William Johnston

Name

Physics 1C
UCLA
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Sivaramakrishnan

Midterm Exam

[redacted]

ID

Problem 1: 25

Problem 2: 15

Problem 3: 25

Problem 4: 25

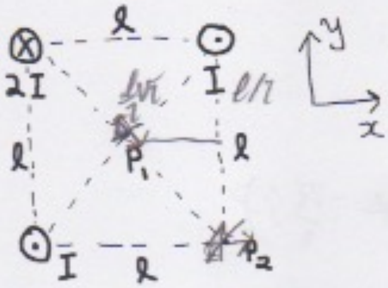
Total: 90 /100

Show your work! Answers are given credit according to justification provided.

Problem 1: [25 points]

5 a) [5pts] Use Ampere's law to calculate the magnitude of the magnetic field a perpendicular distance r from an infinitely-long straight wire carrying current I.

10 b) [10 pts] Now consider the following diagram, in which parallel infinitely-long straight wires are placed at three corners of a square of side length l. The wires opposite one another carry current I out of the page, and the third carries current $2I$ into the page. Find the magnetic field at point P_1, the center of the square.



10

c) [10 pts] Find the magnetic field at point P_2, the fourth corner of the square.

a.) ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{en}$

for long current carrying wire $\vec{B} \cdot d\vec{l} = B\,dl$

$\oint B\,dl = \mu_0 I_{en}$ because of symmetry, magnetism is the same everywhere on amperian loop

$B \cdot 2\pi r = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi r}$

b.) I will use the principle of superposition here.

There is an axis of symmetry between the two wires carrying current out page, and since they are equidistant from P_1, they cancel. Then there is only the wire into page, with magnitude $2I$

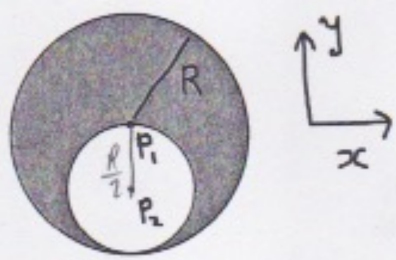
$|B| = \frac{\mu_0 2I}{2\pi \frac{l\sqrt{2}}{2}}$ $\quad r = \sqrt{2}\frac{l}{2}$

$|B| = \frac{\mu_0 I}{\pi \frac{l\sqrt{2}}{2}}$ $\quad \vec{B} = \frac{2\mu_0 I}{\pi l \sqrt{2}} \cdot \langle -\frac{\sqrt{2}}{2}\hat{i}, -\frac{\sqrt{2}}{2}\hat{j} \rangle$

Problem 2: [25 points] Name

a) [5 pts] Suppose a cylindrical wire of radius R has uniform current density with total current I. Find the magnitude of the magnetic field at a perpendicular distance $r < R$ from the center of the wire.

b) [10 pts] Now suppose the cylindrical wire has an off-center cylindrical hole as pictured below, but the current density in the remaining shaded region remains the same as in part a). The hole has diameter R and lies tangent to the circle. What is the magnitude of the magnetic field at point P_1, the center of the circle?



c) [10 pts] What is the magnitude of the field at point P_2, the center of the hole?

a.) $J = \frac{I}{A} = \frac{I}{\pi R^2}$

$J \cdot A$

$I_{enc} = \frac{I}{\pi R^2} \cdot \pi r^2 = \frac{I r^2}{R^2}$

$B \, 2\pi r = \mu_0 \frac{I r^2}{R^2}$

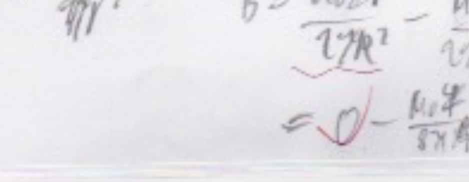
$B(r) = \frac{\mu_0 I r^2}{2\pi r R^2} = \frac{\mu_0 I r}{2\pi R^2}$

$= \frac{I}{\pi R^2} \cdot \pi \left(\frac{R}{2}\right)^2 = \frac{I \pi R^2}{\pi R^2 4} = \frac{I}{4}$

$I_{enc\,small} = -\frac{I}{4}$

$I_{enc} = I$ $\quad I_{small} =$

b.)



