Physics 1C – Spring 2018: Midterm 2

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This exam is closed book and closed notes. Electronics are not permitted, except for one calculator. Please show your full solution in the boxes provided (where the scanners can pick them up). Your solutions will be graded on correctness and coherence; results given with no details will receive zero credit. There is additional scratch paper attached so you can collect your thoughts first. Academic dishonesty is reported to the Office of the Dean of Students.

Problem 1. You have designed a solar space craft of mass m that is accelerated by the force due to the 'radiation pressure' from the sun's light that falls on a perfectly reflective circular sail that is oriented face-on to the sun. The time averaged radiative power of the sun is $< P >$. The gravitational constant is G. The mass of the sun is m_s The speed of light is c. Model the sun's light as a plane electromagnetic wave with the electric field given by $\vec{E}(z,t) = E_0 \cos(kz - \omega t)\hat{i}$. (Recall, Newton's universal law of gravitation: $F = \frac{Gmm_s}{r^2}$)

a. (8 pts) What is the magnetic field associated with this electric field? (7 pts) One of Maxwell's equations in vacuum has the form: $\vec{\nabla} \times \vec{B} = \alpha \frac{d\vec{E}}{dt}$. Using your expression for $\vec{B}$, determine what the constant α on the right hand side must be in terms of c, the speed of light. (Compute it; just writing down the correct answer will receive no credit.) Does your expression for $\vec{B}$ lead you to the correct constant?

$cB = E$, $B = \frac{E}{c} = \frac{E_0}{c} \cos(-kz - \omega t)\hat{j}$

$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & B & 0 \end{vmatrix} = \left(-\frac{\partial B}{\partial z}\right)\hat{i} + \left(\frac{\partial B}{\partial x}\right)\hat{k} = \alpha \frac{\partial E}{\partial t} = \alpha \omega E_0 \sin(kx - \omega t)$

$-\frac{\partial B}{\partial z} = \frac{E_0 k}{c} \sin(kz - \omega t) = \alpha \omega E_0 \sin(kz - \omega t)$

$\omega \alpha = \frac{k}{c}$ $\Rightarrow k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$

$\alpha = \frac{k}{c\omega}$

$\boxed{\alpha = \frac{1}{c^2}}$ $\Rightarrow \frac{k}{\omega} = \frac{2\pi}{\lambda} \cdot \frac{1}{2\pi f} = \frac{1}{c}$

b. (2 pts) What is the magnitude of the time averaged Poynting vector, $<\vec{S}>$, associated with this wave. (4 pts) Use it to find the amplitude of the electric field at your starting point, a distance r from the sun. Constants that may appear in your answer: m , $<P>$, c, m_s, G, k, π, μ_0, ϵ_0 and ω as necessary.

$$\vec{S} = \frac{EB}{\mu_0} \qquad <\vec{S}> = \frac{E_0 B_0}{2\mu_0} = \frac{cE_0^2}{2\mu_0}\hat{k}$$

$$<P> = \int \vec{S} \cdot d\vec{A} = <S> 4\pi r^2 \qquad <\vec{S}> = \frac{<P>}{4\pi r^2} = \frac{c E_0^2}{2\mu_0}$$

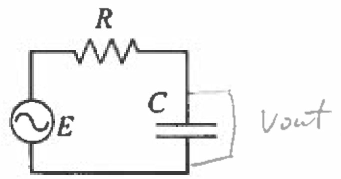
$$E_0 = \sqrt{\frac{2\mu_0 <P>}{4\pi r^2 c}}$$

c. (15 pts) What is the minimum area for the sail in order to exactly balance the gravitational attraction from the sun? Constants that may appear in your answer: m , $<P>$, c, m_s, G, k, π, μ_0, ϵ_0 and ω as necessary.

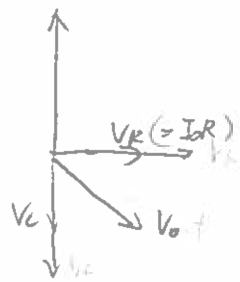
$$F_g = \frac{G m m_s}{r^2} \qquad \frac{2<P>}{c 4\pi r^2} \cdot A = \frac{G m m_s}{r^2}$$

$$A = \frac{G m m_s 4\pi c}{2<P>} \; m^2$$

Problem 2. The circuit shown has an AC generator $V_{in} = V_0 \sin(\omega t)$ connected in series with a resistor R and a capacitor C.



a. (5 pts) Draw a phasor diagram for this circuit, showing the relative phases of the voltages V_R, V_C, and V_0.



b. (5 pts) Does the current lag or lead the voltage? Explain.

