Physics 1C Spring 2018 Final

Anshul Kale

TOTAL POINTS

96.5 / 106

QUESTION 1

1 1 **10 / 10**

- ✓ + **3 pts** Correct expression for I(\phi,\beta)
- ✓ + **2 pts** Correct expression for \phi
- ✓ + **2 pts** Correct expression for \beta
- ✓ + **3 pts** Correct numerical answer
- + **0 pts** Click here to replace this description.
- + **10 pts** Correct *

QUESTION 2

2 2a **5 / 5**

- ✓ + **2 pts** dsin(\theta) = m \lambda
- ✓ + **2 pts** sin(\theta) = y/L
- ✓ + **1 pts** Correct \lambda
- + **0 pts** Click here to replace this description.

QUESTION 3

3 2b **5 / 5**

- ✓ + **1 pts** Correct expression for max
- ✓ + **1 pts** Correct expression for min.
- ✓ + **1 pts** Let correct distances be equal
- ✓ + **2 pts** Solve for smallest integer values
- + **0 pts** Click here to replace this description.

QUESTION 4

4 3a **4 / 4**

- ✓ + **1 pts** x'_1
- ✓ + **1 pts** x'_2
- ✓ + **1 pts** t'_1
- ✓ + **1 pts** t'_2
- + **0 pts** Click here to replace this description.

QUESTION 5

5 3b **6 / 6**

- ✓ + **1.5 pts** Correct time interval in S'
- ✓ + **1.5 pts** Correct time interval in S
- ✓ + **1.5 pts** NOT entirely a result of time dilation
- ✓ + **1.5 pts** Short reason for conclusion (i.e. clocks no longer synchronized/ events took place at different positions in both frames)
- + **0 pts** Click here to replace this description.

QUESTION 6

6 4 **12 / 12**

- ✓ + **3 pts** Correct expression for length in moving frame
- ✓ + **3 pts** Correct \gamma for spaceship B
- ✓ + **3 pts** Let contracted lengths be equal
- ✓ + **3 pts** Solve for correct velocity of spaceship A
- + **0 pts** Click here to replace this description.

QUESTION 7

7 5a **3 / 3**

- ✓ + **2 pts** Write displacement current as a time derivative of electric flux
- ✓ + **1 pts** Correct final expression for displacement current
- + **0 pts** Click here to replace this description.

QUESTION 8

8 5b **3 / 5**

- ✓ + **2 pts** Correct magnitude in terms of displacement current from (a)
- + **2 pts** Correct direction
- + **0 pts** Click here to replace this description.
- ✓ + **1 pts** Ampere's Law

QUESTION 9

9 5c **1 / 2**

- ✓ + **1 pts** B field lines are circles
- + **0 pts** Click here to replace this description.
- + **1 pts** B field goes in the correct direction

QUESTION 10

10 6 **10 / 10**

✓ **+ 10 pts** **Correct**

+ **2 pts** Partial points: Gravity Fg=mg=density*area*length*g

+ **2 pts** Partial points: Force due to magnetic field: Fb=J*area*length*B0

+ **2 pts** Partial points: Fg=Fb

+ **2 pts** Partial points: correct final expression for J. J=density*g/B0

+ **2 pts** Partial points: Direction of current: from west to east

+ **0 pts** no point

QUESTION 11

11 7a **6 / 6**

✓ **+ 6 pts** **Correct**

+ **2 pts** Partial point: length equation

+ **2 pts** Partial point: object distance+image distance=D

+ **0 pts** no point

QUESTION 12

12 7b **4 / 4**

✓ **+ 4 pts** **Correct**

+ **0 pts** no point

QUESTION 13

13 8a **5 / 5**

✓ **+ 5 pts** **Correct**

+ **1 pts** Partial point: q(t)=Qcos(omega*t)

+ **1 pts** Partial point: omega=1/sqrt(LC)

+ **1 pts** Partial point: Ec(t)=q(t)^2/2C

+ **1 pts** Partial point: Ec(T)=(1/4)*Ec(0)

+ **0 pts** no point

QUESTION 14

14 8b **5 / 5**

✓ **+ 5 pts** **Correct**

+ **4 pts** All correct but use wrong value of inductor(should be 20*10^-3 H) or make some mistake when plug in values and calculate.

