20S-PHYSICS1C-1 1CS20 Final Exam

FRANK ZHENG

TOTAL POINTS

124 / 150

QUESTION 1

1 30 pts

1.1 1a **10 / 10**

- ✓ + **3 pts** **Correct Integral setup 1**
- ✓ + **2 pts** **Current r < a**
- ✓ + **3 pts** **Correct integral setup 2**
- ✓ + **2 pts** **Current r > a**
- \+ **0 pts** Incorrect

1.2 1b **5 / 5**

- ✓ + **1 pts** **Correct direction**
- ✓ + **2 pts** **Correct magnetic field inside**
- ✓ + **2 pts** **Correct magnetic field outside**
- \- **1 pts** Mistake carried over
- \+ **0 pts** Incorrect

1.3 1c **4 / 5**

- ✓ + **2 pts** **J**
- ✓ + **1 pts** **Net current**
- ✓ + **1 pts** **Magnetic field**
- \+ **1 pts** Discussion
- \- **1 pts** Mistake carried over
- \+ **0 pts** Incorrect

1.4 1d **5 / 10**

- ✓ + **2.5 pts** **Correct Density**
- \+ **5 pts** Correct integral setup for energy/length
- ✓ + **2.5 pts** **Final answer**
- \+ **0 pts** Incorrect

QUESTION 2

30 pts

2.1 2a **10 / 10**

- \+ **10 pts** Correct
- ✓ + **5 pts** **kirchoffs setup**
- \+ **3 pts** kirchoff partly right Io used initially
- ✓ + **3 pts** **integral setup with the right limits**
- \+ **1.5 pts** integral setup partly right/limits wrong or missing
- ✓ + **2 pts** **right solution**
- \+ **1 pts** some attempt
- \+ **0 pts** no attempt

2.2 2b **5 / 5**

- ✓ + **5 pts** **Correct**
- \+ **3 pts** partly right/why should I2 be zero at t=0
- \+ **1 pts** some attempt
- \+ **0 pts** no attempt

2.3 2c **10 / 10**

- \+ **10 pts** Correct
- ✓ + **4 pts** **voltage**
- ✓ + **3 pts** **current**
- ✓ + **3 pts** **direction**
- \- **2 pts** error carried over/partial credit
- \+ **1 pts** some attempt for the current and voltage
- \+ **0 pts** wrong

2.4 2d (problem) **1 / 3**

- \+ **3 pts** magnitude correct
- \+ **2 pts** identifying I2=0,dI2/dt = e/I2/typo/no explanation
- ✓ + **1 pts** **I2=0/write form of the equation**
- \+ **0.25 pts** some attempt
- \+ **0 pts** no attempt

2.5 2d (polarity) **0 / 2**

- \+ **2 pts** Correct
- ✓ + **0 pts** **no attempt/wrong**

QUESTION 3

3 30 pts

3.1 3a 8 / 10

✓ + **2 pts** **Correct X_C**

✓ + **2 pts** **Correct X_L**

✓ + **4 pts** **Z_LC expression**

+ **2 pts** Modulus

+ **0 pts** Incorrect

3.2 3b 5 / 5

✓ + **3 pts** **Correct Condition on reactance**

✓ + **2 pts** **Correct condition on omega**

- **1 pts** Mistake carried over

+ **0 pts** Incorrect

3.3 3c 10 / 10

✓ + **5 pts** **Correct Impedance**

✓ + **5 pts** **I_max**

- **1 pts** Mistake carried over

+ **0 pts** Incorrect

+ **2 pts** Partial steps in the correct direction

3.4 3d 0 / 5

+ **3 pts** LC voltage lags LC current

+ **2 pts** omega condition

✓ + **0 pts** **Incorrect**

- **1 pts** Mistake carried over

QUESTION 4

4 30 pts

4.1 4a 8 / 10

+ **10 pts** Correct

✓ + **5 pts** **path**

+ **2.5 pts** path is partly right

+ **3 pts** reflected difference

✓ + **1 pts** **reflection mentioned just once (pi instead of 2pi)/zero mentioned but not why or it was wrong**

