Physics 1C
Fall 2018
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Midterm Exam

Problem 1: 20

Problem 2: 25

Problem 3: 24

Problem 4: 25

Total: 94 /100

To get credit for an answer you must show your work!

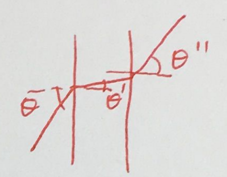
Problem 1: [25 points]

Name

10 a) [15 pts] A light ray is incident on the longer side of a rectangular block of glass at angle θ from the normal. The block has index of refraction $n > 1$, width W, and length $L > W$. Find the path of the ray through the block and out the other side (draw your answer and compute all relevant angles). Is it possible to find a value of n and θ such that no light makes it all the way out the other side (i.e. it is reflected back)? Why or why not?

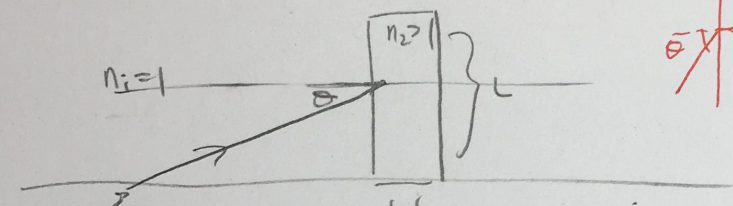
5 b) [5 pts] Suppose instead there is a small light bulb inside the block of glass. Draw a ray diagram to show where the apparent image would be, as viewed by the eye outside the block of glass.

5 c) [5 pts] Now exchange the location of the eye and bulb so the bulb is outside the block and the observer is inside. Draw a ray diagram to show where the apparent image would be.



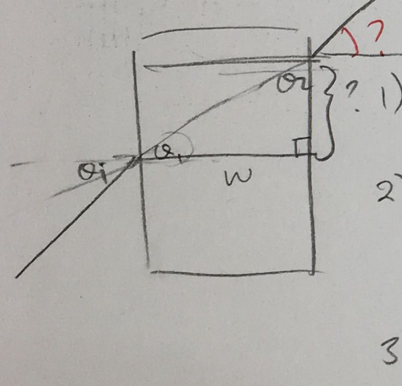
$\theta'' = ?$ −5

a)



$\sin\theta_1 = \frac{w}{hyp}$

$hyp = \frac{w}{\sin\theta_1}$



$\theta_i = \theta$

1) $n_i \sin\theta_i = n \sin\theta_1 \rightarrow \underline{\sin\theta = n\sin\theta_1}$

inside:

2) $\theta_2 + \theta_1 = 90°$

$\sin\theta_2 = \cos\theta_1$ $\qquad \theta_1 = \tan^{-1}\left(\frac{L?}{w}\right)$

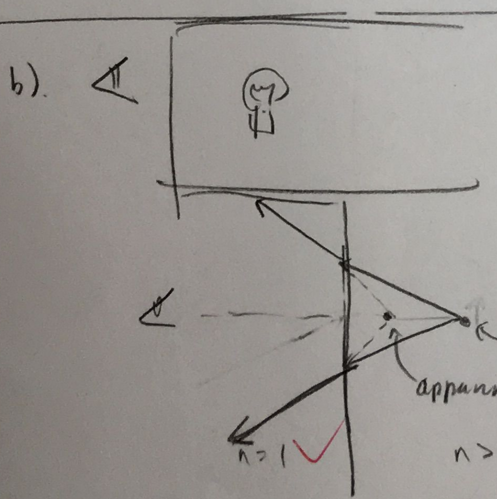
$n \sin\theta_2 = \sin\theta_0$

3) $\underline{n\cos\theta_1 = \sin\theta_0}$ $\qquad \theta_0 = \theta$ outgoing $\rightarrow \underline{\theta_0 = \sin^{-1}(n\cos\theta_1)}$

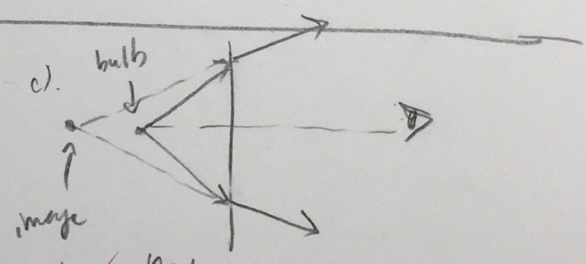
$n_1 < n_2$

Not possible because $n_{incident} < n_{glass}$. Total internal reflection can only happen if the n of the incident ray is greater than n_{glass}.

b).



c).