+ **1 pts** nice try

+ **0 pts** no point

QUESTION 15

15 9a **0 / 4**

+ **4 pts** Correct

+ **2 pts** reason is not so correct. Key point is that first coil needs AC or time-varying current to produce emf.

✓ **+ 0 pts** **no point**

QUESTION 16

16 9b **5 / 5**

✓ **+ 5 pts** **above and correct reason. (derive from VL>VC)**

+ **2 pts** driven above resonant but reason is not correct.

+ **0 pts** no point

QUESTION 17

17 9c **4 / 4**

✓ **+ 2 pts** **Yes**

✓ **+ 2 pts** **correct direction. (CCW if look from above)**

+ **0 pts** no points

QUESTION 18

18 9d **2.5 / 5**

✓ **+ 2.5 pts** **Speed of light is measured to be the same for all inertial abservers**

+ **2.5 pts** The laws of physics are the same for all inertial abservers

- **1 pts** Do not mention inertial observers when saying the laws of physics are the same.

+ **0 pts** no point

QUESTION 19

19 extra **6 / 6**

✓ **+ 6 pts** **correct**

+ **2 pts** Partial points: Length contraction for x direction(direction of velocity)

+ **2 pts** Partial points: length does not change in y direction

+ **0 pts** no point

Physics 1C – Spring 2018: Final Exam

Name Anshul Kale UID 004929892

This exam is closed book and closed notes. Electronics are not permitted, except for one calculator. Please show your full solution in the boxes provided (where the scanners can pick them up). Answers recorded outside of boxes will not be graded. Your solutions will be graded on correctness and coherence; results given with no details will receive zero credit. There is additional scratch paper attached so you can collect your thoughts first. Academic dishonesty is reported to the Office of the Dean of Students. Good Luck!

Problem 1. Parallel rays of monochromatic light with wavelength 568 nm illuminate two identical slits and produce an interference pattern on a screen that is 75.0 cm from the slits. The centers of the slits are 0.640 mm apart and the width of each slit is 0.434 mm. If the intensity at the center of the central maximum is 3.20×10^4 W/m^2, what is the intensity at a point on the screen that is 0.840 mm from the center of the central maximum? (10 pts)

$I_0 = 3.20 \cdot 10^4$; $I = I_0 \cos^2\left(\frac{\phi}{2}\right)\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$; $d = 0.640 \cdot 10^{-3}$

$\tan\theta = \frac{y}{R}$

$R = 75 \cdot 10^{-2}$ m, $y = 0.840 \cdot 10^{-3}$ m

$I = I_0 \cos^2\left(\frac{2\pi d \sin\theta}{2\lambda}\right)\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$

$\sin\theta \approx \tan\theta \approx \theta$; θ is small

$\tan\theta = \frac{0.84 \cdot 10^{-3}}{75 \cdot 10^{-2}}$

$I = I_0 \cos^2\left(\frac{\pi \cdot 0.64 \cdot 10^{-3} \cdot 0.00112}{568 \cdot 10^{-9}}\right)\left(\frac{\sin(2.69)}{2.69}\right)^2$

$= I_0 \cdot 0.462 \cdot 0.026$

$\boxed{I = 389 \text{ W/m}^2}$

$\theta = 0.00112$ rad

$\beta = \frac{2\pi a \sin\theta}{\lambda}$

$= \frac{2\pi \cdot 0.434 \cdot 10^{-3} \cdot 0.00112}{568 \cdot 10^{-9}}$

$= 5.38$ rad.

$\beta/2 = 2.69$ rad.

Problem 2. Consider Young's double slit experiment.

a. Light passes through two slits separated by a distance d=0.8mm, and the observing plane is 1.6m away from the two slits. If the distance between the two consecutive maxima is 5mm, what is the wavelength of the light? (5 pts)

$Y_m = R \cdot 1 \cdot \frac{\lambda}{d} \rightarrow 5 \cdot 10^{-3} = 1.6 \cdot 1 \cdot \frac{\lambda}{0.8 \cdot 10^{-3}}$

$= m_2 - m_1 = 2 - 1 = 1$

$\boxed{\lambda = 0.0000025 \text{ m.}}$

b. Now consider the same setup, but instead of sending through light of the wavelength calculated above, you send in light containing two wavelengths, 450nm and 600nm. What is the least order at which a maximum of one wavelength will fall exactly on a minimum of the other? (You should state the order for both wavelengths that lead to this condition.) (5 pts)