✓ + **2 pts** **ic**

+ **1 pts** some attempt

+ **0 pts** no attempt

4.2 4b 5 / 5

✓ + **5 pts** **Correct**

+ **1.5 pts** some attempt/mention of only in phase not the condition

+ **0 pts** no attempt

4.3 4c 4 / 10

+ **6 pts** right wavelength relation

+ **4 pts** right solution N not replaced/calculation error

+ **2 pts** n-1 and n

✓ + **4 pts** **correct substitution from part b**

- **1 pts** error carried over

+ **2 pts** some attempt in the right direction

+ **0 pts** incorrect

4.4 4d 5 / 5

+ **0 pts** Correct

✓ + **2.5 pts** **large delta**

✓ + **2.5 pts** **proportional q**

+ **1 pts** some attempt/no explanation

+ **0 pts** no attempt

QUESTION 5

30 pts

5.1 5a 5 / 5

✓ + **2.5 pts** **Length contraction**

✓ + **2.5 pts** **Time dilation**

+ **1 pts** Partial steps

+ **0 pts** Incorrect

5.2 5b 10 / 10

✓ + **3 pts** **Realize NOT length contraction and time dilation**

✓ + **1 pts** **Correct rocket frame vector**

✓ + **1 pts** **Correct earth frame vector**

✓ + **3 pts** **Correct transformation matrix**

✓ + **2 pts** **Final answers**

+ **0 pts** Incorrect

+ **1 pts** Partial steps in the correct direction

5.3 5c **10 / 10**

- ✓ **+ 1 pts** Correct parallel velocity wrt Earth
- ✓ **+ 1 pts** Correct perpendicular velocity wrt Earth
- ✓ **+ 1 pts** Parallel formula to change frame
- ✓ **+ 1 pts** Perpendicular formula to change frame
- ✓ **+ 2.5 pts** Correct parallel velocity wrt A
- ✓ **+ 2.5 pts** Correct perpendicular velocity wrt A
- ✓ **+ 1 pts** Final answer
- **+ 0 pts** Incorrect

5.4 5d **4 / 5**

- ✓ **+ 1 pts** part c beta = 0 wrt earth
- ✓ **+ 1 pts** part c beta = 0 wrt A
- ✓ **+ 1 pts** part c beta = 1 wrt earth
- ✓ **+ 1 pts** part c beta = 1 wrt A
- **- 1 pts** Mistake carried over
- **+ 0 pts** Incorrect
- **+ 1 pts** Beta = 0 and beta = 1 sig

1CS20 Final Exam

Full Name (Printed) Frank Zheng

Full Name (Signature) Frank Zheng

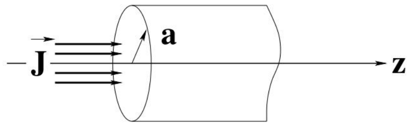
Student ID Number 805-311-093

- The exam is open-book and open notes. You will probably do better to limit yourself to a single page of notes you prepared well in advance.
- **All work must be your own.** You are not allowed to collaborate with anyone else, you are not allowed to discuss the exam with anyone until all the exams have been submitted (after the close of the submissions window for the exam).
- You have **120 minutes** to complete the exam and more than sufficient time to scan the exam and upload it to GradeScope. The exam *must* be uploaded to GradeScope within the time alloted (that is, by the end of the 3-hour finals slot). We will only except submissions through GradeScope and will not accept any exam submitted after the submission window closes (CAE students must contact Corbin for instructions).
- **Given the limits of GradeScope, you must fit your work for each part into the space provided.** You may work on scratch paper, but you will not be able to upload the work you do on scratch paper, so it is essential that you copy your complete solution onto the exam form for final submission. We can only consider the work you submit on your exam form.
- **For full credit the grader must be able to follow your solution from first principles to your final answer.** ***There is a valid penalty for confusing the grader.***
- It is **YOUR** responsibility to make sure the exam is scanned correctly and uploaded before the end of the submission window. The graders may refuse to grade pages that are significantly blurred, solutions to problems that are not written in the correct place, pages submitted in landscape mode and/or work that is otherwise illegible - if any of this occurs, you may not receive *any* credit for the affected parts.
- Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

The following must be signed before you submit your exam:

By my signature below, I hereby certify that all of the work on this exam was my own, that I did not collaborate with anyone else, nor did I discuss the exam with anyone while I was taking it.

