

# Physics 1C Spring 2018 Second Midterm

Anshul Kale

TOTAL POINTS

**78 / 100**

QUESTION 1

**1 1a 8 / 15**

- + 4 pts Correct magnitude for B.
- ✓ + 4 pts Correct direction for B.
- + 7 pts Correctly and convincingly show  $\alpha = 1/c^2$ .
- ✓ + 4 pts Display moderate understanding of material in solving for alpha.
- + 0 pts No points.

QUESTION 2

**2 1b 2 / 6**

- + 2 pts Correctly express the time-averaged magnitude of the Poynting vector  $\langle S \rangle$  in terms of  $E_0$  and constants.
- + 2 pts Correctly express  $\langle S \rangle$  in terms of the time-averaged radiated power  $\langle P \rangle$ ,  $r$ , and constants.
- + 2 pts Correctly solve for  $E_0$ .
- ✓ + 2 pts Miscellaneous partial credit.
- + 0 pts No points.

QUESTION 3

**3 1c 11 / 15**

- ✓ + 8 pts Write a somewhat correct expression for the force on the sail due to radiation.
- + 5 pts Correctly solve for the area of the sail (up to a factor of 2).
- + 2 pts Correctly solve for the area of the sail.
- ✓ + 3 pts Miscellaneous partial credit.
- + 0 pts No points.

QUESTION 4

**4 2a 5 / 5**

- ✓ + 5 pts Correct
- + 2 pts Partial credit: Correct show that  $V_c$  lags

behind  $V_r$  by 90 degree.

- + 3 pts Partial credit: Correct show that  $V_0 = \text{vector sum of } V_c \text{ and } V_r$ .
- + 1 pts nice try
- + 0 pts No points

QUESTION 5

**5 2b 5 / 5**

- ✓ + 2 pts current leads voltage or voltage lags current
- ✓ + 3 pts clearly explain why current leads voltage or voltage lags current.
- + 0 pts no points

QUESTION 6

**6 2c 13 / 16**

- ✓ + 5 pts Correct  $V_{out}/V_{in}$
- + 3 pts Correct limit value of ratio at small  $\omega$ , from your expression of  $V_{out}/V_{in}$ .
- ✓ + 3 pts Correct limit value of ratio at large  $\omega$ , from your expression of  $V_{out}/V_{in}$ .
- ✓ + 3 pts Decide if this is high pass or low pass correctly according to your limit values.
- ✓ + 2 pts Explain why
- + 0 pts no points

QUESTION 7

**7 2d 10 / 10**

- ✓ + 3 pts Correct expression of  $V_{out}/V_{in}$  for the first circuit.
- ✓ + 2 pts Determine if the first circuit is high or low pass according to your  $V_{out}/V_{in}$  correctly and explain.
- ✓ + 3 pts Correct expression of  $V_{out}/V_{in}$  for the second circuit.
- ✓ + 2 pts Determine if the second circuit is high or

low pass according to your  $V_{out}/V_{in}$  correctly and explain.

+ 0 pts No points

QUESTION 8

8 3a 5 / 5

✓ + 5 pts Correct

+ 0 pts Incorrect

QUESTION 9

9 3b 15 / 15

✓ + 4 pts Correct expression for Brewster's angle.

✓ + 4 pts Correct expression for critical angle.

✓ + 4 pts Recognize that one angle must be taken for "internal" reflection (air to n) and the other "externally" (n to air). Or recognize that the naive approach to this problem leads to nonsensical values of n.

✓ + 3 pts Correct expression for n (with a non-zero solution).

+ 0 pts No points.

QUESTION 10

10 3c 4 / 8

+ 8 pts Correct (or very almost correct) analytic expression for n.

✓ + 4 pts Valiant algebraic or geometric attempt in solving for n.

+ 0 pts No points

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This exam is closed book and closed notes. Electronics are not permitted, except for one calculator. Please show your full solution in the boxes provided (where the scanners can pick them up). Your solutions will be graded on correctness and coherence; results given with no details will receive zero credit. There is additional scratch paper attached so you can collect your thoughts first. Academic dishonesty is reported to the Office of the Dean of Students.