$Y_{max} = R \cdot m \cdot \frac{\lambda}{d}$ $\quad Y_{min} = R \cdot (n + 0.5) \cdot \frac{\lambda}{d}$

$\cancel{R} \cdot m \cdot \frac{450\text{nm}}{\cancel{d}} = \cancel{R} \cdot (n + 0.5) \cdot \frac{600\text{nm}}{\cancel{d}}$

$m \cdot 450 = 600n + 300$

$m = 2 \quad n = 1$

The 450 nm wavelength's maximum at order $\boxed{m=2}$ coincides with the 600 nm's minimum at order $n=1$.

Problem 3. A reference frame S' passes a second reference frame S with a velocity of 0.6c in the X direction. Clocks are adjusted in the two frames so that when t = t'= 0 the origins of the two reference frames coincide.

a. An event occurs in S with space-time coordinates $x_1 = 50$m, $t_1 = 2.0 \times 10^{-7}s$. A second event occurs at $x_2 = 10$m, $t_2 = 3.0 \times 10^{-7}$s. What are the coordinates of these two events in S'? (I.e., x'_1, t'_1, x'_2, t'_2). (4 pts)

$u = 0.6c$; $x' = (x - ut)\alpha$; $t' = (t - ux/c^2)\alpha$ $\alpha = \frac{1}{\sqrt{1-0.6^2}} = 1.25$

Event 1: $x_1' = (50 - 0.6c \cdot 2 \cdot 10^{-7}) \cdot 1.25 = \boxed{17.5 \text{ m}}$

$t_1' = (2 \cdot 10^{-7} - 0.6\cancel{c} \cdot 50/c^{\cancel{2}}) \cdot 1.25 = (2 \cdot 10^{-7} - 0.6 \cdot 50/c) \cdot 1.25$

$= \boxed{1.25 \cdot 10^{-7} \text{ s}}$

$x_2' = (10 - 0.6c \cdot 3 \cdot 10^{-7}) \cdot 1.25 = \boxed{-55 \text{ m}}$

$t_2' = (3 \cdot 10^{-7} - \frac{0.6\cancel{c} \cdot 10}{c\cancel{c}}) \cdot 1.25 = \boxed{3.5 \cdot 10^{-7} \text{ s}}$

b. What is the time interval between the events as measured in S (Δt) and S'($\Delta t'$)? Is this difference an example solely of time dilation (I.e., are the two time intervals related by a factor of γ)? Give a short reason for your conclusion. (6 pts)

$S(\Delta t) = \boxed{1 \cdot 10^{-7} \text{ s}} = (3-2) \cdot 10^{-7}$ s.

$S'(\Delta t) = (3.5 - 1.25) \cdot 10^{-7}$ s $= \boxed{2.25 \cdot 10^{-7} \text{ s}}$

This example is not solely an example of time dilation as $\frac{2.25 \cdot 10^{-7} \text{ s}}{1 \cdot 10^{-7}} = 2.25$ which is not a factor of 1.25. Instead, they are related using the Lorentz coordinate transformation which is based on position differences in different frames of reference which is due to length contraction

Problem 4. Suppose we have two spaceships A and B, and the rest length L_A of A is twice the rest length L_B of B. If B is moving at $v_B = c/2$ relative to an observer at rest and A is moving at a speed v_A that makes A appear the same length as B to the same observer at rest, how fast is A moving? (12 pts)

Observer

$$\cancel{L_A}\sqrt{1-\left(\frac{v_A}{c}\right)^2} = \frac{\cancel{L_A}}{2}\sqrt{1-\left(\frac{v_B}{c}\right)^2} = \frac{L_A}{2}\sqrt{1-\frac{v_B^2}{c^2}}$$