Signature Frank Zheng

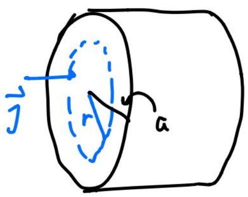


1) An electrical current described by the current density

$$\vec{J} = J_0 (1 - 2\frac{r^2}{a^2})\,\hat{z}$$

flows in a cylindrical region of radius a aligned with and centered on the z-axis as shown.

- 1a) (10 points) How much current is enclosed by a circular loop of radius r centered on the symmetry-axis of the cylinder, oriented perpendicular to the current-density? Consider both $r \le a$ and $r > a$.



$$\vec{J} = J_0(1 - 2\frac{r^2}{a^2})\hat{z}$$

$r \leq a$:

$$I(r) = \int \vec{J}\cdot d\vec{A}$$
$$= \int_0^r J(r)\cdot 2\pi s\, ds$$
$$= \int_0^r J_0(1 - 2\frac{s^2}{a^2})\, 2\pi s\, ds$$
$$= 2\pi J_0\left(\int_0^r s\,ds - \frac{2}{a^2}\int_0^r s^3\,ds\right)$$
$$= 2\pi J_0\left(\frac{1}{2}r^2 - \frac{2}{a^2}\left(\frac{1}{4}r^4\right)\right)$$
$$= \pi J_0 r^2 (1 - r^2/a^2)$$

$r > a$:

$$\pi J_0 a^2 (1 - 1) = \boxed{0}$$

- 1b) (5 points) Find the (vector) magnetic field at a radial distance r from the symmetry-axis of the cylinder for points inside and outside the cylinder.

$r \leq a$:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I(r)$$
$$B\cdot 2\cancel{\pi}\cancel{r} = \mu_0 \cancel{\pi} J_0 r^{\cancel{2}}(1 - \frac{r^2}{a^2})$$
$$B = \frac{1}{2}\mu_0 J_0 r (1 - \frac{r^2}{a^2})$$
$$\boxed{\vec{B} = \frac{1}{2}\mu_0 J_0 r (1 - \frac{r^2}{a^2})\,\hat{\phi}}$$

RH rule

$r > a$

$$\oint \vec{B}\cdot d\vec{s} = 0$$
$$\boxed{\vec{B} = \vec{0}}$$

- 1c) (5 points) At what value of r does $\vec{J}$ change direction? At what value r does the net current through a loop of radius r change direction? At what value of r does $\vec{B}$ change direction? Discuss.

$$|J| = J_0\left(1 - \frac{2r^2}{a^2}\right)$$

$$|J| = 0 \rightarrow \quad a = \sqrt{2}\,r$$

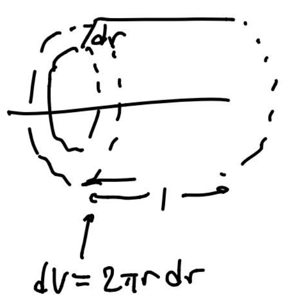
$$\text{or } r = \frac{1}{\sqrt{2}}a$$

$\vec{J}$ changes direction at $r = a/\sqrt{2}$

The net current doesn't actually change direction (if my calculations are correct). $I(a)$ is 0 but $I(r)$ is never negative (if the radius of the cylinder were greater than a, then I would change directions at a). Likewise, $\vec{B}$ does not change directions either.