**Problem 1.** You have designed a solar space craft of mass  $m$  that is accelerated by the force due to the 'radiation pressure' from the sun's light that falls on a perfectly reflective circular sail that is oriented face-on to the sun. The time averaged radiative power of the sun is  $\langle P \rangle$ . The gravitational constant is  $G$ . The mass of the sun is  $m_s$ . The speed of light is  $c$ . Model the sun's light as a plane electromagnetic wave with the electric field given by  $\vec{E}(z, t) = E_0 \cos(kz - \omega t) \hat{i}$ . (Recall, Newton's universal law of gravitation:  $F = \frac{Gm_1 m_2}{r^2}$ )

- a. (8 pts) What is the magnetic field associated with this electric field? (7 pts) One of Maxwell's equations in vacuum has the form:  $\vec{\nabla} \times \vec{B} = \alpha \frac{d\vec{E}}{dt}$ . Using your expression for  $\vec{B}$ , determine what the constant  $\alpha$  on the right hand side must be in terms of  $c$ , the speed of light. (Compute it; just writing down the correct answer will receive no credit.) Does your expression for  $\vec{B}$  lead you to the correct constant?

EM wave travels in  $+z$  direction  
 $\vec{E}$  field in  $\hat{i}$ ,  $\vec{B}$  field in  $\hat{j}$

$$\vec{B}(z, t) = c E_0 \cos(kz - \omega t) \hat{j}$$

$$\vec{\nabla} \times \vec{B} = \alpha \frac{d\vec{E}}{dt}$$

$i$	$j$	$k$	
$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$	
0	$c E_0 \cos(kz - \omega t)$	0	

$$= -kc E_0 \sin(kz - \omega t) \hat{j}$$

$$= kc E_0 \sin(kz - \omega t) \hat{j}$$

$$\frac{d\vec{E}}{dt} = -\omega E_0 \sin(kz - \omega t) \hat{i}$$

$$kc E_0 \sin(kz - \omega t) \hat{j} = \alpha \omega E_0 \sin(kz - \omega t) \hat{j}$$

$\alpha = \frac{kc}{\omega}$

Yes it does

- b. (2 pts) What is the magnitude of the time averaged Poynting vector,  $\langle \vec{S} \rangle$ , associated with this wave. (4 pts) Use it to find the amplitude of the electric field at your starting point, a distance  $r$  from the sun. Constants that may appear in your answer:  $m$ ,  $\langle P \rangle$ ,  $c$ ,  $m_s$ ,  $G$ ,  $k$ ,  $\pi$ ,  $\mu_0$ ,  $\epsilon_0$  and  $\omega$  as necessary.

$$|\vec{S}| = \frac{1}{2\mu_0} E_0 B_0 = \frac{1}{2\mu_0} B_0 \cdot c B_0 = \frac{1}{2\mu_0} c B_0^2$$

$$\frac{1}{2\mu_0}$$

$$E_0 = \frac{c B_0}{\mu_0}$$

- c. (15 pts) What is the minimum area for the sail in order to exactly balance the gravitational attraction from the sun? Constants that may appear in your answer:  $m$ ,  $\langle P \rangle$ ,  $c$ ,  $m_s$ ,  $G$ ,  $k$ ,  $\pi$ ,  $\mu_0$ ,  $\epsilon_0$  and  $\omega$  as necessary.

