Physics 1C
UCLA
Fall 2018
Sivaramakrishnan

Midterm Exam

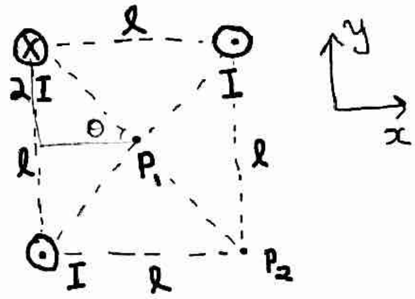
Problem 1:	18
Problem 2:	16
Problem 3:	25
Problem 4:	21
Total:	80 /100

Show your work! Answers are given credit according to justification provided.

Problem 1: [25 points]

5 **a) [5pts]** Use Ampere's law to calculate the magnitude of the magnetic field a perpendicular distance r from an infinitely-long straight wire carrying current I.

10 **b) [10 pts]** Now consider the following diagram, in which parallel infinitely-long straight wires are placed at three corners of a square of side length l. The wires opposite one another carry current I out of the page, and the third carries current $2I$ into the page. Find the magnetic field at point P_1, the center of the square.



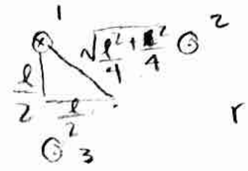
3 **c) [10 pts]** Find the magnetic field at point P_2, the fourth corner of the square.

✓1 a) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$ $\quad (I_{encl} = I)$

$B\,2\pi r = \mu_0 I$

$$\boxed{|\vec{B}| = \frac{\mu_0 I}{2\pi r}}$$

✓ b) $\vec{B}_{P_1} = B_1 + B_2 + B_3$



$r = \sqrt{\frac{l^2}{4}+\frac{l^2}{4}}$

According to symmetry, B_2 and B_3 cancel. Thus,

$\vec{B}_{P_1} = \vec{B}_1$

$$|\vec{B}_{P_1}| = \frac{\mu_0 I}{2\pi\sqrt{\frac{l^2}{4}+\frac{l^2}{4}}} \longrightarrow \boxed{\vec{B}_{P_1} = -\frac{\mu_0 I}{\pi l}\hat{i} - \frac{\mu_0 I}{\pi l}\hat{j}}$$

3 c) Again, B_2 and B_3 cancel. Thus,

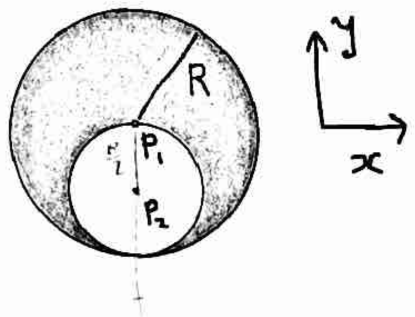
$\vec{B}_{P_2} = \vec{B}_1$

$$\vec{B}_{P_2} = -\frac{\mu_0 I}{2\pi l}\hat{i} - \frac{\mu_0 I}{2\pi l}\hat{j}$$

Problem 2: [25 points] **Name**

a) [5 pts] Suppose a cylindrical wire of radius R has uniform current density with total current I. Find the magnitude of the magnetic field at a perpendicular distance $r < R$ from the center of the wire.

b) [10 pts] Now suppose the cylindrical wire has an off-center cylindrical hole as pictured below, but the current density in the remaining shaded region remains the same as in part a). The hole has diameter R and lies tangent to the circle. What is the magnitude of the magnetic field at point P_1, the center of the circle?



c) [10 pts] What is the magnitude of the field at point P_2, the center of the hole?