Current leads the voltage. This is a RC circuit. The capacitor delays the voltage in a circuit.

c. (5 pts) What is the ratio of the magnitudes of the output signal amplitude to the input signal amplitude $\frac{V_{out,0}}{V_{in,0}}$ if the output is measured across the capacitor only? (3 pts) What is the limiting value of this ratio at very small values of ω? (3 pts) What is the limiting value of this ratio at very large values of ω? (5 pts) Decide if this is a high pass or low pass filter. (Recall that a low pass filter blocks high frequencies.)

$V_{out} = V_c = \frac{Q}{C} = I_o \frac{1}{wC}$ $I_o = \frac{V_o}{Z} = \frac{V_o}{\sqrt{R^2+\frac{1}{w^2C^2}}}$

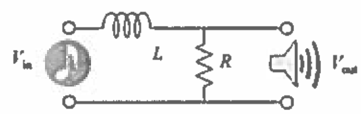
$I_o \frac{1}{wC} = \frac{V_o}{\sqrt{w^2C^2R^2+1}} = V_{out}$

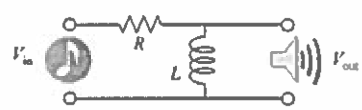
$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{w^2C^2R^2+1}}$

$\lim_{w \to 0} \frac{1}{\sqrt{w^2C^2R^2+1}} = 1$

It's a low pass filter.

d. (10 pts) Low-pass and high-pass filters can also be constructed using inductors rather than capacitors. Determine which of these two circuits would provide a low-pass filter and which would provide a high-pass filter. Explain your answer.

(1) 

(2) 

(1) $V_{out} = I_o R = \frac{V_{in}}{Z} R = \frac{V_{in} R}{\sqrt{R^2+w^2L^2}}$ $\lim_{w \to \infty} \frac{V_{in} R}{\sqrt{R^2+w^2L^2}} = 0$ ratio: 0

(1) is a low pass filter. $\lim_{w \to 0} \frac{V_{in} R}{\sqrt{R^2+w^2L^2}} = V_{in}$ ratio: 1

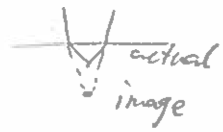
(2) $V_{out} = I_o X_L = I_o wL = \frac{V_{in} wL}{Z} = \frac{V_{in} wL}{\sqrt{R^2+w^2L^2}}$ $\lim_{w \to \infty} \frac{V_{in} wL}{\sqrt{R^2+w^2L^2}} = 1$

(2) is a high pass filter $\lim_{w \to 0} \frac{V_{in} wL}{\sqrt{R^2+w^2L^2}} = 0$

Problem 3. Consider an object immersed in an unknown fluid such that its apparent depth is **larger** than its actual depth when viewed from an observer located outside of the fluid. What can you say about the index of refraction of the fluid? (note: the fluid is not necessarily surrounded by air.) (5 pts)

a. $n_{fluid} > n_{outside}$
b. $n_{fluid} < n_{outside}$
c. $n_{fluid} = n_{outside}$
d. not possible to tell

b. @



Now consider an air-medium interface in which $n_{air} = 1$. The medium is not the one from above.

a. (15 pts) Determine an expression for the value of n of the medium for which Brewsters angle is equal to the critical angle.

$\frac{n_2}{n_1} = \tan\theta_B \Rightarrow n \cos\theta_B = n_1 \sin\theta_B = \sin\theta_B$

critical angle: $n_1 \sin\theta = n$

$\theta = \theta_B \Rightarrow n_1 \sin\theta_B = n \Rightarrow \sin\theta_B = n$

b. (8 pts) Solve your expression from part a for n. (Hint: Use similar triangles. Does your value of n make physical sense? If not, you should reconsider the problem.)

$\sin \theta_B = \tan \theta_B$

$\sin\theta_B = \frac{\sin\theta_B}{\cos\theta_B} \quad \theta = 90°$

$\sin 90° = 0 = n$

Scratch paper