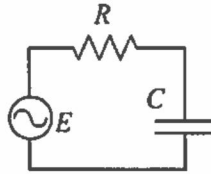
$$\frac{G m_s m}{r^2} = \left( \frac{\langle P \rangle}{4\pi r^2} \cdot \frac{2c}{c} \right) \pi r^2$$

$$\frac{G m_s m}{r^2} = \frac{\langle P \rangle}{2c}$$

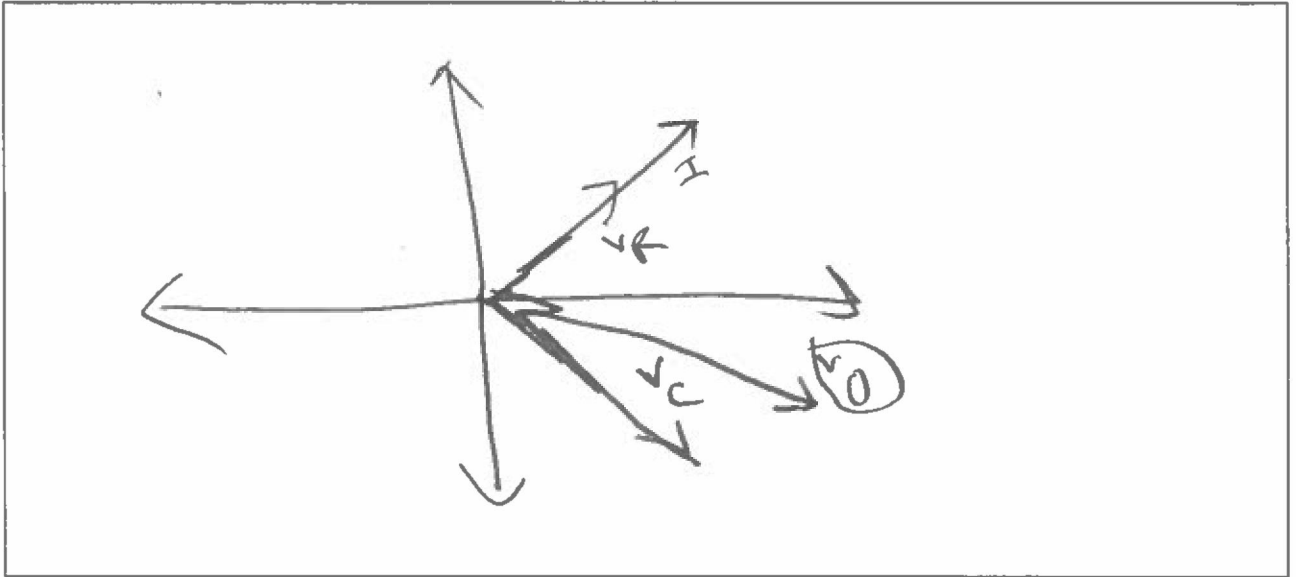
$$r^2 = \frac{G m_s m (2c)}{\langle P \rangle}$$

$$\text{Area sail} = \pi r^2 = \frac{\pi G m_s m (2c)}{\langle P \rangle}$$

**Problem 2.** The circuit shown has an AC generator  $V_{in} = V_0 \sin(\omega t)$  connected in series with a resistor R and a capacitor C.



- a. (5 pts) Draw a phasor diagram for this circuit, showing the relative phases of the voltages  $V_R$ ,  $V_C$ , and  $V_0$ .



- b. (5 pts) Does the current lag or lead the voltage? Explain.

Current leads voltage, since the angle of voltage with respect to current is negative.

- c. (5 pts) What is the ratio of the magnitudes of the output signal amplitude to the input signal amplitude  $\frac{V_{out,0}}{V_{in,0}}$  if the output is measured across the capacitor only? (3 pts) What is the limiting value of this ratio at very small values of  $\omega$ ? (3 pts) What is the limiting value of this ratio at very large values of  $\omega$ ? (5 pts) Decide if this is a high pass or low pass filter. (Recall that a low pass filter blocks high frequencies.)

$$\frac{V_{out,0}}{V_{in,0}} = \frac{V_c}{V_0} = \frac{I X_c}{V_0} = \frac{I \cdot \frac{1}{\omega C}}{V_0} = \frac{1}{\omega C \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

