

Name:

Student ID #:

Discussion day, time:

Physics 1C, Spring 2019, Lecture 2
Midterm 1

Time allotted: 50 minutes
No calculators or notes allowed.
No phones out during the exam.
All work must be your own.
Partial credit will be awarded for correct work.

Problem 1	/20
Problem 2	/20
Problem 3	/20
Total	/60

Inspirational quote:

But still, try, for who knows what is possible? —Michael Faraday (1870)

Potentially useful formulas:

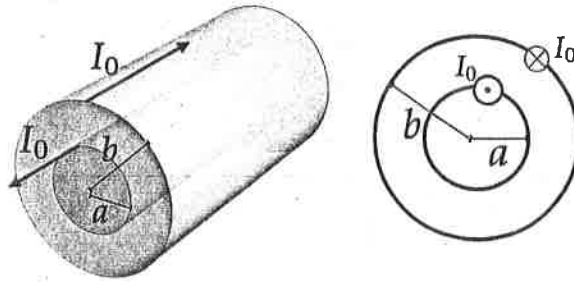
Vacuum magnetic permeability

$$\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1} \quad (1)$$

Field due to a ring (radius a) of current I on axis

$$\mathbf{B}(z) = \frac{\mu_0 I}{2} \frac{a^2}{(z^2 + a^2)^{3/2}} \hat{\mathbf{z}} \quad (2)$$

Problem 1. A coaxial cable consists of a long, cylindrical, conducting shell of radius a surrounded by another, coaxial shell of radius $b > a$. The two shells both carry current I_0 distributed evenly over their surface area, but in opposite directions.



(a) Use Ampere's law to find the magnetic field everywhere as a function of radius.

(b) Calculate the self-inductance L of a length ℓ of such a cable.

(c) A length of this coaxial cable is connected as an inductor to a fixed capacitance C , forming a series LC circuit. Explain how you might vary the geometry of the cable to make the LC oscillation frequency ω_0 large.

(a)

Amperes law
 $r < a$:
 $I_{enc} = 0 \rightarrow \int \vec{B} \cdot d\vec{\ell} = 0 \rightarrow \boxed{B=0}$

$a < r < b$:
 $I_{enc} = I_0$

amperian loop is circle of radius r
 $\int \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{enc} = \mu_0 I_0$

$\boxed{B = \frac{\mu_0 I_0}{2\pi r}}$

$r > b$:
 $I_{enc} = I_0 - I_0 = 0 \rightarrow \boxed{B=0}$

(b)

$$L = \frac{\Phi}{I}$$



$$\Phi = \int \vec{B} \cdot d\vec{a} = \int_0^l dz \int_a^b dr \left(\frac{\mu_0 I_0}{2\pi r} \right)$$

$$= \frac{\mu_0 I_0 l}{2\pi} \int_a^b dr \frac{1}{r}$$

$$= \frac{\mu_0 I_0 l}{2\pi} \log\left(\frac{b}{a}\right)$$

$$L = \Phi / I_0 = \frac{\mu_0 l}{2\pi} \log\left(\frac{b}{a}\right)$$

(c)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

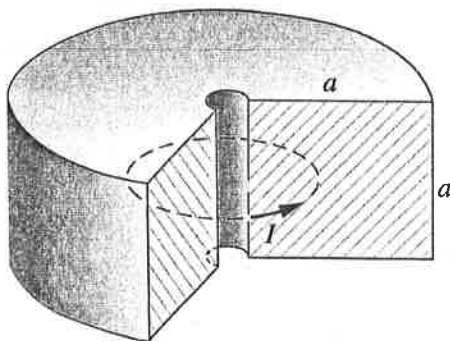
to make ω_0 large,
make L small

$$L \sim l \log\left(\frac{b}{a}\right)$$

- could make l small, but ~~what~~ what if we need a certain length of cable?
- more useful: make b very close to a , then $\log\left(\frac{b}{a}\right)$ becomes small. This makes L small for any length of cable.

