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MIDTERM #3
Physics 1BH
Prof. David Saltzberg
February 25, 2016

Time: 50 minutes. Closed Notes. Closed Book. Allowed the standard "cheat sheet". Calculators are allowed. Show your work.

If a problem is confusing or ambiguous, notify the professor. Clarifications will be written on the blackboard. Check the board.

Extra workspace is given and extra paper is at the front of the room.

Problem	Points
1	0 /25
2	0 /25
3	0 /25
4	0 /25
TOTAL	0 /100

1) Current runs through the length of a long, thick wire. The magnetic field inside the wire has a magnitude $B(r) = \frac{B_0}{r} exp\,(-\frac{r^2}{a^2})$ where a and B_0 are constants and r is the distance from the central axis of the wire.

Find the current per area as a function of r that produces this field by using one of Maxwell's equations <u>in differential form</u>.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}(r)$$

Let $\vec{J} = J_z \hat{z}$ that means $\vec{B} = |B| \hat{\theta}$

So using only term of curl in cylindrical coordinates that is non-zero:

Normally the mu0 is part of the B field, not 1/mu0 in front of J as we have here. But okay, just a constant.

$$\frac{1}{\mu_0 r}\left[\frac{\partial (r B_\theta)}{\partial r}\right] = J_z$$

$$\frac{1}{\mu_0 r}\frac{\partial}{\partial r}\left[\exp(-r^2/a^2)\right] = J_z$$

$$\frac{1}{\mu_0 r}\left[-\frac{2r}{a^2}\exp(-r^2/a^2)\right] = J_z$$

$$-\frac{2}{\mu_0 a^2}\exp(-r^2/a^2) = J_z$$

we did not grade the sign

$$\boxed{\vec{J} = -\frac{2}{\mu_0 a^2}\exp(-r^2/a^2)\,\hat{z}}$$

You may notice that the sign is backwards from the right hand rule. It turns out that because this B field diverges at r=0 the correct answer will be more complicated for J(r) as r--->0. Luckily that did not seem to confuse anyone.

2) Two streetlights, A and B, that are 400 meters apart on the ground both are turned on by the power company at exactly the same time according to clocks on the ground. An astronaut flies by in a rocket at 3/5 the speed of light relative to the ground along the direction between the lights, passing A first and then B. In the astronaut's frame, which light turns on first, and how many microseconds sooner? (Hint: We are asking about time in the frame of reference, and you should not include the time it takes for the light to reach the eyes of the astronaut.)

F = frame of lights

F' = frame of astronaut

$x_A \equiv 0 \qquad t_A \equiv 0$

$x_B = +400 \qquad t_B = 0$

$x'_A = \gamma(x_A - vt_A) = 0$

$x'_B = \gamma(x_B - vt_B) = \gamma(400)$

$t'_B = \gamma\left(t_B - \frac{x_B v}{c^2}\right) = \gamma\left(-\frac{(400)(3/5\,c)}{c^2}\right) = -\frac{400\gamma}{c}\left(\frac{3}{5}\right)$

$t'_A = \gamma\left(t_A - \frac{x_A v}{c^2}\right) = 0$

$\gamma = \frac{1}{\sqrt{1-(3/5)^2}} = \frac{1}{\sqrt{1-9/25}} = \frac{1}{\sqrt{\frac{16}{25}}} = \frac{5}{4}$

$t'_B < t'_A$ so B turns on first

$\Delta t = |t_B - t_A| = \frac{400}{c}\left(\frac{5}{4}\right)\left(\frac{3}{5}\right)$

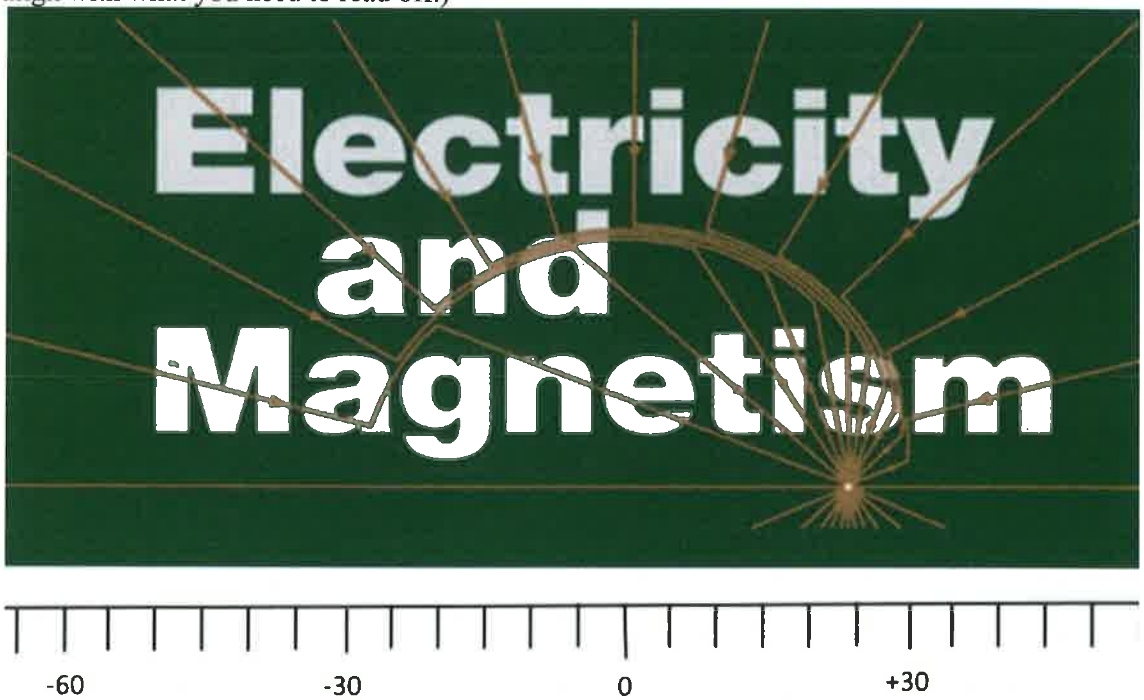
$= \frac{300}{3\times10^8\,m/s} = \frac{10^2}{10^8}\,m/s = 10^{-6}\,sec$