I_{enc} with hole $= \frac{3}{4} I$

$J = \frac{I}{\pi r^2}$

$\vec{B} = \frac{\mu_0 I r}{2\pi R^2} - \frac{\mu_0 \frac{I}{4}}{2\pi r}$

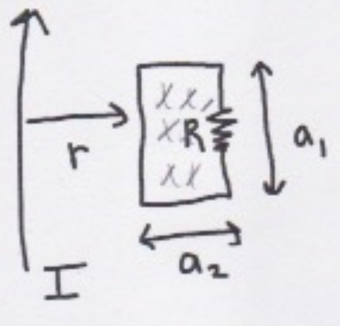
$= 0 - \frac{\mu_0 I}{8\pi R} = -\frac{\mu_0 I}{4\pi R} \hat{i}$

$|B| = \frac{\mu_0 I}{4\pi R}$

Will Fehrnstrom

Problem 3: [25 points] **Name**

A infinite straight wire carries current I. A rectangular loop is placed a distance r from the wire. In this problem, ignore any self-inductance effects (if you don't know what these are, don't worry, we haven't learnt this yet).



a) [**10 pts**] Suppose that $a_1 = a_2 = a$. What is the magnetic flux through the loop?

b) [**10 pts**] Suppose now that the current in the straight wire is time dependent, $I = I(t) = I_0 e^{-bt}$, where $b > 0$. If the loop has resistance R, what current will flow through the loop and in which direction?

c) [**5 pts**] In addition to the time-dependence of $I(t)$ above, suppose also that the loop's length changes in time according to $a_1(t) = af(t)$. What is the sign of $f'(t)$ (i.e. should the loop should grow or shrink) so that there is no induced current? Justify with a brief explanation or by finding $f'(t)$.

a.) $\Phi = B \cdot A$

$d\Phi = \frac{\mu_0 I}{2\pi r} dr \cdot a$

$\Phi = a \int_r^{a+r} \frac{\mu_0 I}{2\pi r} dr$.

$\Phi_{total} = \frac{a \mu_0 I}{2\pi} \ln \frac{a+r}{r}$

b.) $I(t) = I_0 e^{-bt}$

I decreasing with time

by lenz's law, current induced opposes decrease will flow clockwise

$\frac{di}{dt} = -b I_0 e^{-bt}$

$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{a\mu_0}{2\pi}(-bI_0 e^{-bt}) \ln \frac{(a+r)}{r}$

