Final Exam 1CF20-2

Full Name (Printed) ______________________________

Full Name (Signature) ____________________________

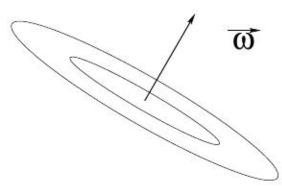
Student ID Number ______________________________

- The exam is open-book and open notes. You will probably do better to limit yourself to a single page of notes you prepared well in advance.
- **All work must be your own.** You are not allowed to collaborate with anyone else, you are not allowed to discuss the exam with anyone until all the exams have been submitted (after the close of the submissions window for the exam).
- You have **120 minutes** to complete the exam and more than sufficient time to scan the exam and upload it to GradeScope. The exam *must* be uploaded to GradeScope within the time alloted (that is, by the end of the 3-hour finals slot). We will only except submissions through GradeScope and will not accept any exam submitted after the submission window closes (CAE students must contact Corbin for instructions).
- **Given the limits of GradeScope, you must fit your work for each part into the space provided.** You may work on scratch paper, but you will not be able to upload the work you do on scratch paper, so it is essential that you copy your complete solution onto the exam form for final submission. We can only consider the work you submit on your exam form.
- **For full credit the grader must be able to follow your solution from first principles to your final answer.** ***There is a valid penalty for confusing the grader.***
- It is **YOUR** responsibility to make sure the exam is scanned correctly and uploaded before the end of the submission window. The graders may refuse to grade pages that are significantly blurred, solutions to problems that are not written in the correct place, pages submitted in landscape mode and/or work that is otherwise illegible - if any of this occurs, you may not receive *any* credit for the affected parts.
- Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

The following must be signed before you submit your exam:

By my signature below, I hereby certify that all of the work on this exam was my own, that I did not collaborate with anyone else, nor did I discuss the exam with anyone while I was taking it.

Signature ____



- 1a) (5 points) A thin ring of surface electric charge density $\sigma(r)$ extends from a radius r to a radius $r + dr$ from its center. It rotates about a perpendicular axis through its center with an angular velocity ω, similar to what is shown above. Find the electric current associated with the ring. Be clear about what you are doing.

- 1b) (10 points) Replace the ring with a washer that has a surface electric charge density given by

$$\sigma(r) = \sigma_0 \big(1 - \frac{r}{a}\big)\big(1 - \frac{r}{b}\big)$$

and extends from a radius a to a radius b from its center. Find the magnitude and direction of the dipole moment associated with the washer.

- 1c) (10 points) Find the magnitude and the direction of the magnetic field at the center of the washer.

- 1d) (5 points) Assuming the mass of the washer is uniformly distributed, it has a rotational inertia about the rotational axis given by:

$$I = \frac{1}{2}\,m\,(a^2 + b^2)$$

If the washer is placed in a uniform magnetic field $\vec{B}$, how rapidly will it precess?

2) A current I flows to the right down the center conductor (radius a) of a long coaxial cable and returns along the thin outer conductor (radius b). The (normalized) current density in the center conductor is given by:

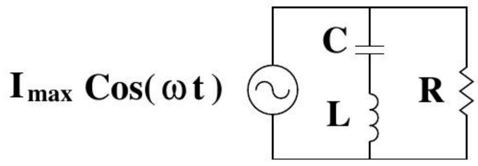
$$|\vec{J}| = \frac{(n+2)}{2}\frac{I}{\pi a^2}\left(\frac{r}{a}\right)^n$$

- 2a) (10 points) Find the magnitude of the magnetic field for all values of r.

- 2b) (15 points) Find the energy per unit length stored in a coaxial cylinder of radius r. Consider all values of r. (there's more room to work on the next page)

- 2b) *(continued)*

- 2c) (5 points) Find the inductance per unit of this cable. Is the value you obtained larger or smaller than the value obtained in the usual approximation (where all the inner current flows down the surface of the central wire)? Explain.



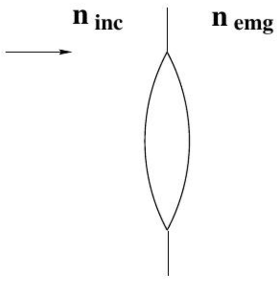
3) A current source $I(t) = I_{max}\cos(\omega t)$ is connected across a resistor, capacitor and inductor, as shown.

- 3a) (10 points) How large is the impedance seen by the source?

- 3b) (5 points) By how much will the current drawn from the source lead or lag the voltage measured across the source? Under what conditions will that current lead the voltage? Under what conditions will it lag? (The more correct details you provide, the more points you'll receive.)

- 3c) (5 points) How large is the voltage amplitude measured across the source?

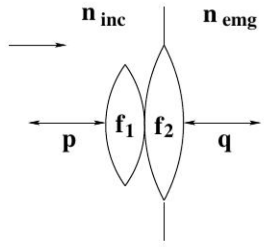
- 3d) (10 points) How large is the voltage amplitude measured across the inductor? What happens to that voltage amplitude when/if the driving frequency from the source becomes equal to the natural frequency for the LC combination?



- 4a) (5 points) Write an equation that correctly relates p, q, n_{inc}, n_{emg} and f_{emg} for a thin lens that straddles two media. (Be careful not to over-think this!)

- 4b) (5 points) While most people can bring very distant objects into focus (within the limits of resolution), near-sighted people have difficulty bringing anything into focus that is located beyond their *far-point*. Use the equation you derived in part a to show that this is consistent with the light from distant objects focusing on a point in front of the retina, too close to the lens.

- 4c) (5 points) Let's evaluate a rather extreme case of near-sightedness. Suppose light from distant objects focuses approximately $\frac{8}{9}$ of the way to someone's retina. If the diameter of the eye is D, $n_{air} = 1$, $n_{eye} = 4/3$, where is this person's far-point?



- 4d) (5 points) Suppose a second thin lens, fully-immersed in medium n_{inc}, is placed right in front of the lens that straddles n_{inc} and n_{emg}. Write an equation that correctly relates the initial object distance (p), the final image distance (q), n_{inc}, n_{emg}, the focal length of the front lens (f_1) and the focal length of the straddling lens ($f_{emg} = f_2$).

- 4e) (10 points) Calculate the focal length of the corrective lens required to restore distant vision to the eye mentioned in part c. Is the corrective lens converging or diverging? Why does this seem reasonable? Compare the focal length of the corrective lens to the far-point of the uncorrected eye - how are they connected (that is, what do the values say about how the lens does what it does).

5) You're in a rocket (**S'**) heading towards the Earth (**S**) at a speed βC. At some point, you launch a wine bottle with a speed u' towards home (gotta give it a chance to breathe) and it arrives a time $\Delta t'$ seconds later. Please note that all primed quantities are measured in the frame of the rocket.

- 5a) (5 points) There are two events of interest here: the bottle leaves the rocket and the bottle arrives on Earth. Using the information provided, write out the space-time interval between those two events as seen in the rocket frame.

- 5b) (10 points) According to the folks back home, how long was the bottle in transit? How far did it travel?

- 5c) (10 points) Using an inner-product, evaluate the proper-time between the two events as seen in the rocket, and again, as seen on the Earth. Discuss.

- 5d) (5 points) While the bottle was in transit, you aged a time $\Delta t'$. How much did your friends on Earth age? How much did the wine in the bottle age? Discuss.