- 1d) (10 points) Find the energy per unit-length associated with the current.

$$u_B = \frac{1}{2\mu_0}B^2$$



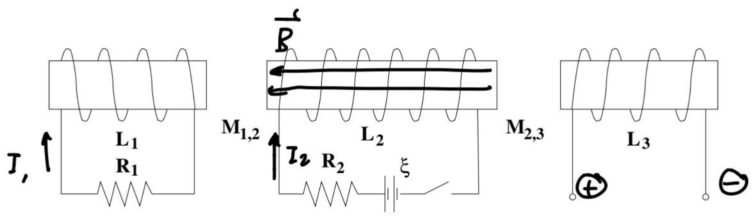
$$U = \int u_B \, dV$$

$$= \int_0^a \frac{1}{2\mu_0}\left(\frac{1}{2}\mu_0 J_0 r\left(1 - \frac{r^2}{a^2}\right)\right)^2 dr$$

$$= \frac{1}{8}\mu_0^2 J_0^2 \int_0^a \left(r^2 - \frac{2r^4}{a^2} + \frac{r^6}{a^4}\right) dr$$

$$= \frac{1}{8}\mu_0^2 J_0^2 \left(\frac{1}{3}a^3 - \frac{2}{5}a^3 + \frac{1}{7}a^3\right)$$

$$= \boxed{\frac{1}{8}\mu_0^2 J_0^2 a^3 \left(\frac{1}{3} - \frac{2}{5} + \frac{1}{7}\right)}$$



2) All the values shown in the diagram above are known. The switch has been open a long time. It will be closed at $t = 0$.

- 2a) (10 points) Use one of Kirchoff's laws to find the current through and the electromotive-force across L_2 as a function of time elapsed after the switch was closed. Don't make any assumptions about the initial current in the circuit just yet, call it I_0 for now.

EMF $\mathcal{E}_1$, R_2, L_2 in series

$$\mathcal{E} = R_2 I + L_2 \frac{dI}{dt}$$

$$\mathcal{E} - R_2 I = L_2 \frac{dI}{dt}$$

$$\int dt = \int \frac{L_2}{\mathcal{E} - R_2 I} dI$$

$$t = \frac{-L_2}{R_2} \ln\left(\frac{\mathcal{E} - R_2 I}{\mathcal{E} - R_2 I_0}\right)$$

$$e^{-\frac{R_2}{L_2}t} = \frac{\mathcal{E} - R_2 I}{\mathcal{E} - R_2 I_0}$$

$$\mathcal{E} - R_2 I = (\mathcal{E} - R_2 I_0) e^{-\frac{R_2}{L_2}t}$$

$$\boxed{I = \frac{1}{R_2}\left[\mathcal{E} - (\mathcal{E} - R_2 I_0) e^{-\frac{R_2}{L_2}t}\right]}$$

$$\mathcal{E}_i = L_2 \frac{dI}{dt}$$

$$= -\frac{L_2}{R_2} (\mathcal{E} - R_2 I_0) e^{-\frac{R_2}{L_2}t} \cdot \frac{-R_2}{L_2}$$

$$\boxed{\mathcal{E}_i = (\mathcal{E} - R_2 I_0) e^{-\frac{R_2}{L_2}t}}$$

- 2b) (5 points) Take a close look at the Kirchoff equation you wrote for the circuit and explain why I_0 has to be zero. It may be a good idea, at this point, to re-write your answers to part a with this new information taken into account.

If I were to jump from $0 \to I_0 \neq 0$ at $t=0$, $\mathcal{E}_i(t=0)$ would be really high, exceeding $\mathcal{E}$ (battery).

$$I(t) = \frac{\mathcal{E}}{R_2}\left(1 - e^{-\frac{R_2}{L_2}t}\right)$$

$$\mathcal{E}_i = \mathcal{E} e^{-\frac{R_2}{L_2}t}, \quad \frac{dI}{dt} = \frac{\mathcal{E}}{L_2} e^{-\frac{R_2}{L_2}t}$$

* I in above problems are referred to as I_2 below

- 2c) (10 points) Find the potential difference across and the current through R_1 as a function of time elapsed after the switch was closed. In what direction does the current flow through R_1? Explain.