$I_{induced} = \frac{a\mu_0}{2\pi R}(-bI_0 e^{-bt}) \ln \frac{(a+r)}{r}$

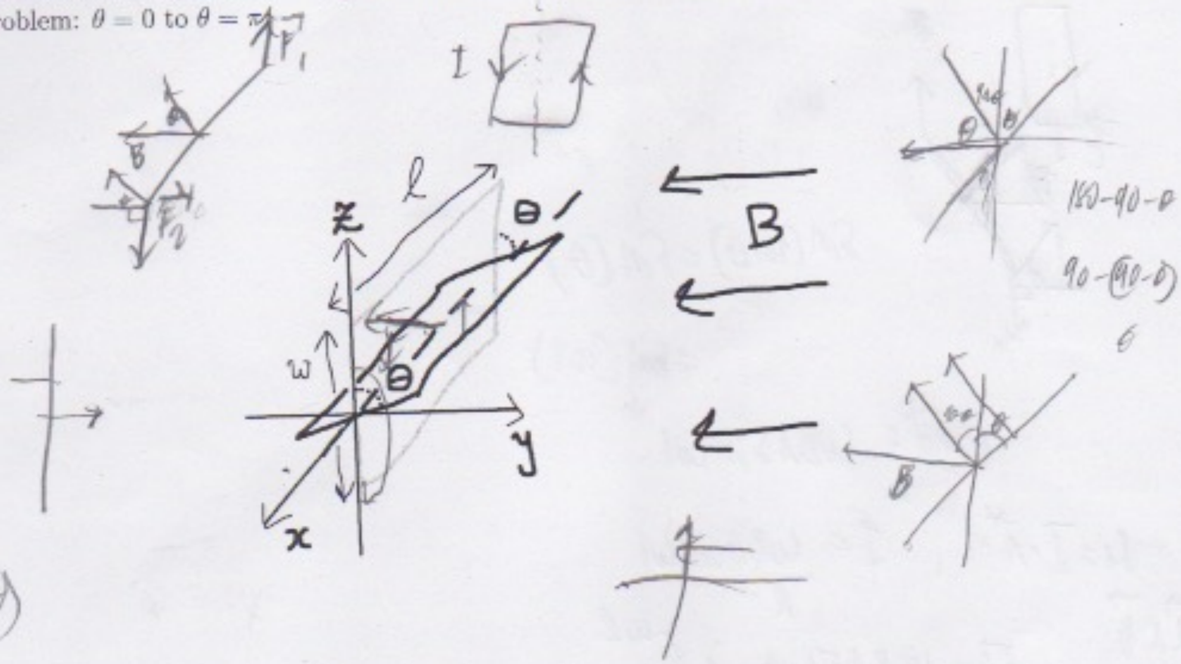
c.) on back

William Fehnstrom

Name

Problem 4: [25 points]

The rectangular loop of wire with length l and width w pictured below is rotating about its center in a constant magnetic field $\vec{B} = -B\hat{y}$. The angular speed of rotation is fixed by hand to be $\omega\frac{rad}{s}$ and the axis of rotation is aligned with the x-axis as pictured. At $t = 0$, the loop is oriented at $\theta = 0$, in the $x - z$ plane. We will only consider half a revolution of the wire in this problem: $\theta = 0$ to $\theta = \pi$.



$\cos(90+\theta)$

a) [10 pts] As a function of time t, what is the induced emf in the circuit?

b) [5 pts] Now suppose the wire has resistance R. What is the net force acting on the wire as a result of the external magnetic field as a function of t?

c) [10 pts] What is the net torque about the axis of rotation? To specify the direction, recall that $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ points from the axis of rotation to the point at which $\vec{F}$ acts.

a.) $\mathcal{E} = \oint(\vec{v}\times\vec{B})\cdot\vec{dl}$ strongest emf $\Phi = -B\ell w \cdot \cos(\omega t)$ $A = w\ell$

$\mathcal{E} = -\frac{d\Phi}{dt} = -B \cdot dA$ $\frac{d\Phi}{dt} = B\ell w\omega \sin(\omega t)$

$= -B\ell w \cdot \cos(t)$

$=$

$\mathcal{E} = \omega BA\sin(\omega t)$ where $A = w\ell$

omega

b.) The net external force applied to the wire is 0, since the loop is not translated, only rotated.

c.) ~~$\tau = Fd\sin\theta$~~ with $\mathcal{E} = vBL\frac{w}{2}$

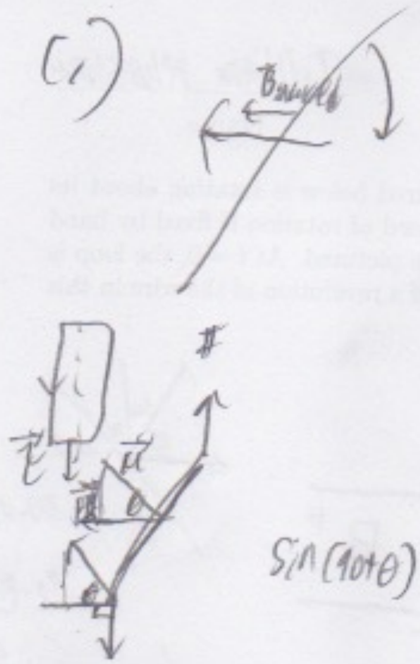
~~with $\omega\ell\frac{w}{2}\sin\omega t = \left(\frac{w}{2}\right)^2 \omega\ell\sin(\omega t)$~~ $v = rw$ 8

$d = \frac{w}{2}$ $\phi = 90+\theta$

~~net torque in $+\hat{?}$ direction~~

c.) on back

(c) decreasing flux through coil



$\sin(90+\theta) = \sin(\theta)$

$= \sin(\omega t)$

$emf = \omega BA \sin\omega t$

$\mu = \vec{I}\cdot\vec{A}$ $\quad I = \frac{\omega BA \sin\omega t}{R}$

$\tau = \vec{\mu}\times\vec{B}$

$\vec{\mu} = \frac{\omega BA \sin\omega t}{R} \cdot A$ we

$= \frac{\omega BA^2 \sin\omega t}{R}$ $\quad \vec{\mu}\times\vec{B} = \mu B \sin(\omega t)$

$\tau = \vec{\mu}\times\vec{B}$ net torque in $+\hat{i}$ direction

$= \frac{\omega (BA)^2 \sin(\omega t) \cdot \sin(\omega t)}{R}$ ✓

~~direct~~