Name

2a) $I = JA$

$I = J\pi R^2$

$\frac{I}{\pi R^2} = J$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$I_{enc} = JA_{enc} = \frac{I}{\cancel{\pi} R^2} \cdot \cancel{\pi} r^2$

$B \cdot 2\pi r = \frac{\mu_0 I r^2}{R^2}$

$B = \frac{\mu_0 I r^{\cancel{2}}}{2\pi \cancel{r} R^2}$

$\boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$ ✓

b) $|\vec{B}_{tot}| = |\vec{B}_{big} - \vec{B}_{small}|$

$= |\cancel{B} \quad \vec{B}_{small}|$ Because $R = 0$

$\boxed{|B_{P_1}| = 0 - \frac{\mu_0 I}{\pi R} = \boxed{\frac{\mu_0 I}{2\pi R}}}$ −3

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$B 2\pi \quad \mu_0 I$

$B = \frac{\mu_0 I}{2\pi R}$

small

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$\oint B \cdot d\vec{l} = \mu_0 \frac{\mu_0 I}{\cancel{2}\cancel{\pi} R}$

$2\pi \frac{R}{2}$

$B \cdot \cancel{2}\pi \frac{R}{\cancel{2}} = \mu_0 \left(\frac{\mu_0 I (\frac{R}{2})}{2\pi R^2} \right) \left(\pi \left(\frac{R}{2}\right)^2 \right)$

$B \pi R = \frac{\mu_0^2 I R}{4\cancel{\pi}} \cdot \frac{\cancel{\pi}}{4}$

$B = \frac{\mu_0^2 I}{16\pi}$

$\frac{\mu_0 I (R)}{2\pi R^2} = \frac{\mu_0 I}{2\pi R}$

$\oint \vec{B} = \mu_0 \frac{I}{\cancel{\pi} R^2} (\cancel{\pi} (\frac{R}{2})^2$

$\frac{\mu_0 I}{2 \cdot 2\pi \frac{R}{2}}$

c) $|B_{tot}| = \left| \frac{\mu_0 I (\frac{R}{2})}{2\pi R^2} - B_{small} \right|$

$= \left| \frac{\mu_0 I}{4\pi R} - \frac{\mu_0 I}{2\pi R} \right| \neq 0.$

$= \left| -\frac{\mu_0 I}{4\pi R} \right|$ −6

$\boxed{|B_{P_2}| = \frac{\mu_0 I}{4\pi R}}$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot \frac{I}{\pi R^2} \cdot 2\pi \left(\frac{R}{2}\right)^2$

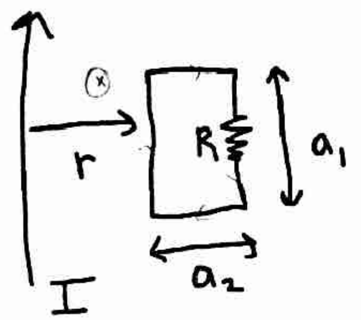
$B 2\pi R = \frac{\mu_0 I}{2}$

$B \cancel{2}\pi \frac{R}{\cancel{2}} = \frac{\mu_0 I}{2\pi R}$

$B = \mu_0 I$

Problem 3: [25 points] Name

A infinite straight wire carries current I. A rectangular loop is placed a distance r from the wire. In this problem, ignore any self-inductance effects (if you don't know what these are, don't worry, we haven't learnt this yet).



a) [10 pts] Suppose that $a_1 = a_2 = a$. What is the magnetic flux through the loop?

b) [10 pts] Suppose now that the current in the straight wire is time dependent, $I = I(t) = I_0 e^{-bt}$, where $b > 0$. If the loop has resistance R, what current will flow through the loop and in which direction?

c) [5 pts] In addition to the time-dependence of $I(t)$ above, suppose also that the loop's length changes in time according to $a_1(t) = af(t)$. What is the sign of $f'(t)$ (i.e. should the loop should grow or shrink) so that there is no induced current? Justify with a brief explanation or by finding $f'(t)$.

Name

1 a) $B = \frac{\mu_0 I}{2\pi r}$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\Phi_B = \int_r^{r+a} \frac{\mu_0 I}{2\pi r} \cdot a\, dr$

$= \frac{\mu_0 I a}{2\pi} \int_r^{r+a} \frac{1}{r}\, dr$

$$\boxed{\Phi_B = \frac{\mu_0 I a}{2\pi} \ln\left|\frac{r+a}{r}\right|}$$

b) $I(t) = I_0 e^{-bt}$

$\frac{dI}{dt} = I_0 - be^{-bt}$

$\varepsilon = IR$

$I = \frac{\varepsilon}{R}$

$\frac{d\Phi_B}{dt} = \frac{\mu_0 a}{2\pi} \ln\left|\frac{r+a}{r}\right| \cdot \frac{dI}{dt}$