Problem 2. Magnetic fields inside good conductors cannot change quickly. We found that current in a simple inductive circuit decays exponentially with characteristic time L/R . In a large conducting body such as the core of the earth, a circuit is not easy to identify. Nevertheless we can find an order-of-magnitude estimate for the decay time τ with some simplifying approximations.

We will model the core of the earth as a conducting "donut" of square cross section (side length a) and total resistance R .



This image shows a portion of the donut cut out to display the cross-section. In your answer, consider the full donut.

(a) As an estimate of the magnitude B of the magnetic field, calculate the field at the center of a conducting ring of radius $a/2$ carrying current I .

(b) Treat the magnetic field in the donut as uniform, with the magnitude you found in (a). Estimate the magnetic field energy U stored in the donut. Treat the donut as a cylinder for the purpose of finding its volume.

(c) Suppose the magnetic energy stored in the donut depends on time roughly as $U \approx U_0 e^{-t/\tau}$, with U_0 constant. Use the fact that the power is

$$\frac{dU}{dt} = -I^2 R \quad (3)$$

and your answer to (b) to obtain an expression for τ in terms of a , R , and fundamental constants. Verify that your expression is dimensionally correct.

(a) From formula sheet:

$$B_{\text{ring}} = \frac{\mu_0 I}{2} \frac{r^2}{(z^2 + r^2)^{3/2}}$$

at center, $z = 0$. For us, $r = a/2$

$$B_{\text{ring}} = \frac{\mu_0 I}{2} \frac{(a/2)^2}{((a/2)^2)^{3/2}} = \frac{\mu_0 I}{2} \left(\frac{1}{a/2} \right)$$

~~$$= \frac{\mu_0 I}{a}$$~~

$$= \frac{\mu_0 I}{a}$$

(b)

$$u = \frac{1}{2\mu_0} B^2$$
$$= \frac{1}{2\mu_0} \left(\frac{\mu_0 I}{a} \right)^2 = \frac{\mu_0 I^2}{2 a^2}$$

u is uniform, so $U = uV$

$V =$ volume of cylinder

$$= (\pi a^2) a$$

$$= \pi a^3$$

$$U = \frac{\mu_0 I^2}{2} \pi a^3 = \boxed{\frac{\mu_0 I^2 \pi a^3}{2}}$$

(c)

$$U = U_0 e^{-t/\tau} \quad \text{so}$$

$$\frac{dU}{dt} = -\frac{1}{\tau} U = -I^2 R$$

$$\Rightarrow \tau = \frac{U}{I^2 R} = \frac{\mu_0 I^2 \pi a^3}{2 I^2 R}$$

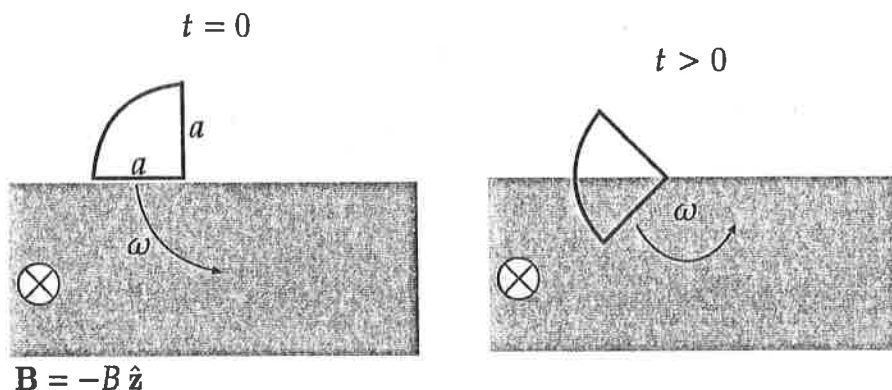
$$= \boxed{\frac{\mu_0 \pi a^3}{2 R}}$$

dimensions:

$$[\text{time}] = \frac{[\text{inductance}] [\text{length}]}{[\text{resistance}]}$$

$$= \frac{[\text{inductance}]}{[\text{resistance}]} = [\text{time}] \quad \checkmark$$

Problem 3. A conducting wire loop in the shape of a quadrant of a circle with radius a rotates at constant angular velocity $d\theta/dt = \omega$ as shown in the figure below. The shaded area indicates a constant, uniform magnetic field of magnitude B directed into the page.



