Physics 1C
UCLA
Fall 2018
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Midterm Exam

Problem 1: 25

Problem 2: 15

Problem 3: 20

Problem 4: 17

Total: 77 /100

Show your work! Answers are given credit according to justification provided.

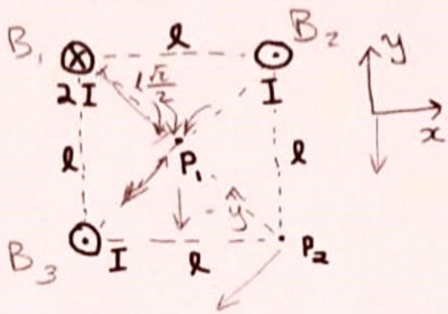
Problem 1: [25 points]

5

a) [5pts] Use Ampere's law to calculate the magnitude of the magnetic field a perpendicular distance r from an infinitely-long straight wire carrying current I.

10

b) [10 pts] Now consider the following diagram, in which parallel infinitely-long straight wires are placed at three corners of a square of side length l. The wires opposite one another carry current I out of the page, and the third carries current $2I$ into the page. Find the magnetic field at point P_1, the center of the square.



10

c) [10 pts] Find the magnetic field at point P_2, the fourth corner of the square.

$r = l\frac{\sqrt{2}}{2}$

a) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

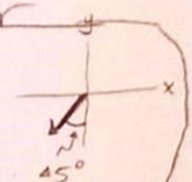
$B(2\pi r) = \mu_0 I$

$\boxed{B = \frac{\mu_0 I}{2\pi r}}$

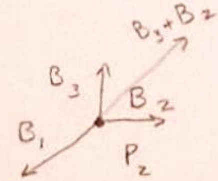
b) $|B_{net}| = B_1 + B_2 - B_3$

$= \frac{\mu_0 I}{\pi r} + \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi r}$

$\boxed{|B_{net}| = \frac{2\mu_0 I}{\sqrt{2}\pi l}}$ pointing in the angle →



c)



$\boxed{\Sigma B_{net} = \vec{0}}$ @ P_2

$|B_1|$ @ P_2 ... $\frac{\mu_0 I}{\pi l\sqrt{2}}$

$|B_2|$ @ P_2 ... $\frac{\mu_0 I}{2\pi l}$

$|B_3|$ @ P_2 = $\frac{\mu_0 I}{2\pi l}$

$|\vec{B}_3 + \vec{B}_2| = \sqrt{2\left(\frac{\mu_0 I}{2\pi l}\right)^2} = \frac{\sqrt{2}\mu_0 I}{2\pi l}$

$\therefore |B_1| = |\vec{B}_3{}^2 + \vec{B}_2|$

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

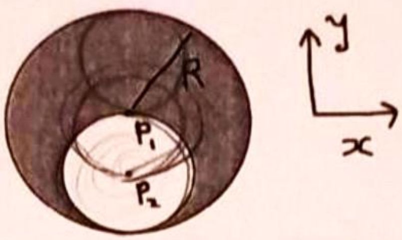
∴ cancels!

Problem 2: [25 points]

Name

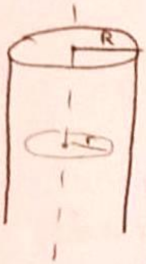
a) [5 pts] Suppose a cylindrical wire of radius R has uniform current density with total current I. Find the magnitude of the magnetic field at a perpendicular distance $r < R$ from the center of the wire.

b) [10 pts] Now suppose the cylindrical wire has an off-center cylindrical hole as pictured below, but the current density in the remaining shaded region remains the same as in part a). The hole has diameter R and lies tangent to the circle. What is the magnitude of the magnetic field at point P_1, the center of the circle?



c) [10 pts] What is the magnitude of the field at point P_2, the center of the hole?

a)



$\oint B \cdot d\vec{l} = \mu_0 I_{enc}$

$B(2\pi r) = \mu_0 \cdot \frac{\pi r^2}{\pi R^2} I \rightarrow B = \frac{\mu_0 r^{\not{2}} I}{2\pi \not{r} R}$

$$|B| = \frac{\mu_0 r I}{2\pi R^2}$$ ✓

b)

← omperian loop

$\oint B \cdot d\vec{l} = \mu_0 I_{enc}$

$B\left(\not{2}\pi \frac{R}{\not{2}}\right) = \mu_0 \cdot \frac{\pi\left(\frac{R}{2}\right)^2}{\pi R^2} I = \mu_0 \cdot \frac{I}{4}$ ✓

$$|B| = \frac{\mu_0 I}{4\pi R}$$

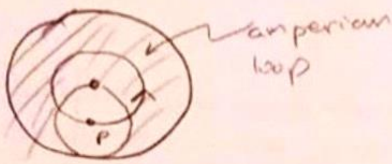
Name

c)

$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I' d\vec{l} \times \hat{r}}{r^2}$ → always perpendicular

$B = \int dB = \frac{\mu_0 I'}{4\pi r^2} \int_0^{2\pi r} dl = \frac{\mu_0 I'}{2r} \rightarrow \frac{\mu_0 I'}{2\left(\frac{R}{2}\right)}$

$B = \frac{\mu_0 I}{R}$



$\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

For solid wire w/ $r < R$: $B = \frac{\mu_0 r I'}{2\pi R^2}$

let $r = \frac{R}{2}$, and $I' = \frac{I}{2}$

I_{enc} is halved... → current is cut in half

$\therefore B = \frac{\mu_0}{2\pi R^2} \cdot \frac{R}{2} \cdot \frac{I}{2}$

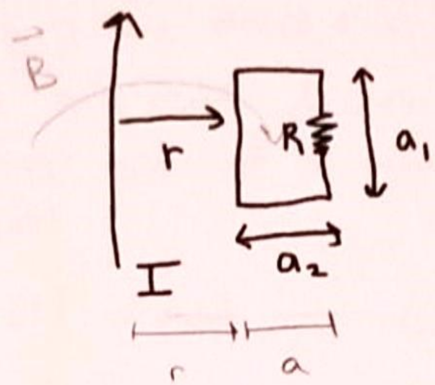
$$\boxed{B = \frac{\mu_0 I}{8\pi R}}$$

✗.