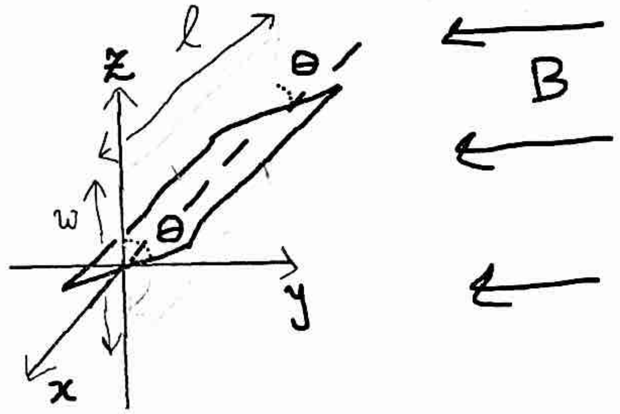
$\varepsilon = -\frac{d\Phi_B}{dt} = +\frac{\mu_0 a}{2\pi} \ln\left|\frac{r+a}{r}\right| I_0\, be^{bt}$

$$\boxed{I_{loop} = \frac{\mu_0 a I_0 b e^{-bt}}{2\pi R} \ln\left|\frac{r+a}{r}\right| \text{ in the clockwise direction}}$$

c) $f(t)$ should be positive. currently, our flux change is decreasing due to decreasing $I_{\text{straight wire}}$, so to counteract this, we need $f(t)$ to increase, which increases area, and therefore increases flux. Thus, $\frac{d\Phi_B}{dt} = 0$, and $\varepsilon = 0$.

Problem 4: [25 points] **Name**

The rectangular loop of wire with length l and width w pictured below is rotating about its center in a constant magnetic field $\vec{B} = -B\hat{y}$. The angular speed of rotation is fixed by hand to be $\omega\frac{rad}{s}$ and the axis of rotation is aligned with the x-axis as pictured. At $t = 0$, the loop is oriented at $\theta = 0$, in the $x - z$ plane. We will only consider half a revolution of the wire in this problem: $\theta = 0$ to $\theta = \pi$.



7 **a)** [**10 pts**] As a function of time t, what is the induced emf in the circuit?

5 **b)** [**5 pts**] Now suppose the wire has resistance R. What is the net force acting on the wire as a result of the external magnetic field as a function of t?

9 **c)** [**10 pts**] What is the net torque about the axis of rotation? To specify the direction, recall that $\vec{\tau} = \vec{r} \times \vec{F}$, where $\vec{r}$ points from the axis of rotation to the point at which $\vec{F}$ acts.

Name

4a) $\Phi_B = Bw\ell\cos\theta$

$\frac{d\Phi}{dt} = \frac{d\Phi}{d\theta} \cdot \frac{d\theta}{dt}$

$\frac{d\Phi_B}{d\theta} = -Bw\ell\sin\theta$

$\frac{d\Phi_B}{dt} = -B_w\ell\sin\theta \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \omega$

$\omega = \frac{rad}{Time}$

$\theta \cdot \frac{Time}{rad} = t$

$$\boxed{\varepsilon = -Bw\ell\cos(\omega t) \text{ in the counterclockwise direction}}$$

X

b) $\vec{F} = I\vec{\ell}\times\vec{B}$

$\varepsilon = IR$

$I = \frac{-Bw\ell\cos(\omega t)}{R}$

$\vec{F} = \frac{-Bw\ell\cos(\omega t)}{R}[2\ell B]$

$\boxed{\text{The net force is zero! (uniform mag. field)}}$

c) $\vec{\tau} = \vec{r}\times\vec{F}$

$\vec{F} = +\frac{Bw\ell\cos(\omega t)}{R}\ell B$

$\vec{\tau} = \frac{w}{2}\left[\frac{Bw\ell\cos(\omega t)}{R}\right]\ell B$ ✓ one side only, so ×2

$$\boxed{\vec{\tau} = \frac{B^2w^2\ell^2\cos(\omega t)}{R} \text{ ✓ in the clockwise direction ?}}$$