$= 1\,\mu sec$

$\boxed{\text{B turns on } 1\,\mu\text{sec sooner in } F'}$

* It really does! This is not an optical illusion or time delay to the astronaut *

3. The figure below is from the cover of your book. Distances in centimeters have been added with tick marks. You see an electron at time t=2.0 ns moving along the x-axis and the associated electric field lines at that instant. The outermost (radial) field lines continue in straight lines out to infinite distance. Except for sudden acceleration(s), the electron only moves at a constant speed. (Assume the tick marks are equally spaced and align with what you need to read off.)



Hint #1: to keep the numbers simple, note that c=30 centimeters/nanosecond.

a) What was the electron's initial speed?

Outer field lines are radial

$\Rightarrow$ initially at rest

(at origin)

Some of you answered this as what was its initial velocity once it started moveing. I did not take off for that.

Note, the electron did not start moving at t=0, It starts moving at t=1nsec.

b) At the moment shown, what is the magnitude of the electric field at the origin?

$$E = \frac{-e}{4\pi\epsilon_0 r^2} \frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}$$

At origin $\theta = 180° \Rightarrow \sin\theta = 0$

$$E = -\frac{e}{4\pi\epsilon_0 r^2}(1-\beta^2)$$

Need to know β of electron. Radius of transverse field is 30 cm $\Rightarrow$ started moving 1nsec earlier

Fields point to origin $\Rightarrow v = \frac{\Delta x}{\Delta t} = \frac{25\,cm}{1\,nsec} = \frac{5}{6}c$

$\Rightarrow \beta = 5/6$

$$E = -\frac{(1.6\times10^{-19})}{4\pi(8.85\times10^{-12})} \frac{1}{(0.25)^2}\left(1-(5/6)^2\right)$$

$E = 7.04\times10^{-9}\ V/m$

Another way to do this is to tranform E|| to E||'. That is the same. But you than also have to transform r to r' which gives a factor gamma^2. That is the same as the factor (1-beta^2) above.

4) Multiple choice, with explanation

The magnetic field in a certain region of space is given by $\mathbf{B}=\mu_0 nI\hat{z}$. (We subsequently learned that this is the B field inside a long solenoid with n turns per meter and current I.) Which one of the following could be the vector potential in that region?

A. $(A_r, A_\theta, A_z)=(\frac{\mu_0 nIr}{2}, 0, 0)$

B. $(A_r, A_\theta, A_z)=(0, \frac{\mu_0 nIr}{2}, 0)$

C. $(A_r, A_\theta, A_z)=(0, 0, \frac{\mu_0 nIr}{2})$

D. $(A_r, A_\theta, A_z)=(\frac{\mu_0 nI}{r}, 0, 0)$

E. $(A_r, A_\theta, A_z)=(0, \frac{\mu_0 nI}{r}, 0)$

F. $(A_r, A_\theta, A_z)=(0, 0, \frac{\mu_0 nI}{r})$

Hint: if you are making a long calculation for each one, you are not being efficient.

Explain:

$\vec{B} = \vec{\nabla} \times \vec{A}$ By units A must be Tesla × meters

$\Rightarrow$ $\cancel{D}$ $\cancel{E}$ $\cancel{F}$ out!

$\vec{A}$ is parallel to $\vec{J}$. $\vec{J} = J_0 \hat{\theta}$ so answer is likely B.

Try it $\nabla \times \left(0, \frac{\mu_0 nIr}{2}, 0\right)$ in cylindrical coordinates:

$$= 0\hat{r} + 0\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{r^2 \mu_0 nI}{2}\right)\right]\hat{z}$$

$$= \frac{1}{r}\left[\frac{2r\mu_0 nI}{2}\right]\hat{z}$$

$$= \mu_0 nI\hat{z} = \vec{B}\ !$$

✓✓✓

Yay.