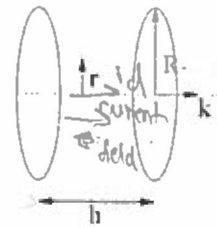
$$\sqrt{1-\frac{v_A^2}{c^2}} = \frac{1}{2}\sqrt{1-\frac{v_B^2}{c^2}} \to 1-\frac{v_A^2}{c^2} = \frac{1}{4}\left(1-\frac{v_B^2}{c^2}\right)$$

$$1-\frac{v_A^2}{c^2} = \frac{1}{4}\left(1-\frac{\left(\frac{c}{2}\right)^2}{c^2}\right) = \frac{1}{4}\left(1-\frac{\frac{c^2}{4}}{c^2}\right) = \frac{1}{4}\left(1-\frac{1}{4}\right) = \frac{3}{16}$$

$$-\frac{v_A^2}{c^2} = -\frac{13}{16}; \quad \frac{v_A^2}{c^2} = \frac{13}{16}; \quad v_A = \sqrt{\frac{13c^2}{16}} = c\sqrt{\frac{13}{16}} =$$

$$\boxed{\frac{c\sqrt{13}}{4}}$$

Problem 5. A cylindrical region of space of radius R and length h contains a non-uniform time-varying electric field $\vec{E}$ give by $\vec{E} = E_0(1 - \frac{r}{R}sin(\omega t))\hat{k}$ where $\hat{k}$ is the unit vector along the axis of the cylinder, E_0 is a positive constant and r is the radial distance from the axis of the cylindrical region.



And r=R

a. Find the displacement current, i_d in the cylindrical region (r<R). (3 pts)

$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$ $\quad \frac{d\vec{E}}{dt} = E_0\left(\frac{+r}{R}\cos(\omega t)\cdot\omega\right) = \frac{-rE_0\omega\cos(\omega t)}{R}$

$i_d = \varepsilon_0 \int_0^r -\frac{r}{R}E_0\omega\cos(\omega t)\cdot 2\pi r\,dr = \frac{2\pi\varepsilon_0 E_0\omega\cos(\omega t)}{R}\int_0^r -r^2\,dr$

$= \frac{2\pi\varepsilon_0 E_0\omega\cos(\omega t)}{R}\left(\frac{-r^3}{3}\right)$

$I_d \to$ magnitude $= \boxed{\frac{2\pi\varepsilon_0 E_0\omega\cos(\omega t)\,r^3}{3R}}$

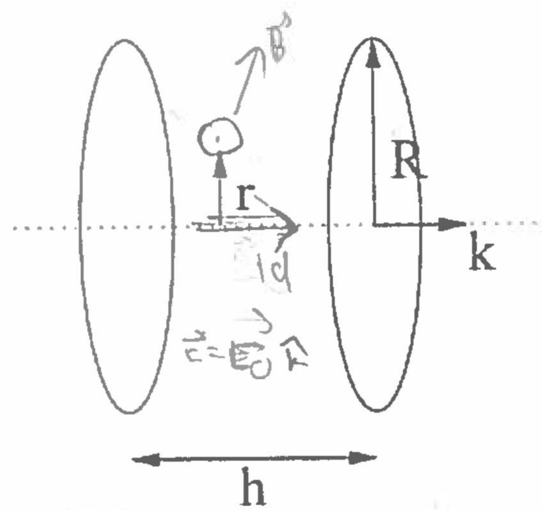
b. Find the magnetic field associated with the electric field in the cylindrical region. (Hint: choose coordinates which make your life easier.) (5 pts)

$B \cdot 2\pi r = \mu_0(\cancel{I_{encl}} + I_d)$

amperes law $\leftarrow$ $B = \frac{\mu_0 i_d}{2\pi r}$ $\rightarrow$ $|\vec{B}| = \frac{\varepsilon_0 E_0\omega\cos(\omega t)\,r^2\mu_0}{3R}$

c. Indicate/sketch the direction of $\vec{B}$ at the radial distance r at t=0 in the figure. (2 pts)

At $t=0$,
$\vec{E}=-E_0\hat{k}$

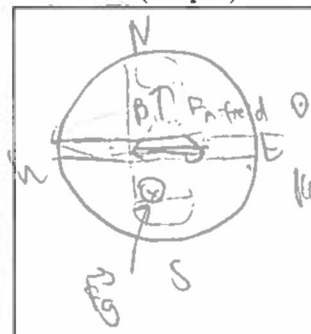


I found this using rH rule., with thumb in direction of displacement current. Thus, $\vec{B}$ points out of page.. Assuming that E_0 is positive. This is simply a model of Ampere's law.