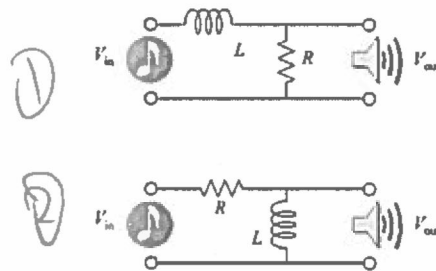
$$= \frac{1}{\omega C \sqrt{R^2 + \frac{1}{\omega^2 C^2}}} = \frac{1}{\omega C \sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$\lim_{\omega \rightarrow 0} \frac{1}{\omega \sqrt{R^2 \omega^2 + \frac{1}{C^2}}} = \frac{1}{\sqrt{0 + \frac{1}{C^2}}} = \frac{1}{\frac{1}{C}} = C$$

$$\lim_{\omega \rightarrow \infty} \frac{1}{\omega \sqrt{R^2 \omega^2 + \frac{1}{C^2}}} = \frac{1}{\omega \sqrt{R^2 \omega^2}} = \frac{1}{\omega R \omega} = \frac{1}{R \omega^2} \rightarrow 0$$

This is a low-pass filter since as  $\omega \rightarrow \infty$  the ratio approaches 0.

d. (10 pts) Low-pass and high-pass filters can also be constructed using inductors rather than capacitors. Determine which of these two circuits would provide a low-pass filter and which would provide a high-pass filter. Explain your answer.



$$\lim_{\omega \rightarrow 0} \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = 1$$

As  $\omega \rightarrow 0$  for ①, the ratio is 1. Thus, ① is a low pass filter since the ratio is non-zero when frequency  $\rightarrow 0$ .

$$\lim_{\omega \rightarrow \infty} \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \lim_{\omega \rightarrow \infty} \frac{L}{\sqrt{\frac{R^2}{\omega^2} + L^2}} = 1$$

② is a high pass filter since as  $\omega \rightarrow \infty$  the ratio approaches 1.

**Problem 3.** Consider an object immersed in an unknown fluid such that its apparent depth is larger than its actual depth when viewed from an observer located outside of the fluid. What can you say about the index of refraction of the fluid? (note: the fluid is not necessarily surrounded by air.) (5 pts)

- a.  $n_{\text{fluid}} > n_{\text{outside}}$
- b.  $n_{\text{fluid}} < n_{\text{outside}}$
- c.  $n_{\text{fluid}} = n_{\text{outside}}$
- d. not possible to tell

Now consider an air-medium interface in which  $n_{\text{air}} = 1$ . The medium is not the one from above.

- a. (15 pts) Determine an expression for the value of  $n$  of the medium for which Brewster's angle is equal to the critical angle.

$\sin^{-1}(1/n) = \tan^{-1}(1)$

slow  $n_a$  vacuum

air  $\rightarrow$

medium  $n$  fast

$\theta_{\text{crit}} = \sin^{-1}(n/1)$

$\theta_B = \tan^{-1}(1)$

$n^2 = 1 + 1$

$\sin^{-1}(1/n) = \tan^{-1}(1)$

$\sin^{-1}(1/n) = \tan^{-1}(n)$

- b. (8 pts) Solve your expression from part a for n. (Hint: Use similar triangles. Does your value of n make physical sense? If not, you should reconsider the problem.)

The student's work is contained within a rectangular box. At the top left, there is a right-angled triangle with a hypotenuse of length 1 and an angle  $\theta$  at the bottom-left vertex. A vertical line segment of length  $n$  is drawn from the top vertex to the hypotenuse. To the right of this triangle, the angle is given as  $\theta = 45^\circ$ . Below this, a rectangular box contains the equation  $n = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ , which is crossed out with a large 'X'. To the left of this box, the word "Ignore" is written with an arrow pointing to the crossed-out equation. To the right, the word "keep" is written with an arrow pointing to the same equation. Below these are two more right-angled triangles. The first has a hypotenuse of length  $\sqrt{2}$ , a vertical leg of length 1, and an angle  $\theta$  at the bottom-left vertex. The second has a hypotenuse of length  $\sqrt{2}$ , a horizontal leg of length 1, and an angle  $\theta$  at the bottom-left vertex.