Problem 2: [25 points] Name

A circuit contains an inductor L, capacitor C, resistor R, and open switch in series. The capacitor initially carries change Q_0. The switch is closed. Applying Kirchhoff's loop rule, we have the relationship $Q/C + IR + L\frac{dI}{dt} = 0$ where Q is the charge on the capacitor.

14

a) [15 pts] Suppose the charge takes the form $Q(t) = Ae^{t/\tau}\cos(\omega t)$ for some values of τ, ω, A. If $R = 0$, find the value of ω, τ, A such that this is a valid solution to $Q/C + \cancel{IR} + L\frac{dI}{dt} = 0$ with $R = 0$.

5

b) [5 pts] The solution for ω with $R \neq 0$ is $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$. This solution is only valid for ω real, but at fixed L, ω becomes imaginary for large enough R or C. What do you expect to be the qualitative difference of $I(t)$ between the ω real and imaginary cases? How does this compare to the behavior in an LR circuit (or if you prefer, an RC circuit)?

RL LC

5

c) [5 pts] What is I_{rms} in the case of $R = 0$?

a) LC circuit.

$$\frac{q}{C} + L\frac{di}{dt} = 0 \qquad i = \frac{dq}{dt}$$

$$\underbrace{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0}_{\text{simple harmonic oscillator}} \rightarrow \frac{d^2q}{dt^2} = -\frac{1}{LC}q \rightarrow \boxed{\omega = \sqrt{\frac{1}{LC}}}$$

$$Q(t) = Q_0\cos(\omega t)$$

$$Ae^{t/T} = Q_0$$

$$\boxed{A = Q_0}, \quad \boxed{T = \infty} \quad \text{since } Ae^{t/\infty} = Ae^0 = Q_0$$

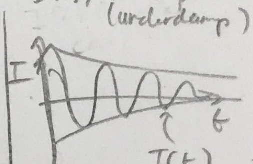
$$I(t) = -\omega Q_0 \sin(\omega t)$$

b) ω becomes imaginary when $\frac{1}{LC} - \frac{R^2}{4L^2} < 0$.

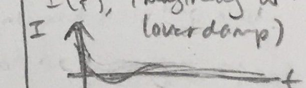
$\frac{1}{LC} < \frac{R^2}{4L^2}$. very large R or C will make this true.

For real ω, $I(t)$ will show underdamped behavior. If R is really large, $I(t)$ will have an imaginary ω and show overdamped behavior.

I(t), real w. (underdamp)



I(t), imaginary w: (overdamp)



In comparison, $I(t)$ in LC circuit oscillates in accordance to $I(t) = -\omega Q_0 \sin(\omega t)$, in which $\omega = \sqrt{\frac{1}{LC}}$ and is not imaginary. For LR, $I(t)$ is exponential decay.

c) $I_{rms} = \frac{I_0}{\sqrt{2}}$ $\qquad I(t) = -\omega Q_0 \sin(\omega t)$, where $I_0 = \omega Q_0$

$$I_{rms} = \boxed{\frac{\omega Q_0}{\sqrt{2}} = \sqrt{\frac{1}{LC}} \cdot \frac{Q_0}{\sqrt{2}}}$$

Problem 3: [25 points]

Name

8 **a) [8 pts]** Consider an object with some non-zero height (for example a rectangle or arrow) on the optical axis of a mirror. Draw four ray diagrams showing how to find the image for the cases of a concave or convex mirror, with the object inside or outside the focal point. State whether the image is inverted or not, and whether the image is real or virtual.

8 **b) [8 pts]** Do the same for a diverging and converging lens.