Problem 6. A copper wire of diameter d carries a current density **J** at the earths equator where the earths magnetic field is horizontal, points north, and has magnitude B_0. The wire lies in a plane that is parallel to the surface of the earth and is oriented in the east-west direction. The density of copper is ρ_{cu}. Find an expression for the magnitude of **J** and the direction it must flow in order to levitate the wire. (10 pts)

current/area

$I = J \cdot \pi \left(\frac{d}{2}\right)^2 = J \cdot \pi \frac{d^2}{4} = J \frac{\pi d^2}{4}$ $\quad r = \frac{d}{2}$



$\rho_{cu} = \frac{mass}{volume}$

mass of wire $= \rho_{cu} \cdot \frac{\pi d^2}{4} h \rightarrow$ height of wire

$F_g = F_{\text{magnetic field}} \rightarrow$

$F_g = mg \qquad F_{\text{magnetic field}} = I h B_0$

$$\rho_{cu} \cdot \frac{\cancel{\pi d^2}\cancel{h}}{\cancel{4}} g = J \frac{\cancel{\pi d^2}}{\cancel{4}} \cdot \cancel{h} \cdot B_0$$

$\rightarrow g = 9.8\ m/s^2$

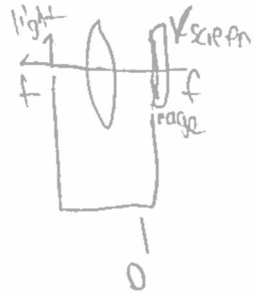
$$\boxed{|\vec{J}| = \frac{\rho_{cu} \cdot g}{B_0}}$$

It must flow in the $\boxed{\text{east}}$ $\boxed{\text{direction}}$ to levitate the wire, using the right hand rule

east × north = out of page

Problem 7. A converging lens with focal length f is placed between a light source and a screen. The distance between the light source and the screen is D. (can include D & f)

a. Find the two locations of the converging lens such that the image is formed at the screen. Give answers in terms of D. (6 pts) & f



$i + o = D$; $i = D - o$

$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$; $\frac{1}{f} = \frac{1}{D-o} + \frac{1}{o}$

$\frac{1}{f} = \frac{o + D - o}{o(D-o)} = \frac{D}{oD - o^2}$; $oD - o^2 = Df$

$o^2 = oD - Df$

$o^2 - oD + Df = 0$; $o = \frac{D \pm \sqrt{D^2 - 4Df}}{2}$

$$\boxed{o = \frac{D + \sqrt{D^2 - 4Df}}{2}, \quad \frac{D - \sqrt{D^2 - 4Df}}{2}}$$

b. What happens if D < 4f ? Can you still get an image on the screen? (4 pts)

If $D = 4f$, $o = \frac{4f \pm \sqrt{16f^2 - 4 \cdot 4f \cdot f}}{2} = \frac{4f \pm \sqrt{0}}{2} = \frac{4f \pm 0}{2}$

If you go lower (than 4f) and $D < 4f$, then the term under square root is negative. Thus, we get imaginary values for o so we can't get an image on the screen. For example, if $D = 3f < 4f$

$o = \frac{4f \pm \sqrt{9f^2 - 4 \cdot 3f \cdot f}}{2} = \frac{4f \pm \sqrt{9f^2 - 12f^2}}{2} = \frac{4f \pm \sqrt{-3f^2}}{2}$

This is impossible since we can't have complex numbers for our object distance. Thus, we $\boxed{\text{can't get an image on the screen}}$ at this distance.

Problem 8.

1. Initially, the capacitor in a series LC circuit is charged. A switch is closed, allowing the capacitor to discharge, and after time T the energy stored in the capacitor is one fourth its initial value. Determine L if C and T are known (in terms of C and T). (5 pts)

$i = -\omega Q \sin(\omega t + \phi) \to$ LC Circuit... $\quad q_{later} = \frac{1}{2}Q$

$C_{iE} = \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{2C} \qquad C_{fe} = \frac{1}{8}\frac{Q^2}{C} = \frac{Q^2}{8C} = \frac{1}{2}\frac{(q_{later})^2}{C}$

$\phi = 0$

$q = Q\cos(\omega t + \phi)$ at $t = 0$ $q = Q$

$\frac{1}{2}Q = Q\cos(\omega T)$; $\frac{1}{2} = \cos(\omega T)$; $\omega T = \frac{\pi}{3}$; $\omega = \frac{\pi}{3T}$

