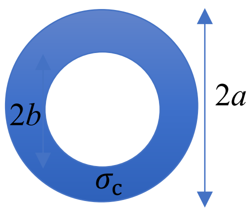
Physics 1BH – Winter 2021
Prof. J. Rosenzweig
Final Exam – March 19, 2021

Show all of your work for full credit. Write your name on each sheet of paper in your answers, then submit through Gradescope. You have 4 hours minutes to complete this exam including the submission to Gradescope.
You are should have prepared up to two sheets of paper as notes, with writing on both sides permitted. Please submit these "cheat sheets" to receive 10 points credit.

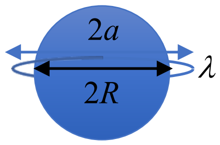
1. Consider a long (l>>a) resistor having conductivity σ_{c1} with a voltage V applied across it, as shown below, where you may in the usual manner approximate the current density along the resistor J_z as uniform and Ohm's law applies $J_z = \sigma_c E_z$ and σ_c is the conductivity

(a) (5 pts) What is the current density J_z? (5 pts) What is the resistance R?
(b) (10 pts) Approximating the current flow as infinite in length, what is the magnetic field inside of the cylinder, for r<a?
(c) Now consider a cylindrical resistor of the same length, but with a hole cut in it, up to the radius r=b and resistive material from b<r<a (conductivity σ_c), as shown.



(10 pts) What is the resistance of this new geometry? This can be calculated by directly looking at the total current flow, or by subtracting the "missing resistor" (10 pts) Again approximating the current density flow as infinite in length, what is the magnetic field for all r<a?

2. Consider a grounded spherical conductor of radius R that has a ring of charge placed symmetrically around its midplane ($z = 0$) at a radius a>R, having linear charge density λ, as shown below.



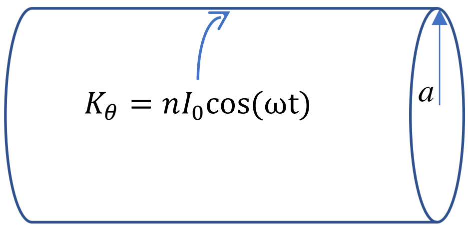
(a) We wish to solve for the potential in this case using image charges; one can take inspiration from the case of a point charge q outside of the conducting sphere at radius a, where the image charge of strength $q' = -q\left(\frac{R}{a}\right)$ is placed at $r' = R\left(\frac{R}{a}\right)$.
There should be in our case the possibility of placing an image charge *ring* at r' in order to obtain an equipotential at the sphere's surface. (10 pts) What is the total charge $Q'$and line charge density λ' in this ring?

(b) (10 pts) What is the electric potential ϕ outside of the sphere on the z-axis (the axis of symmetry of the charge ring and its image ring)? Check to make sure you obtain the correct potential on this axis at $z = \pm R$.

(c) (5 pts) What are the electric field values just outside the surface on the z-axis at $z = \pm R$? (5 pts) What is the charge density σ at these points?

(d) (10 pts) Looking at a distance z-axis at $z \gg R$, the potential $\phi \propto r^{-n}$, where n is the leading (lowest multipole) order of the dependence, find n.

3. An infinitely long cylindrical solenoid made up of a uniform surface current density has a total time-varying azimuthal current $I(t) = I_0 \cos(\omega t)$ located at the radius $r = a$.

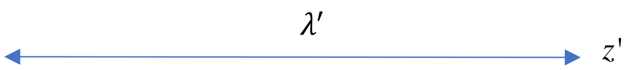


(a) (10 pts) If we simply consider that is the magnetic field inside of the solenoid is the same as that from a static current, but with sinusoidal variation $\cos(\omega t)$, what is the magnetic field $\vec{B}$ inside of the solenoid?

(b) (10 pts) Find a cylindrically symmetric magnetic vector potential $\vec{A}$ inside the solenoid.

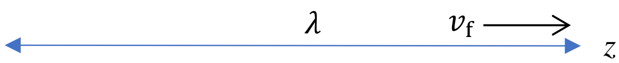
(c) (10 pts) What is the induced electric field inside of the solenoid? You may for **extra credit** (10 pts) do this two ways – using Faraday's law of induction, and/or directly through $\vec{A}$.

4. An infinite uniform line charge lying on the z'-axis is observed to be stationary in its rest frame, and has linear charge density λ' .



(a) (10 pts) What is the electrostatic potential associated with this static linear charge density?

(b) Now consider the charge to be in motion in the positive z direction with speed $v_f = 0.8c$. (10 pts) What are the observed line charge density and currents in this "lab" frame, considering the Lorentz transformation-induced length contraction? (10 pts) What is the longitudinal component of the vector potential A_z? You can answer this either from direct calculation of the vector potential from knowledge of the current, or to Lorentz transform the scalar potential using the $\left(\vec{A}, \frac{\phi}{c}\right)$ 4-vector.



(c) (15 pts) A particle of charge q is travelling parallel to this line charge at a radial distance of $r = b$, in the same direction and with the same speed v_f. What is the net force on this charge?

5. Consider a static magnetic field component, of the form B_y=$B_0 \cos(kz) \cosh(ky)$, existing in a region where the current density $\vec{J} = 0$. Note: $\frac{\mathrm{d}}{du}\cosh(u) = \sinh(u)$, and $\frac{\mathrm{d}}{du}\sinh(u) = \cosh(u)$.
   (a) Show that for this field component the *static* Laplacian $\vec{\nabla}^2 B_y$=0 is obeyed. (5 pts)
   (b) What is the form of B_z that must accompany this component B_y? (5 pts)
   (c) Find a vector potential $\vec{A}$ that can produce these magnetic field components. (10 pts)
   (d) Now consider a magnetic field of wave form $\vec{B}$=$B_0 \cos(kz - \omega t)\hat{y}$. If this field obeys the electromagnetic wave equation, what relationship exists between k and ω? (5 pts)
   (e) Find a vector potential $\vec{A}$ that can produce the wave field $\vec{B}$. (5 pts) Find the electric field component that is associated with this $\vec{A}$. (5 pts)