} = \frac{\xi_{max}}{Z}$$

$$I_{max} = \frac{\xi_{max}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

~~$$I = I_{max} \left( \cos(\omega t) + \tan^{-1} \left( \frac{-1}{\omega C R} \right) \right)$$~~

Current leads voltage ( $\xi$  is negative)  
by  $\tan^{-1} \left( \frac{-1}{\omega C R} \right)$

- 3b) (10 points) What is the voltage amplitude across the resistor? Will the sum of these voltage amplitudes equal the voltage amplitude of the source? Explain.

$$V_R = IR$$

$$V_{Rmax} = \frac{\xi_{max} R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

$$V_C = I X_C$$

$$V_{Cmax} = \frac{\xi_{max}}{\omega C \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

No.

The voltages reach max amplitude at different times (In phase w/  $\xi$  for  $V_R$  but not for  $V_C$ ) so they won't add up to equal  $\xi_{max}$