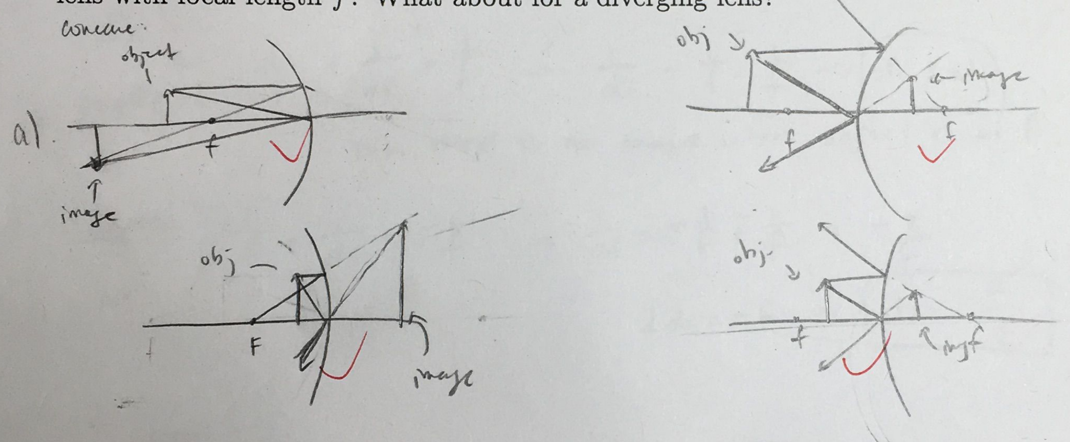
5 **c) [5 pts]** Using ray diagrams and/or the thin lens equation, briefly justify the statement that "solving the converging lens is equivalent to solving the converging mirror" and the same for a diverging lens/mirror.

1 **d) [2 pts]** Where is the image located when the object is placed at the focal point of a converging lens with focal length f? What about for a diverging lens?

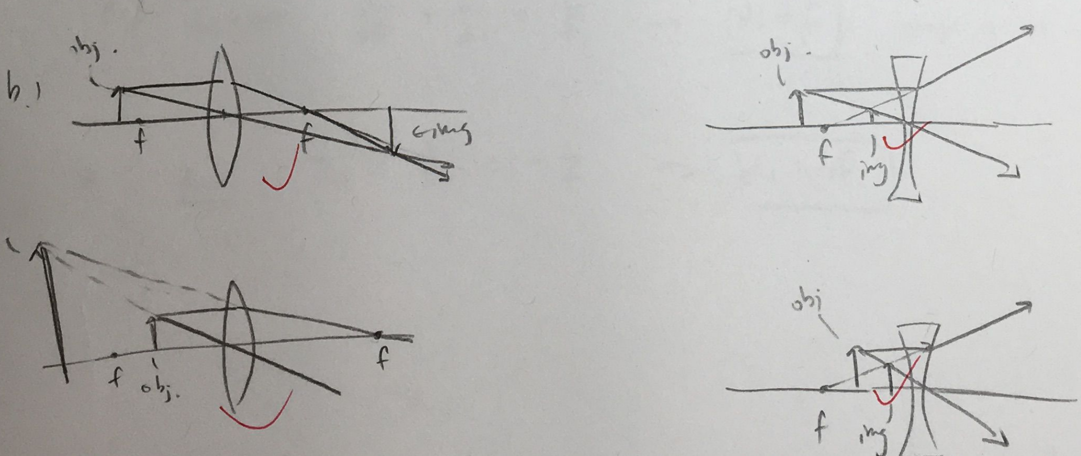
2 **e) [2 pts]** Where is the image located when the object is placed infinitely far from a converging lens with focal length f? What about for a diverging lens?

concave:

a).



b.)



c). $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$.

For converging lens/mirror, for an object the same distance d_o to the left, and for the same focal point f, d_i will be the same thing.

This is because for $d_o > f$, for lens the image is real (opposite side of lens) and $d_i > 0$.
For mirrors, the image is also real (same side as mirror) so $d_i > 0$ also.
The same applies for $d_o < f$, except for both lens and mirrors $d_i < 0$ since the image is now virtual. → Thus, d_o, d_i, and f are the same for both, so both equations are equivalent.

Name

c). The same applies for diverging lens/mirrors.

$$\frac{1}{d_o} + \frac{1}{d_i} = -\frac{1}{f} \text{ for diverging.}$$

For mirrors/lens with the same f, the image ~~is both~~ is virtual for both, and $d_i < 0$ for both lens and mirrors.