$$\mathcal{E}_i^{(1)} = M_{1,2}\, dI_2/dt = \frac{\mathcal{E} M_{1,2}}{L_2} e^{-\frac{R_2}{L_2}t}$$

$$\Delta V_{R_1} = \boxed{\frac{\mathcal{E} M_{1,2}}{L_2} e^{-\frac{R_2}{L_2}t}}$$

$$I_1 = \frac{\Delta V_{R_1}}{R_1} = \boxed{\frac{\mathcal{E} M_{1,2}}{R_1 L_2} e^{-\frac{R_2}{L_2}t}}$$

The current flows CW

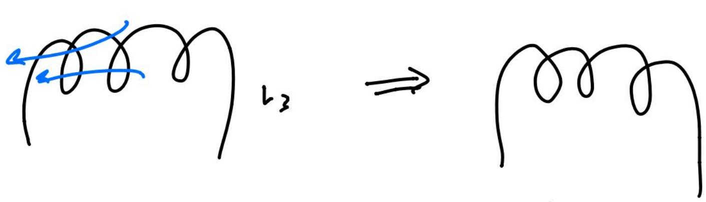
This is b/c:

- * current in circuit 2 (middle) flows CW due to the battery.
- * $\vec{B}$ in solenoid L_2 points left from RH rule.
- * Faraday's + Lenz's Law produces an opposing EMF (RH rule points right) in circuit 1 (left circuit) that points to the right (CW).

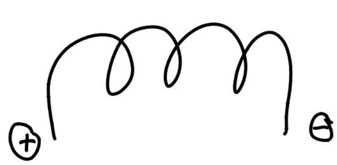
- 2d) (5 points) After a long time, the switch is once again opened. At that instant, estimate the magnitude of the electromotive force across L_3 and mark the polarity of that electromotive force on the open terminals in the diagram.

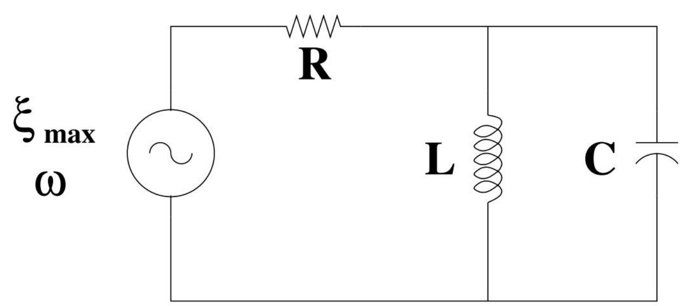
Current in circuit 2 instantly drops from $\frac{\mathcal{E}}{R_2} \rightarrow 0$.

Magnetic flux



EMF from R→L opposes this $d\Phi_B/dt$:





3) The quantities shown in the diagram above are all known. When you answer the following questions, make sure your final answers are written in terms of ω (in other words, don't leave any X's lying around).

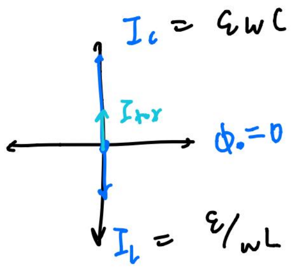
- 3a) (10 points) Find the impedance presented to the circuit by the L-C portion of the circuit.

$$\Delta V_L = \Delta V_C = \mathcal{E}'_{max}\cos(\omega t + \phi_0) \qquad I = \mathcal{E}/\tilde{Z}$$

ELI, ICE

$$I_L = \frac{\mathcal{E}'_{max}}{\omega L}\cos(\omega t + \phi_0 - \pi/2)$$

$$I_C = \mathcal{E}'_{max}\,\omega C\cos(\omega t + \phi_0 + \pi/2)$$



$$I_{tot} = \mathcal{E}/\tilde{Z}$$

$$\rightarrow \frac{1}{Z} = \omega C - 1/\omega L$$

$$= \boxed{\frac{1}{\omega C - 1/\omega L}}$$

- 3b) (5 points) Under what conditions will the voltage across the L-C portion lead the current through the L-C portion of the circuit?

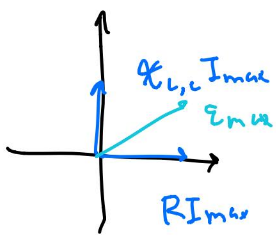
If $1/X_C = \omega C > 1/X_L = 1/\omega L$, current leads voltage.