$i = -\frac{\pi}{3T} Q \sin\left(\frac{\pi}{3}\right) = -\frac{\pi}{3T} Q \cdot \frac{\sqrt{3}}{2} = \frac{-\pi Q\sqrt{3}}{6T}$

$16\overline{)144}$

$\frac{Q^2}{2C} - \frac{Q^2}{8C} = \frac{1}{2}Li^2$

$\frac{12Q^2}{16C} = Li^2 = L \cdot \pi^2 \cdot Q^2 \cdot 3 / 36T^2$

$L = \frac{12}{16C} \cdot \frac{12T^2}{\pi^2} = \boxed{\frac{9T^2}{C\pi^2}}$

2. An LC circuit consists of a 20.0-mH inductor and a 0.500-F capacitor. If the maximum instantaneous current is 0.100 A, what is the greatest potential difference across the capacitor? (5 pts)

Energy is conserved...

$E_{tot} = \frac{1}{2}LI_{max}^2 = \frac{1}{2}CV_{max}^2$

$\cancel{\frac{1}{2}} \cdot 20 \cdot 10^{-3} \cdot 0.1^2 = \cancel{\frac{1}{2}} \cdot 0.5 \cdot V_{max}^2$

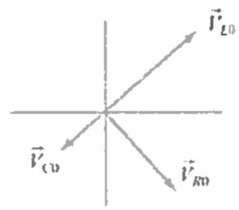
$\boxed{V_{max} = 0.02\text{ V}}$

Problem 9. Short conceptual problems

a. Can a battery be used as the primary voltage source for a transformer? Explain using a sentence or two. (4 pts)

Yes, a battery can be used as a primary voltage source for a transformer. This is because the battery provides the input voltage that may be transformed into an output voltage using the concept of inductance. However on the output side, no battery is needed, since voltage & current is already present through the output circuit, & this current can be used to power electric devices.

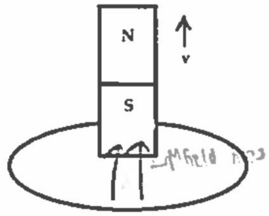
b. Consider the phasor diagram shown below for a driven RLC circuit. Is the driving frequency above or below the resonant frequency? Briefly explain. (5 pts)



In this diagram, $X_L > X_C$ by observation since $IX_L > IX_C$.

$\omega L > \frac{1}{\omega C}$; $\omega^2 > \frac{1}{LC}$; $\omega > \frac{1}{\sqrt{LC}}$, Thus, the driving frequency is above the resonant frequency $= \frac{1}{\sqrt{LC}}$.

c. Consider a bar magnet moving with respect to a circular loop of wire as shown below. Will a current be induced in the wire? If so, in which direction as viewed from above? (4 pts)



If the magnet moves upward, the magnetic flux (at current loop) in the upward direction decreases as the magnet is moving farther from the loop, so the field line strength decreases.
With Lenz's law, we know the EMF oppose this decrease in upward flux with an increase in upward flux. Using Right hand rule with thumb up, fingers curl curl CCW.
Thus, current is induced in CCW (counter-clockwise) direction.

d. State the two postulates Einstein used to formulate the special theory of relativity. (It is possible to state these concisely in only one sentence. This is not how we did it in class, and if you choose to do so, be careful.) (6 pts)

① One postulate is that time is not absolute, so the position, velocity, & length, time of certain objects may appears to be different in different frames. No one measurement is "correct". However, the (length/proper) is called the proper length in the frame of the object being measured.
② Another postulate is that the speed of light measured in any inertial reference frame is always the same. This was needed to calculate the γ factor.

Extra Credit (6 pts)

Suppose a sailboat moves at speed v relative to an observer on the shore. The sailboat has a mast of length L that is anchored near the front of the boat and makes an angle (when the boat is at rest) of θ with the deck of the boat. What angle will the observer on the shore see?

$$\sin\theta' = \frac{h}{L}$$

$$\alpha = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_{person} = L\sqrt{1-\frac{v^2}{c^2}}$$

$$h_{person} = h\sqrt{1-\frac{v^2}{c^2}}$$

$$\tan\theta = \frac{h}{L\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{h}{L\sqrt{1-\frac{v^2}{c^2}}}\right)}$$

Scratch paper