(image on same side as lens) (image on opposite side of mirror)

d).

converg. $d_o = f$ $\quad \frac{1}{f} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{f} \rightarrow \boxed{d_i = 0}$ $\quad d = \infty$

Thus, there is no image when object is at f.

diverge $\quad \frac{1}{f} + \frac{1}{d_i} = -\frac{1}{f} \rightarrow \frac{1}{d_i} = -\frac{1}{f} - \frac{1}{f} = -\frac{2}{f}$

$$2d_i = -f \rightarrow \boxed{d_i = -\frac{f}{2}}$$

e.) conv. $d_o = \infty$ $\quad \frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \boxed{d_i = f}$ image is at f.

div. $\quad \frac{1}{\infty} + \frac{1}{d_i} = -\frac{1}{f} \rightarrow \boxed{d_i = -f}$

Problem 4: [25 points]

Name

A plane electromagnetic wave is travelling in the z direction. The wavenumber is $k = 2\pi/\lambda = 2m^{-1}$. The magnetic field is found to point in the x direction and has amplitude $B = 1 \times 10^{-8}T$.

$$\text{Recall}: \quad c = 3 \times 10^8 m/s, \quad \mu_0 = (4\pi) \times 10^{-7} T \cdot m/A, \quad \epsilon_0 = 8.85 \times 10^{-12} C^2/N \cdot m^2$$

a) [5 pts] What is the angular frequency ω (recall $\omega = 2\pi f$)?

b) [5 pts] What is the direction and amplitude of the electric field?

c) [5 pts] What is the direction and magnitude of the *average* energy flux (energy flow per area per unit time)?

d) [10 pts] Consider a hypothetical plane electromagnetic wave which had a nonzero electric field as usual, but a vanishing magnetic field everywhere. Which of Maxwell's equations would this violate?

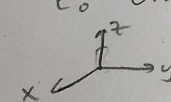
Note: Give numerical answers as appropriate, but don't worry about giving precise answers with multiple decimal places.

a). $\omega = 2\pi f$. $\quad c = \frac{\omega}{k} \quad \omega = ck = 3\times10^8 \cdot 2 = \boxed{6\times10^8 \frac{rad}{sec}}$ 5

b). $E_0 = cB_0 = 3\times10^8 \cdot 1\cdot10^{-8} = \boxed{3\ T\cdot\frac{m}{s}}$ $\quad$ $\boxed{\text{dir of } E: -\hat{j}}$

dir $= \vec{E}\times\vec{B}$

$\hat{z} = -\hat{j} \times \hat{i}$ $\quad$ 5



$\frac{m}{s}\cdot\frac{N}{A\cdot m}$

$\frac{N\cdot s}{m^2}$ kg $\quad \frac{J/C\cdot s}{m^2}$

$P = \int S\cdot dA$

$P = \frac{I}{A}$ $\quad I = \langle S \rangle = \frac{E_0 B_0}{2\mu_0} = .0119 \frac{W}{m^2}$

c) Φ_{avg}^{energy} $\quad \frac{\text{energy flow}}{A\cdot\text{time}}$ $\quad$ $\boxed{\text{dir of } \Phi = \text{dir of } \langle S\rangle = +\hat{z}}$ 5

$E = E_0\cos(kx - \omega t)$

$B = B_0\cos(kx-\omega t)$ $\quad \Phi = BA$ $\quad$ similar units as power

energy density: $u = \varepsilon_0 E^2$

d) Maxwell's.

1. $\nabla\cdot E = \int E\cdot dA = \frac{q_{enc}}{\varepsilon_0}$
2. $\nabla\cdot B = \int B\cdot dA = 0$
3. $\nabla\times E = \int E\cdot dl = -\frac{d\Phi_B}{dt}$
4. $\nabla\times B = \int B\cdot dl = \frac{1}{c^2}\frac{d\Phi_E}{dt}$

Would violate these

3. $\nabla\times B = \frac{1}{c^2}\frac{dE}{dt} \Rightarrow B \sim \frac{dE}{dt}, E \sim \frac{dB}{dt}$
4. $\nabla\times E = -\frac{dB}{dt}$

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if $E \neq 0$ can't have vanishing B-field everywhere.