If $\boxed{\omega C < 1/\omega L}$ voltage leads current.

- 3c) (10 points) How large is the amplitude of the current drawn out of the source?

$$\Delta V_{L,C} = \Delta V_{L,C\,max} \cos(\omega t \pm \pi/2)$$

$$\Delta V_R = \Delta V_R \cos(\omega t)$$



$$Z = \sqrt{R^2 + X_{L,C}^2}$$

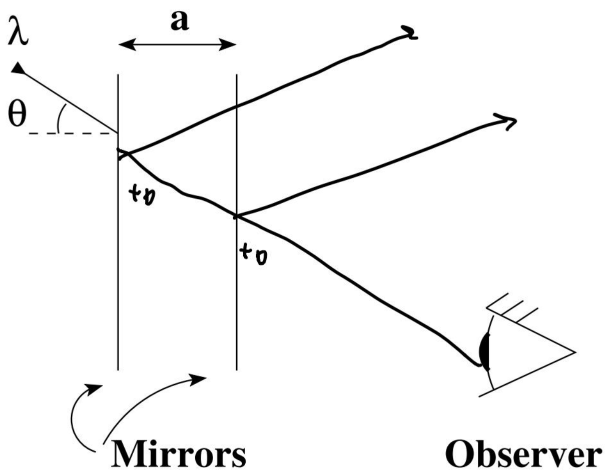
$$= \sqrt{R^2 + 1/(\omega C - 1/\omega L)^2}$$

$$\frac{\mathcal{E}_{max}}{Z} = I_{max}$$

$$= \frac{\mathcal{E}}{\sqrt{R^2 + 1/(\omega C - 1/\omega L)^2}}$$

- 3d) (5 points) Under what conditions will the current drawn from the source lead the voltage delivered by the source? Explain.

The answer is the same as in 3b. This is apparent in 3c's phasor diagram; I drew it so $\mathcal{E}$ leads I but could have made the $\Delta V_{L,C}$ phasor point down depending on the sign of $X_{L,C}$ (ans. to 3a).

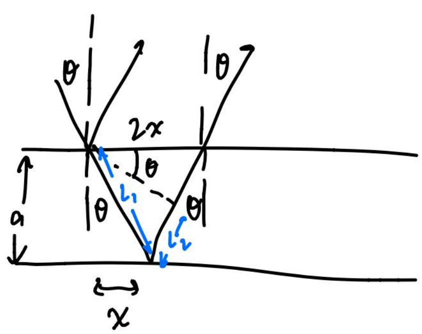


4) A Fabry-Perot interferometer consists of two parallel, half-silvered mirrors that are separated by a small distance a. Each of the mirrors transmits 50% of the incident light intensity and reflects the other 50% of the incident intensity. For the following, assume the angle of incidence is given by θ.

- 4a) (10 points) Derive the contributions to total phase difference at the location of the observer made by path difference, initial conditions and reflection. Be careful - the angles are not necessarily small.

$$\Delta\theta_{tot} = \Delta\theta_{path} + \overset{\swarrow 0}{\Delta\theta_{ic}} + \overset{\swarrow 0}{\Delta\theta_{ref}}$$

$$= \Delta\theta_{path} = \Delta(kL) = k\Delta L \quad \text{(n does not change)}$$



$$L_1 \cos\theta = a$$

$$L_1 = a/\cos\theta$$

$$x = a\tan\theta$$

$$L_2 = L_1 - 2x\sin\theta$$

$$= a/\cos\theta - 2a\tan\theta\sin\theta$$

$$\Delta L = L_1 + L_2 = 2L_1 - 2x\sin\theta$$

$$k\Delta L = \frac{2ka}{\cos\theta}(1-\sin^2\theta)$$

$$= \boxed{2ka\cos\theta}$$

- 4b) (5 points) Under what conditions will the light observed have maximum intensity?