Problem 3: [25 points]

Name

A infinite straight wire carries current I. A rectangular loop is placed a distance r from the wire. In this problem, ignore any self-inductance effects (if you don't know what these are, don't worry, we haven't learnt this yet).



a) [10 pts] Suppose that $a_1 = a_2 = a$. What is the magnetic flux through the loop?

b) [10 pts] Suppose now that the current in the straight wire is time dependent, $I = I(t) = I_0 e^{-bt}$, where $b > 0$. If the loop has resistance R, what current will flow through the loop and in which direction?

c) [5 pts] In addition to the time-dependence of $I(t)$ above, suppose also that the loop's length changes in time according to $a_1(t) = af(t)$. What is the sign of $f'(t)$ (i.e. should the loop should grow or shrink) so that there is no induced current? Justify with a brief explanation or by finding $f'(t)$.

a) B of infinite wire: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = a\int_r^{r+a} B\,dr' = a\int_r^{r+a} \frac{\mu_0 I}{2\pi r'}dr' = \frac{a\mu_0 I}{2\pi} \ln r' \Big|_r^{r+a}$$

$$= \frac{a\mu_0 I}{2\pi}\left(\ln(r+a) - \ln(r)\right) = \boxed{\ln\left(\frac{r+a}{r}\right)\cdot\frac{a\mu_0 I}{2\pi}}$$

b)
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\ln\left(\frac{r+a}{r}\right)\cdot\frac{a\mu_0 I_0 e^{-bt}}{2\pi}\right) = -\ln\left(\frac{r+a}{r}\right)\cdot\frac{a\mu_0 I_0}{2\pi}\cdot\left(-be^{-bt}\right)$$

$$= \ln\left(\frac{r+a}{r}\right)\cdot\frac{ab\mu_0 I_0}{2\pi}\cdot e^{-bt}$$

I gets smaller w/ time, B induced should point into page

$$\boxed{I = \frac{\mathcal{E}}{R} = \ln\left(\frac{r+a}{r}\right)\cdot\frac{ab\mu_0 I_0}{2\pi R}\cdot e^{-bt},\ \text{clockwise}}$$

[redacted]

c) Want flux to stay constant, since emf is induced only when there's a change in flux. Since current decreases with time, the loop should shrink to keep the flux constant.

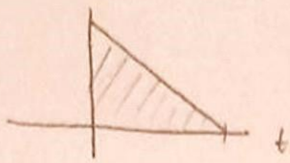
I think $f'(t)$ should be negative.

[redacted]

Name

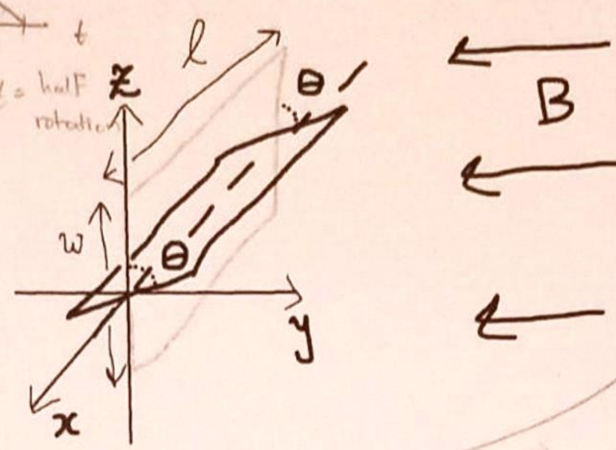
Problem 4: [25 points]

The rectangular loop of wire with length l and width w pictured below is rotating about its center in a constant magnetic field $\vec{B} = -B\hat{y}$. The angular speed of rotation is fixed by hand to be $\omega \frac{rad}{s}$ and the axis of rotation is aligned with the x-axis as pictured. At $t = 0$, the loop is oriented at $\theta = 0$, in the $x - z$ plane. We will only consider half a revolution of the wire in this problem: $\theta = 0$ to $\theta = \pi$.



$A_0 = \frac{lw}{2}$ t=0

$A_f = 0$ t = half rotation



$A = \left|\frac{lw}{2}\cos t\right|$

$A' = \left|\frac{lw}{2}\sin t\right|$

7 **a)** [10 pts] As a function of time t, what is the induced emf in the circuit?

5 **b)** [5 pts] Now suppose the wire has resistance R. What is the net force acting on the wire as a result of the external magnetic field as a function of t?

5 **c)** [10 pts] What is the net torque about the axis of rotation? To specify the direction, recall that $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ points from the axis of rotation to the point at which $\vec{F}$ acts.

a) $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}B(A(t))$

see above for eq. for A $= \boxed{-BA'(t)} = \frac{Blw}{2}|\sin t|$?

b) $\Sigma F = \boxed{0N}$, since it's a closed loop

c) $\tau = Iwl\sin\phi = \boxed{\frac{-BA'(t)}{R} \cdot wl\sin(90° - \theta)} = \frac{Blw|\sin t|}{2R} \cdot wl\sin(90° - \theta)$

$I = \frac{\mathcal{E}}{R}$

=