(a) Calculate the emf $\mathcal{E}(t)$ around the wire as a function of time for one full period, $0 \leq t < 2\pi/\omega$. Your answer will be a piecewise function with four time segments. The area of a circular sector of radius r subtending angle θ is $\text{Area} = r^2\theta/2$.

(b) For some time segments of its motion, the loop will experience a magnetic force. Without detailed calculations, give the direction of this force, if any, during each segment of time.

(a) ① $0 \leq t < \frac{\pi}{2\omega}$: loop entering field
~~flux through loop increasing~~

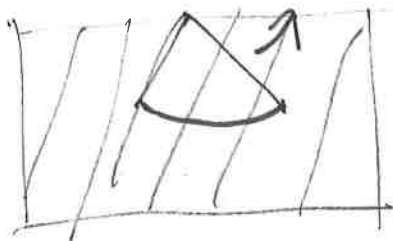
$$\Phi = \vec{B} \cdot \vec{A} = -BA \rightarrow \frac{d\Phi}{dt} = -B \frac{dA}{dt}$$

$$A = \frac{a^2\theta}{2}, \quad \frac{dA}{dt} = \frac{a^2}{2} \frac{d\theta}{dt} = \frac{a^2\omega}{2}$$

$$\mathcal{E} = -\frac{d\Phi}{dt} = \boxed{\frac{Ba^2\omega}{2}}$$

② $\frac{\pi}{2\omega} \leq t < \frac{\pi}{\omega}$: loop completely in field

$$\frac{d\Phi}{dt} = 0 \Rightarrow \mathcal{E} = 0$$



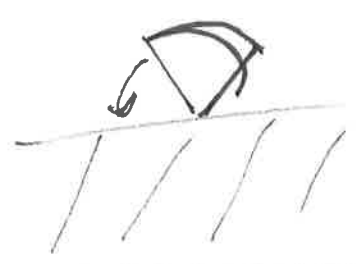
③ $\frac{\pi}{\omega} \leq t < \frac{3\pi}{2\omega}$: loop exiting field

Now $\frac{dA}{dt}$ is negative, b/c area in field region decreasing.

$\frac{d\Phi}{dt} = -B \frac{dA}{dt}$, $\frac{dA}{dt} = -\frac{a^2 \omega}{2}$ (same as 1st time segment, but negative)

$\mathcal{E} = -\frac{d\Phi}{dt} = \boxed{-\frac{Ba^2\omega}{2}}$

④ $\frac{3\pi}{2\omega} \leq t < \frac{2\pi}{\omega}$: loop completely out of field



$\frac{d\Phi}{dt} = 0 \Rightarrow \mathcal{E} = 0$

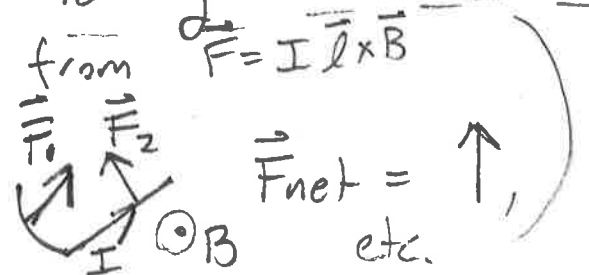
(b) Induced emf opposes change in flux.

For ②, ④, no force.

For ①, force points away from field region, opposing $\Delta\Phi$ caused by entering field

For ③, force points into field region, opposing $\Delta\Phi$ caused by leaving field.

(could also argue



$\vec{F}_{net} = \uparrow$, etc.

from $\vec{F} = I \vec{l} \times \vec{B}$