$$\Delta\theta_{tot} = \cancel{2}N\cancel{\pi} = 2ka\cos\theta = 2\cdot\frac{\cancel{2\pi}}{\lambda}a\cos\theta$$

$$\boxed{\lambda = \frac{2a\cos\theta}{N}} \quad \text{for } N = 1, 2, \ldots$$

- 4c) (10 points) If light of wavelength λ_0, incident at an angle θ, produces a signal of maximum intensity at the location of the observer, how far from λ_0 will the next longer wavelength that produces a signal of maximum intensity at an angle of θ appear (that is, what is $\Delta\lambda$, if $\Delta\lambda \equiv \lambda_{next} - \lambda_0$)?

Assuming λ_0 is when $N=1$,

$$\lambda_{next} = \frac{2a\cos\theta}{2}$$

$$\Delta\lambda = \frac{2a\cos\theta}{2} - \frac{2a\cos\theta}{1}$$

$$= \boxed{a\cos\theta}$$

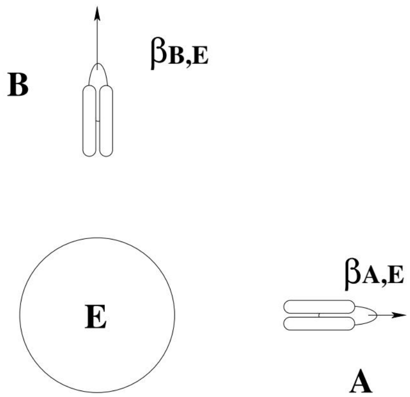
If λ_0 is when $N =$ anything,

$$\Delta\lambda = \frac{2a\cos\theta}{N+1} - \frac{2a\cos\theta}{N}$$

$$= \boxed{2a\cos\theta\left(\frac{1}{N+1} - \frac{1}{N}\right)}$$

- 4d) (5 points) Suppose you wanted to design a filter to extract light of wavelength λ_0 from a mixture with other signals flowing down an optical fiber. Would you want $\Delta\lambda$ to be large or small? Explain. If one were to define a quality factor for a Fabry-Perot filter, would it be proportional or inversely proportional to $\Delta\lambda$?

You would want a large $\Delta\lambda$. In the extreme case where $\Delta\lambda$ is REALLY small and the "mixture of signals" has signals in some range $(\lambda_{min}, \lambda_{max})$, the filter would extract light of not just λ_0, but also $\lambda_0 \pm \Delta\lambda$, and probably many other wavelengths. If $\Delta\lambda >> \lambda_{max} - \lambda_{min}$, this wouldn't be a problem. Thus the quality of such a filter would be $\boxed{\text{proportional}}$ to $\Delta\lambda$



5) Rocket A is moving with a velocity $(\beta_{A,E}\, C)\,\hat{x}$ relative to the Earth and Rocket B is moving with a velocity $(\beta_{B,E}\, C)\,\hat{y}$ relative to the Earth. Make sure your final answers to the following questions are in terms of the given information (no γ's).

- 5a) (5 points) A stick of length L_A lies along the x axis in Rocket A. How long is the stick in the Earth's frame? If the stick ages a year in Rocket A, how much has it aged in the Earth's frame?

Time dilation:

$\Delta t' = 1$ year

$\Delta x' = 0$

$\Delta t, \Delta x = ???$

$$\begin{pmatrix} c\Delta t \\ \Delta x \end{pmatrix} = \gamma_{A,E} \begin{pmatrix} 1 & \beta_{A,E} \\ \beta_{A,E} & 1 \end{pmatrix} \begin{pmatrix} c \\ 0 \end{pmatrix}$$

$$c\Delta t = \gamma_{A,E}\, c$$

$$\Delta t = \gamma_{A,E} = \boxed{1/\sqrt{1-\beta_{A,E}^2}} \text{ years}$$

Length contraction

$\Delta t = 0$

$\Delta x' = L_A$

$\Delta t', \Delta x = ???$

$$\begin{pmatrix} 0 \\ \Delta x \end{pmatrix} = \gamma_{A,E} \begin{pmatrix} 1 & \beta_{A,E} \\ \beta_{A,E} & 1 \end{pmatrix} \begin{pmatrix} c\Delta t' \\ L_A \end{pmatrix}$$

$$0 = \gamma_{A,E}(c\Delta t' + \beta_{A,E} L_A)$$

$$\Delta x = \gamma_{A,E}(\beta_{A,E}\, c\Delta t' + L_A) = \gamma_{A,E} L_A (1-\beta_{A,E}^2) = \boxed{L_A\sqrt{1-\beta_{A,E}^2}}$$

- 5b) (10 points) It takes a bug a time Δt_A to walk along the stick (back-to-front) in (and relative to) Rocket A. How long did it take the bug in the Earth frame? How far did the bug travel in the Earth frame?

$\Delta x' = L_A$

$\Delta t' = \Delta t_A$

$\Delta x, \Delta t = ???$

$$\begin{pmatrix} c\Delta t \\ \Delta x \end{pmatrix} = \gamma_{A,E} \begin{pmatrix} 1 & \beta_{A,E} \\ \beta_{A,E} & 1 \end{pmatrix} \begin{pmatrix} c\Delta t_A \\ L_A \end{pmatrix}$$

$$c\Delta t = \gamma_{A,E}(c\Delta t_A + \beta_{A,E} L_A)$$

$$\Delta t = \boxed{\frac{\Delta t_A + \beta_{A,E}/c\, L_A}{\sqrt{1-\beta_{A,E}^2}}}$$

$$\Delta x = \gamma_{A,E}(\beta c\Delta t_A + L_A) = \boxed{\frac{1}{\sqrt{1-\beta_{A,E}^2}}(\beta c\Delta t_A + L_A)}$$

- 5c) (10 points) How fast is Rocket B moving in Rocket A's frame?

$$u_{\parallel} = \frac{\beta c + u'_{\parallel}}{1 + \beta u'_{\parallel}/c}$$

$u_{\parallel}$ = x-vel. of rocket B in Earth frame $= 0$

βc = x-vel. of rocket A in Earth frame $= \beta_{A,E}\, c$

$u'_{\parallel}$ = x-vel. of rocket B in rocket A's frame

$$0 = \frac{\beta_{A,E}\, c + u'_{\parallel}}{1 + \frac{\beta_{A,E}\, u'_{\parallel}}{c}} \rightarrow u'_{\parallel} = -\beta_{A,E}\, c$$

$$\boxed{|\vec{u}'| = c\sqrt{\beta_{A,E}^2 + \beta_{B,E}^2 (1-\beta_{A,E}^2)}}$$

$$u_{\perp} = \frac{u'_{\perp}}{\gamma(1+\beta u'_{\parallel}/c)}$$

$$\beta_{B,E}\, c = \frac{u'_{\perp}}{\gamma(\underbrace{1-\beta_{A,E}^2}_{\frac{1}{\gamma^2}})}$$

$$u'_{\perp} = \frac{\beta_{B,E}\, c}{\gamma} = \beta_{B,E}\, c\sqrt{1-\beta_{A,E}^2}$$

- 5d) (5 points) What is the significance of $\beta = 0$ and $\beta = 1$? Evaluate your answer to part c in the limits $\beta_{B,E} = 0$ and $\beta_{B,E} = 1$ and discuss. . .

$\beta = 0$ means a rocket is at rest W.R.T some other frame,

$\beta = 1$ means it's a photon! (or something else at the speed of light which I think is impossible)

$$\beta_{B,E} = 0 \rightarrow |\vec{u}'| = c\sqrt{\beta_{A,E}^2 + 0} = \boxed{c\beta_{A,E}}$$

W.R.T. the Earth

As expected, $\beta_{B,E} = 0$ means rocket B is at rest, so the speed of B relative to A is just the speed of A relative to the Earth.

$$\beta_{B,E} = 1 \rightarrow |\vec{u}'| = c\sqrt{\beta_{A,E}^2 + 1 - \beta_{A,E}^2} = c\sqrt{1} = \boxed{c}$$

As expected, B acts like a photon and has speed c in all frames.